Model Predictive Control Toolbox™

User's Guide

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Model Predictive Control Toolbox™ User's Guide

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Introduction

Specifying Scale Factors

Overview

Recommended practice includes specification of scale factors for each plant input and output variable, which is especially important when certain variables have much larger or smaller magnitudes than others.

The scale factor should equal (or approximate) the span of the variable. *Span* is the difference between its maximum and minimum value in engineering units, that is, the unit of measure specified in the plant model. Internally, MPC divides each plant input and output signal by its scale factor to generate dimensionless signals.

The potential benefits of scaling are as follows:

- Default MPC tuning weights work best when all signals are of order unity. Appropriate scale factors make the default weights a good starting point for controller tuning and refinement.
- When choosing cost function weights, you can focus on the relative priority of each term rather than a combination of priority and signal scale.
- Improved numerical conditioning. When values are scaled, round-off errors have less impact on calculations.

Once you have tuned the controller, changing a scale factor is likely to affect performance and the controller may need retuning. Best practice is to establish scale factors at the beginning of controller design and hold them constant thereafter.

Defining Scale Factors

To identify scale factors, estimate the span of each plant input and output variable in engineering units.

- If the signal has known bounds, use the difference between the upper and lower limit.

- If you do not know the signal bounds, consider running open-loop plant model simulations. You can vary the inputs over their likely ranges, and record output signal spans.
- If you have no idea, use the default scale factor (=1).

You can define scale factors at the command line and using the MPC Designer app.

Once you have set the scale factors and have begun to tune the controller performance, hold the scale factors constant.

Using Commands

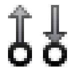
After you create the MPC controller object using the `mpc` command, set the scale factor property for each plant input and output variable.

For example, the following commands create a random plant, specify the signal types, and define a scale factor for each signal.

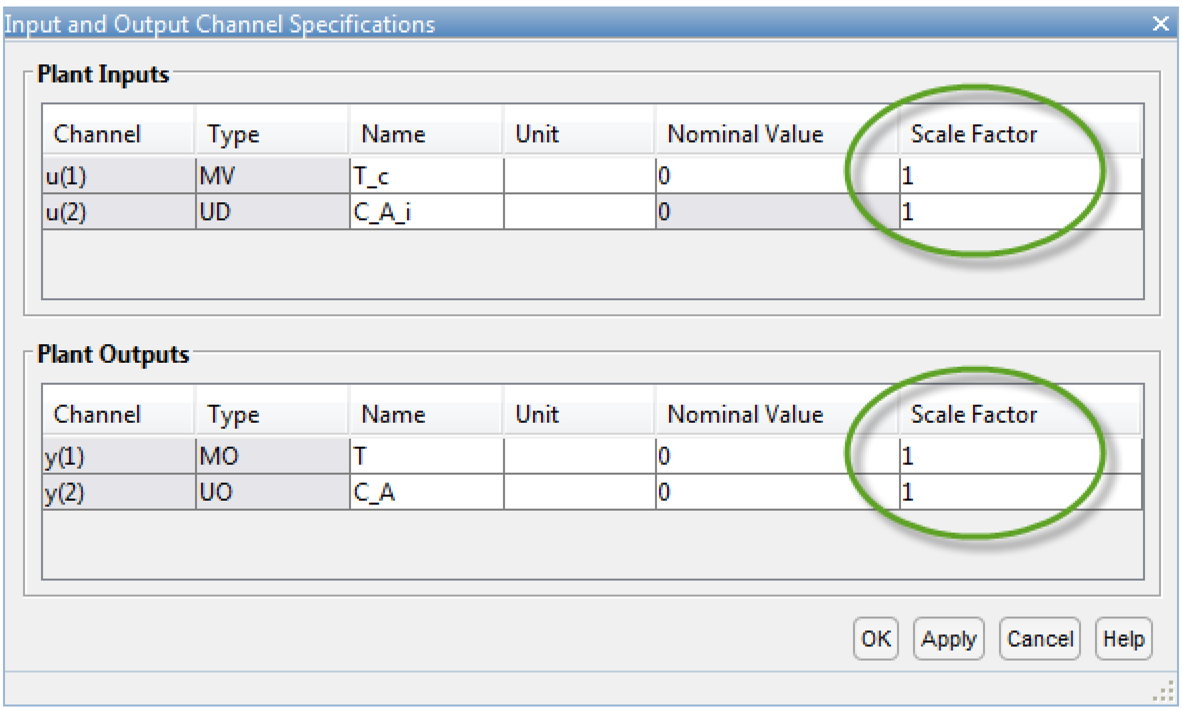
```
% Random plant for illustrative purposes: 5 inputs, 3 outputs
Plant = drss(4,3,5);
Plant.InputName = {'MV1','UD1','MV2','UD2','MD'};
Plant.OutputName = {'UO','MO1','MO2'};

% Example signal spans
Uspan = [2, 20, 0.1, 5, 2000];
Yspan = [0.01, 400, 75];

% Example signal type specifications
iMV = [1 3];
iMD = 5;
iUD = [2 4];
iDV = [iMD,iUD];
Plant = setmpcsignals(Plant,'MV',iMV,'MD',iMD,'UD',iUD, ...
    'MO',[2 3],'UO',1);
Plant.d(:,iMV) = 0;   % MPC requires zero direct MV feed-through

% Controller object creation.  Ts = 0.3 for illustration.
MPCobj = mpc(Plant, 0.3);

% Override default scale factors using specified spans
for i = 1:2
    MPCobj.MV(i).ScaleFactor = Uspan(iMV(i));
end
```

```
% NOTE:  DV sequence is MD followed by UD
for i = 1:3
    MPCobj.DV(i).ScaleFactor = Uspan(iDV(i));
end
for i = 1:3
    MPCobj.OV(i).ScaleFactor = Yspan(i);
end
```

Using MPC Designer App

After opening MPC Designer and defining the initial MPC structure, in the MPC Designer tab, click **I/O Attributes** .

In the Input and Output Channel Specifications dialog box, specify a **Scale Factor** for each input and output signal.



Click **OK** to update the controller settings.

See Also

`mpc` | MPC Designer

Related Examples

- Using Scale Factor to Facilitate Weight Tuning

More About

- "Choosing Sample Time and Horizons" on page 1-6

Choosing Sample Time and Horizons

In this section...

Sample Time

Duration

Recommended practice is to choose the control interval duration (controller property T_s) initially, and then hold it constant as you tune other controller parameters. If it becomes obvious that the original choice was poor, you can revise T_s. If you do so, you might then need to retune other settings.

Qualitatively, as T_s decreases, rejection of unknown disturbance usually improves and then plateaus. The T_s value at which performance plateaus depends on the plant dynamic characteristics.

However, as T_s becomes small, the computational effort increases dramatically. Thus, the optimal choice is a balance of performance and computational effort.

In Model Predictive Control, the prediction horizon, p is also an important consideration. If one chooses to hold the prediction horizon duration (the product $p*T_s$) constant, p must vary inversely with T_s. Many array sizes are proportional to p. Thus, as p increases, the controller memory requirements ans QP solution time increase.

Consider the following when choosing T_s:

- As a rough guideline, set T_s between 10% and 25% of your minimum desired closed-loop response time.
- Run at least one simulation to see whether unmeasured disturbance rejection improves significantly when T_s is halved. If so, consider revising T_s.
- For process control, T_s >> 1 s is common, especially when MPC supervises lower-level single-loop controllers. Other applications, such as automotive or aerospace), can

require $T_s < 1$ s. If the time needed for solving the QP in real time exceeds the desired control interval, consider the "Explicit MPC" on page 6-2 option.

- For plants with delays, the number of state variables needed for modeling delays is inversely proportional to T_s.
- For open-loop unstable plants, if p^*T_s is too large, such that the plant step responses become infinite during this amount of time, key parameters needed for MPC calculations become undefined, generating an error message.

Units

The controller inherits its time unit from the plant model. Specifically, the controller uses the `TimeUnit` property of the plant model LTI object. This property defaults to seconds.

Prediction Horizon

Suppose that the current control interval is k. The *prediction horizon*, p, is the number of future control intervals the MPC controller must evaluate by prediction when optimizing its MVs at control interval k.

Tips

- Recommended practice is to choose p early in the controller design and then hold it constant while tuning other controller settings, such as the cost function weights. In other words, do not use p adjustments for controller tuning. Rather, the value of p should be such that the controller is internally stable and anticipates constraint violations early enough to allow corrective action.
- If the desired closed-loop response time is T and the control interval is T_s, try p such that $T \approx pT_s$.
- Plant delays impose a lower bound on the possible closed-loop response times. Choose p accordingly. To check for a violation of this condition, ue the `review` command.
- Recommended practice is to increase p until further increases have a minor impact on performance. If the plant is open-loop unstable, the maximum p is the number of control intervals required for the open-loop step response of the plant to become infinite. $p > 50$ is rarely necessary unless T_s is too small.
- Unfavorable plant characteristics combined with a small p can generate an internally unstable controller. To check for this condition, use the `review` command, and increase p if possible. If p is already large, consider the following:
  - Increase T_s.

- Increase the cost function weights on MV increments.
- Modify the control horizon or use MV blocking (see “Manipulated Variable Blocking” on page 2-35).
- Use a small *p* with terminal weighting to approximate LQR behavior (See “Terminal Weights and Constraints” on page 2-30).

Control Horizon

The control horizon, *m*, is the number of MV moves to be optimized at control interval *k*. The control horizon falls between 1 and the prediction horizon *p*. The default is $m = 2$. Regardless of your choice for *m*, when the controller operates, the optimized MV move at the beginning of the horizon is used and any others are discarded.

Tips

Reasons to keep $m << p$ are as follows:

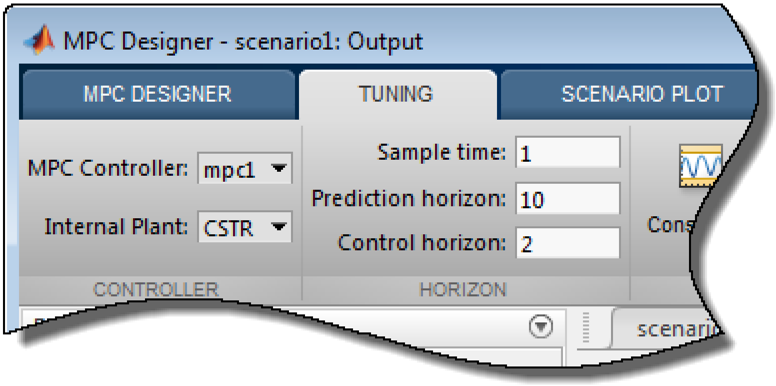
- Small *m* means fewer variables to compute in the QP solved at each control interval, which promotes faster computations.
- If the plant includes delays, $m < p$ is essential. Otherwise, some MV moves might not affect any of the plant outputs before the end of the prediction horizon, leading to a singular QP Hessian matrix. To check for a violation of this condition, use the `review` command.
- Small *m* promotes (but does not guarantee) an internally stable controller.

Defining Sample Time and Horizons

You can define the sample time, prediction horizon, and control horizon when creating an `mpc` controller at the command line. After creating a controller, `mpcObj`, you can modify the sample time and horizons by setting the following controller properties:

- Sample time — `mpcObj.Ts`
- Prediction horizon — `mpcObj.p`
- Control horizon — `mpcObj.m`

Also, when designing an MPC controller using the MPC Designer app, in the **Tuning** tab, in the **Horizon** section, you can modify the sample time and horizons.



See Also

`mpc` | MPC Designer

More About

- "Specifying Constraints" on page 1-10

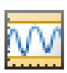
Specifying Constraints

In this section...

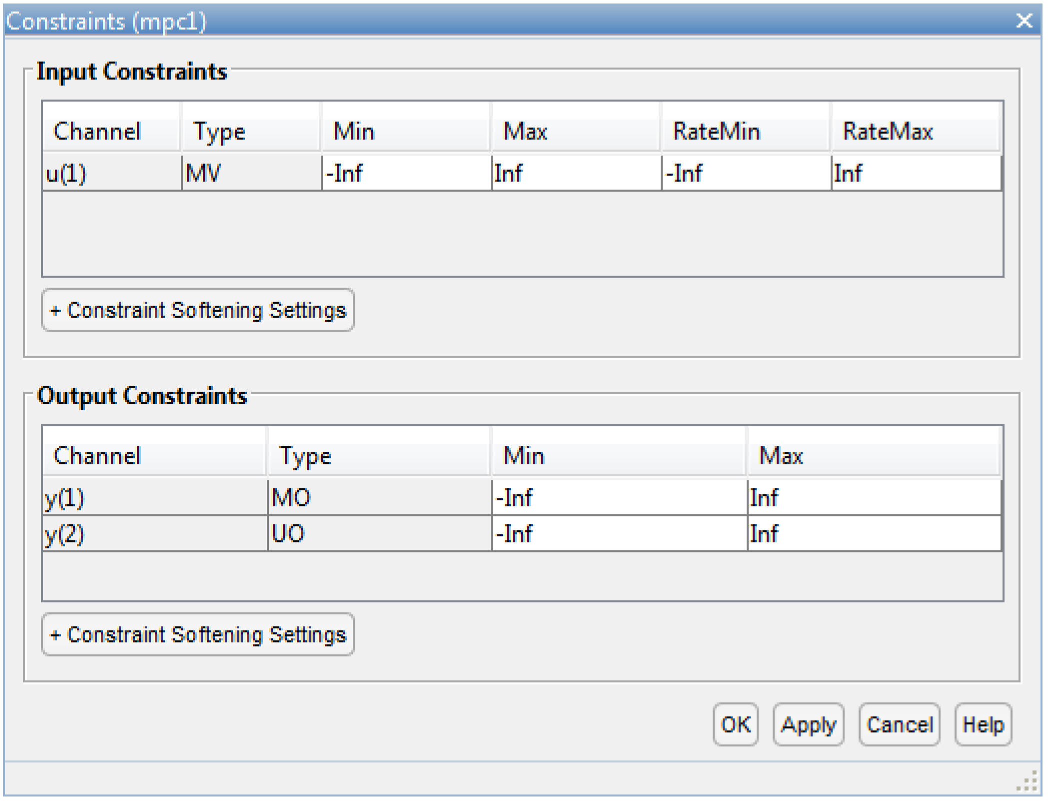
Input and Output Constraints

By default, when you create a controller object using the `mpc` command, no constraints exist. To include a constraint, set the appropriate controller property. The following table summarizes the controller properties used to define most MPC Toolbox constraints. (MV = plant manipulated variable; OV = plant output variable; MV increment = $u(k) - u(k - 1)$).

To include this constraint	Set this controller property	Soften constraint by setting
Lower bound on *i*th MV	`MV(i).Min > -Inf`	`MV(i).MinECR > 0`
Upper bound on *i*th MV	`MV(i).Max < Inf`	`MV(i).MaxECR > 0`
Lower bound on *i*th OV	`OV(i).Min > -Inf`	`OV(i).MinECR > 0`
Upper bound on *i*th OV	`OV(i).Max < Inf`	`OV(i).MaxECR > 0`
Lower bound on *i*th MV increment	`MV(i).RateMin > -Inf`	`MV(i).RateMinECR > 0`
Upper bound on *i*th MV increment	`MV(i).RateMax < Inf`	`MV(i).RateMaxECR > 0`

To set the controller constraint properties using the MPC Designer app, in the **Tuning** tab, click **Constraints** . In the Constraints dialog box, specify the constraint values.

See “Constraints” on page 2-7 for the equations describing the corresponding constraints.



Tips

For MV bounds:

- Include known physical limits on the plant MVs as hard MV bounds.
- Include MV increment bounds when there is a known physical limit on the rate of change, or your application requires you to prevent large increments for some other reason.
- Do not include both hard MV bounds and hard MV increment bounds on the same MV, as they can conflict. If both types of bounds are important, soften one.

For OV bounds:

- Do not include OV bounds unless they are essential to your application. As an alternative to setting an OV bound, you can define an OV reference and set its cost function weight to keep the OV close to its setpoint.
- All OV constraints should be softened.
- Consider leaving the OV unconstrained for some prediction horizon steps. See "Time-Varying Weights and Constraints" on page 2-26.
- Consider a time-varying OV constraint that is easy to satisfy early in the horizon, gradually tapering to a more strict constraint. See "Time-Varying Weights and Constraints" on page 2-26.
- Do not include OV constraints that are impossible to satisfy. Even if soft, such constraints can cause unexpected controller behavior. For example, consider a SISO plant with five sampling periods of delay. An OV constraint before the sixth prediction horizon step is, in general, impossible to satisfy. You can use the `review` command to check for such impossible constraints, and use a time-varying OV bound instead. See "Time-Varying Weights and Constraints" on page 2-26.

Constraint Softening

Hard constraints are constraints that the quadratic programming (QP) solution must satisfy. If it is mathematically impossible to satisfy a hard constraint at a given control interval, k, the QP is *infeasible*. In this case, the controller returns an error status, and sets the manipulated variables (MVs) to $u(k) = u(k–1)$, i.e., no change. If the condition leading to infeasibility is not resolved, infeasibility can continue indefinitely, leading to a loss of control.

Disturbances and prediction errors are inevitable in practice. Therefore, a constraint violation could occur in the plant even though the controller predicts otherwise. A feasible QP solution does not guarantee that all hard constraints will be satisfied when the optimal MV is used in the plant.

If the only constraints in your application are bounds on MVs, the MV bounds can be hard constraints, as they are by default. MV bounds alone cannot cause infeasibility. The same is true when the only constraints are on MV increments.

However, a hard MV bound with a hard MV increment constraint can lead to infeasibility. For example, an upset or operation under manual control could cause the actual MV used in the plant to exceed the specified bound during interval $k–1$. If the controller is in automatic during interval k, it must return the MV to a value within the hard bound. If the MV exceeds the bound by too much, the hard increment constraint can make correcting the bound violation in the next interval impossible.

When there are hard constraints on plant outputs, or hard custom constraints (on linear combinations of plant inputs and outputs, and the plant is subject to disturbances, QP infeasibility is a distinct possibility.

All Model Predictive Control Toolbox™ constraints (except slack variable nonnegativity) can be *soft*. When a constraint is soft, the controller can deem an MV optimal even though it predicts a violation of that constraint. If all plant output, MV increment, and custom constraints are soft (as they are by default), QP infeasibility does not occur. However, controller performance can be substandard.

To soften a constraint, set the corresponding ECR value to a positive value (zero implies a hard constraint). The larger the ECR value, the more likely the controller will deem it optimal to violate the constraint in order to satisfy your other performance goals. The Model Predictive Control Toolbox software provides default ECR values but, as for the cost function weights, you might need to tune the ECR values in order to achieve acceptable performance.

To understand how constraint softening works, suppose that your cost function uses $w_{i,j}^{u} = w_{i,j}^{\Delta u} = 0$, giving both the MV and MV increments zero weight in the cost function. Only the output reference tracking and constraint violation terms are nonzero. In this case, the cost function is:

$$J(z_k) = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \left\{ \frac{w_{i,j}^{y}}{s_j^{y}} \left[r_j(k+i|k) - y_j(k+i|k) \right] \right\}^2 + \rho \quad {}_{k}^{2}.$$

Suppose that you have also specified hard MV bounds with $V_{j,min}^{u}(i) = 0$ and $V_{j,max}^{u}(i) = 0$. Then these constraints simplify to:

$$\frac{u_{j,min}(i)}{s_j^u} \le \frac{u_j(k+i-1|k)}{s_j^u} \le \frac{u_{j,max}(i)}{s_j^u}, \; i = 1:p, \quad j = 1:n_u.$$

Thus, the slack variable, ϵ_k, no longer appears in the above equations. You have also specified soft constraints on plant outputs with $V_{j,min}^{y}(i) > 0$ and $V_{j,max}^{y}(i) > 0$.

$$\frac{y_{j,min}(i)}{s_j^y} - {}_k V_{j,min}^y(i) \le \frac{y_j(k+i|k)}{s_j^y} \le \frac{y_{j,max}(i)}{s_j^y} + {}_k V_{j,max}^y(i),\ i = 1:p, \quad j = 1:n_y.$$

Now, suppose that a disturbance has pushed a plant output above its specified upper bound, but the QP with hard output constraints would be feasible, that is, all constraint violations could be avoided in the QP solution. The QP involves a trade-off between output reference tracking and constraint violation. The slack variable, $\in_k$, must be nonnegative. Its appearance in the cost function discourages, but does not prevent, an optimal $\in_k > 0$. A larger $\rho_{\in}$ weight, however, increases the likelihood that the optimal $\in_k$ will be small or zero.

If the optimal $\in_k > 0$, at least one of the bound inequalities must be active (at equality). A relatively large $V_{j,max}^y(i)$ makes it easier to satisfy the constraint with a small $\in_k$. In that case,

$$\frac{y_j(k+i|k)}{s_j^y}$$

can be larger, without exceeding

$$\frac{y_{j,max}(i)}{s_j^y} + {}_k V_{j,max}^y(i).$$

Notice that $V_{j,max}^y(i)$ does not set an upper limit on the constraint violation. Rather, it is a tuning factor determining whether a soft constraint is easy or difficult to satisfy.

Tips

- Use of dimensionless variables simplifies constraint tuning. Define appropriate scale factors for each plant input and output variable. See “Specifying Scale Factors” on page 1-2.
- To indicate the relative magnitude of a tolerable violation, use the ECR parameter associated with each constraint. Rough guidelines are as follows:
  - 0 — No violation allowed (hard constraint)

  - 0.05 — Very small violation allowed (nearly hard)
  - 0.2 — Small violation allowed (quite hard)
  - 1 — average softness
  - 5 — greater-than-average violation allowed (quite soft)
  - 20 — large violation allowed (very soft)
- Use the overall constraint softening parameter of the controller (controller object property: `Weights.ECR`) to penalize a tolerable soft constraint violation relative to the other cost function terms. Set the `Weights.ECR` property such that the corresponding penalty is 1–2 orders of magnitude greater than the typical sum of the other three cost function terms. If constraint violations seem too large during simulation tests, try increasing `Weights.ECR` by a factor of 2–5.

  Be aware, however, that an excessively large `Weights.ECR` distorts MV optimization, leading to inappropriate MV adjustments when constraint violations occur. To check for this, display the cost function value during simulations. If its magnitude increases by more than 2 orders of magnitude when a constraint violation occurs, consider decreasing `Weights.ECR`.
- Disturbances and prediction errors can lead to unexpected constraint violations in a real system. Attempting to prevent these violations by making constraints harder often degrades controller performance.

See Also

`review`

More About

- "Time-Varying Weights and Constraints" on page 2-26
- "Terminal Weights and Constraints" on page 2-30
- "Optimization Problem" on page 2-2

Tuning Weights

In this section...

A model predictive controller design usually requires some tuning of the cost function weights. This topic provides tuning tips. See "Optimization Problem" on page 2-2 for details on the cost function equations.

Initial Tuning

- Before tuning the cost function weights, specify scale factors for each plant input and output variable. Hold these scale factors constant as you tune the controller. See "Specifying Scale Factors" on page 1-2 for more information.
- During tuning, use the `sensitivity` and `review` commands to obtain diagnostic feedback. The `sensitivity` command is intended to help with cost function weight selection.
- Change a weight by setting the appropriate controller property, as follows:

To change this weight	Set this controller property	Array size
OV reference tracking (w^y)	`Weights.OV`	p-by-n_y
MV reference tracking (w^u)	`Weights.MV`	p-by-n_u
MV increment suppression ($w^{\Delta u}$)	`Weights.MVRate`	p-by-n_u

Here, MV is a plant manipulated variable, and n_u is the number of MVs. OV is a plant output variable, and n_y is the number of OVs. Finally,p is the number of steps in the prediction horizon.

If a weight array contains $n < p$ rows, the controller duplicates the last row to obtain a full array of p rows. The default ($n = 1$) minimizes the number of parameters to be tuned, and is therefore recommended. See "Time-Varying Weights and Constraints" on page 2-26 for an alternative.

Tips for Setting OV Weights

- Considering the n_y OVs, suppose that n_{yc} must be held at or near a reference value (setpoint). If the ith OV is not in this group, set `Weights.OV(:,i)` = 0.
- If $n_u \geq n_{yc}$, it is usually possible to achieve zero OV tracking error at steady state, if at least n_{yc} MVs are not constrained. The default `Weights.OV = ones(1,ny)` is a good starting point in this case.

  If $n_u > n_{yc}$, however, you have excess degrees of freedom. Unless you take preventive measures, therefore, the MVs may drift even when the OVs are near their reference values.

  - The most common preventive measure is to define reference values (targets) for the number of excess MVs you have, $n_u - n_{yc}$. Such targets can represent economically or technically desirable steady-state values.
  - An alternative measure is to set $w_{\Delta u} > 0$ for at least $n_u - n_{yc}$ MVs to discourage the controller from changing them.
- If $n_u < n_{yc}$, you do not have enough degrees of freedom to keep all required OVs at a setpoint. In this case, consider prioritizing reference tracking. To do so, set `Weights.OV(:,i) > 0` to specify the priority for the ith OV. Rough guidelines for this are as follows:

  - 0.05 — Low priority: Large tracking error acceptable
  - 0.2 — Below-average priority
  - 1 — Average priority – the default. Use this value if $n_{yc} = 1$.
  - 5 — Above average priority
  - 20 — High priority: Small tracking error desired

Tips for Setting MV Weights

By default, `Weights.MV = zeros(1,nu)`. If some MVs have targets, the corresponding MV reference tracking weights must be nonzero. Otherwise, the targets are ignored. If the number of MV targets is less than ($n_u - n_{yc}$), try using the same weight for each. A suggested value is 0.2, the same as below-average OV tracking. This value allows the MVs to move away from their targets temporarily to improve OV tracking.

Otherwise, the MV and OV reference tracking goals are likely to conflict. Prioritize by setting the `Weights.MV(:,i)` values in a manner similar to that suggested for

Weights.OV (see above). Typical practice sets the average MV tracking priority lower than the average OV tracking priority (e.g., 0.2 < 1).

If the *i*th MV does not have a target, set Weights.MV(:,i) = 0 (the default).

Tips for Setting MVRate Weights

- By default, Weights.MVRate = 0.1*ones(1,nu). The reasons for this default include:
  - If the plant is open-loop stable, large increments are unnecessary and probably undesirable. For example, when model predictions are imperfect, as is always the case in practice, more conservative increments usually provide more robust controller performance, but poorer reference tracking.
  - These values force the QP Hessian matrix to be positive-definite, such that the QP has a unique solution if no constraints are active.

  To encourage the controller to use even smaller increments for the *i*th MV, increase the Weights.MVRate(:,i) value.
- If the plant is open-loop unstable, you might need to decrease the average Weight.MVRate value to allow sufficiently rapid response to upsets.

Tips for Setting ECR Weights

See "Constraint Softening" on page 1-12 for tips regarding the Weights.ECR property.

Testing and Refinement

To focus on tuning individual cost function weights, perform closed-loop simulation tests under the following conditions:

- No constraints.
- No prediction error. The controller prediction model should be identical to the plant model. Both the MPC Designer app and the sim function provide the option to simulate under these conditions.

Use changes in the reference and measured disturbance signals (if any) to force a dynamic response. Based on the results of each test, consider changing the magnitudes of selected weights.

One suggested approach is to use constant Weights.OV(:,i) = 1 to signify "average OV tracking priority," and adjust all other weights to be relative to this value. Use the `sensitivity` command for guidance. Use the `review` command to check for typical tuning issues, such as lack of closed-loop stability.

See “Adjusting Disturbance and Noise Models” on page 2-15 for tests focusing on the disturbance rejection ability of the controller.

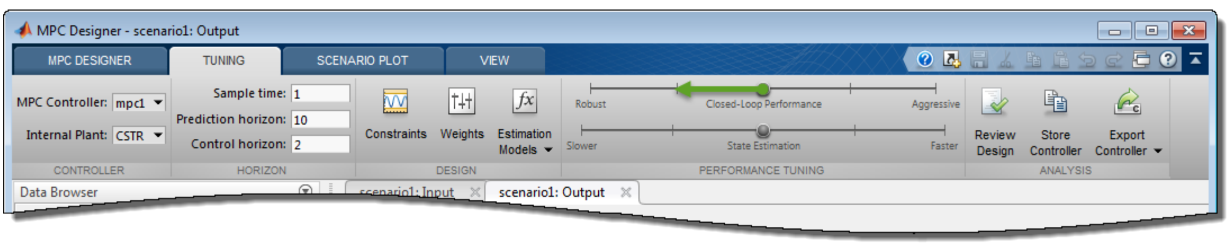
Robustness

Once you have weights that work well under the above conditions, check for sensitivity to prediction error. There are several ways to do so:

- If you have a nonlinear plant model of your system, such as a Simulink® model, simulate the closed-loop performance at operating points other than that for which the LTI prediction model applies.
- Alternatively, run closed-loop simulations in which the LTI model representing the plant differs (such as in structure or parameter values) from that used at the MPC prediction model. Both the MPC Designer app and the `sim` function provide the option to simulate under these conditions. See “Test Controller Robustness” for an example.

If controller performance seems to degrade significantly in comparison to tests with no prediction error, for an open-loop stable plant, consider making the controller less aggressive.

In the MPC Designer app, on the **Tuning** tab, you can do so using the **Closed-Loop Performance** slider.



Moving towards more robust control decreases OV/MV weights and increases MV Rate weights, which leads to relaxed control of outputs and more conservative control moves.

At the command line, you can make the following changes to decrease controller aggressiveness:

- Increase all `Weight.MVRate` values by a multiplicative factor of order 2.
- Decrease all `Weight.OV` and `Weight.MV` values by dividing by the same factor.

After adjusting the weights, reevaluate performance both with and without prediction error.

- If both are now acceptable, stop tuning the weights.
- If there is improvement but still too much degradation with model error, increase the controller robustness further.
- If the change does not noticeably improve performance, restore the original weights and focus on state estimator tuning (see “Adjusting Disturbance and Noise Models” on page 2-15).

Finally, if tuning changes do not provide adequate robustness, consider one of the following options:

- “Adaptive MPC” on page 5-2
- “Gain-Scheduled MPC” on page 7-2

Related Examples

- Tuning Controller Weights
- “Setting Targets for Manipulated Variables” on page 4-105

More About

- “Optimization Problem” on page 2-2
- “Specifying Constraints” on page 1-10
- “Adjusting Disturbance and Noise Models” on page 2-15

2

Model Predictive Control Problem Setup

Optimization Problem

In this section...

Overview

Model Predictive Control solves an optimization problem – specifically, a quadratic program (QP) – at each control interval. The solution determines the manipulated variables (MVs) to be used in the plant until the next control interval.

This QP problem includes the following features:

- The objective, or "cost", function — A scalar, nonnegative measure of controller performance to be minimized.
- Constraints — Conditions the solution must satisfy, such as physical bounds on MVs and plant output variables.
- Decision — The MV adjustments that minimizes the cost function while satisfying the constraints.

The following sections describe these features in more detail.

Standard Cost Function

The standard cost function is the sum of four terms, each focusing on a particular aspect of controller performance, as follows:

$$J\left(z_k\right) = J_y\left(z_k\right) + J_u\left(z_k\right) + J_{\Delta u}\left(z_k\right) + J\quad\left(z_k\right).$$

Here, z_k is the QP decision. As described below, each term includes weights that help you balance competing objectives. MPC controller provides default weights but you will usually need to adjust them to tune the controller for your application.

Output Reference Tracking

In most applications, the controller must keep selected plant outputs at or near specified reference values. MPC controller uses the following scalar performance measure:

$$J_y\left(z_k\right)=\sum_{j=1}^{n_y}\sum_{i=1}^{p}\left\{\frac{w_{i,j}^y}{s_j^y}\left[r_j\left(k+i|k\right)-y_j\left(k+i|k\right)\right]\right\}^2.$$

Here,

- k — Current control interval.
- p — Prediction horizon (number of intervals).
- n_y — Number of plant output variables.
- z_k — QP decision, given by:

$$z_k^T=\left[u(k\,|\,k)^T \quad u(k+1\,|\,k)^T \quad \cdots \quad u(k+p-1\,|\,k)^T \quad {}_k\right].$$

- $y_j(k+i\,|\,k)$ — Predicted value of jth plant output at ith prediction horizon step, in engineering units.
- $r_j(k+i\,|\,k)$ — Reference value for jth plant output at ith prediction horizon step, in engineering units.
- s_j^y — Scale factor for jth plant output, in engineering units.
- $w_{i,j}^y$ — Tuning weight for jth plant output at ith prediction horizon step (dimensionless).

The values n_y, p, s_j^y, and $w_{i,j}^y$ are controller specifications, and are constant. The controller receives $r_j(k+i\,|\,k)$ values for the entire prediction horizon. The controller uses the state observer to predict the plant outputs. At interval k, the controller state estimates and MD values are available. Thus, J_y is a function of z_k only.

Manipulated Variable Tracking

In some applications, i.e. when there are more manipulated variables than plant outputs, the controller must keep selected manipulated variables (MVs) at or near specified target values. MPC controller uses the following scalar performance measure:

$$J_u(z_k) = \sum_{j=1}^{n_u} \sum_{i=0}^{p-1} \left\{ \frac{w_{i,j}^u}{s_j^u} \left[u_j(k+i|k) - u_{j,target}(k+i|k) \right] \right\}^2 .$$

Here,

- k — Current control interval.
- p — Prediction horizon (number of intervals).
- n_u — Number of manipulated variables.
- z_k — QP decision, given by:

$$z_k^T = \left[u(k|k)^T \quad u(k+1|k)^T \quad \cdots \quad u(k+p-1|k)^T \quad k \right].$$

- $u_{j,target}(k+i\,|\,k)$ — Target value for jth MV at ith prediction horizon step, in engineering units.
- s_j^u — Scale factor for jth MV, in engineering units.
- $w_{i,j}^u$ — Tuning weight for jth MV at ith prediction horizon step (dimensionless).

The values n_u, p, s_j^u, and $w_{i,j}^u$ are controller specifications, and are constant. The controller receives $u_{j,target}(k+i\,|\,k)$ values for the entire horizon. The controller uses the state observer to predict the plant outputs. Thus, J_u is a function of z_k only.

Manipulated Variable Move Suppression

Most applications prefer small MV adjustments (*moves*). MPC uses the following scalar performance measure:

$$J_{\Delta u}(z_k) = \sum_{j=1}^{n_u} \sum_{i=0}^{p-1} \left\{ \frac{w_{i,j}^{\Delta u}}{s_j^u} \left[u_j(k+i|k) - u_j(k+i-1|k) \right] \right\}^2 .$$

Here,

Here,

- k — Current control interval.
- p — Prediction horizon (number of intervals).
- n_u — Number of manipulated variables.
- z_k — QP decision, given by:

$$z_k^T = \left[u(k \mid k)^T \quad u(k+1 \mid k)^T \quad \cdots \quad u(k+p-1 \mid k)^T \qquad {}_k \right].$$

- s_j^u — Scale factor for jth MV, in engineering units.
- $w_{i,j}^{\Delta u}$ — Tuning weight for jth MV movement at ith prediction horizon step (dimensionless).

The values n_u, p, s_j^u, and $w_{i,j}^{\Delta u}$ are controller specifications, and are constant. $u(k–1 \mid k) = u(k–1)$, which are the known MVs from the previous control interval. $J_{\Delta u}$ is a function of z_k only.

In addition, a control horizon $m < p$ (or MV blocking) constrains certain MV moves to be zero.

Constraint Violation

In practice, constraint violations might be unavoidable. Soft constraints allow a feasible QP solution under such conditions. MPC controller employs a dimensionless, nonnegative slack variable, ε_k, which quantifies the worst-case constraint violation. (See “Constraints” on page 2-7) The corresponding performance measure is:

$$J \ \left(z_k \right) = \rho \quad {}_k^2.$$

Here,

- z_k — QP decision, given by:

$$z_k^T = \begin{bmatrix} u(k \mid k)^T & u(k+1 \mid k)^T & \cdots & u(k+p-1 \mid k)^T & {}_k \end{bmatrix}.$$

- ε_k — Slack variable at control interval k (dimensionless).
- ρ_ϵ — Constraint violation penalty weight (dimensionless).

Alternative Cost Function

You can elect to use the following alternative to the standard cost function:

$$J(z_k) = \sum_{i=0}^{p-1} \left\{ \left[e_y^T(k+i) Q e_y(k+i) \right] + \left[e_u^T(k+i) R_u e_u(k+i) \right] + \left[\Delta u^T(k+i) R_{\Delta u} \Delta u(k+i) \right] \right\} + \rho \quad {}_k^2.$$

Here, Q (n_y-by-n_y), R_u, and $R_{\Delta u}$ (n_u-by-n_u) are positive-semi-definite weight matrices, and:

$$\begin{aligned}
e_y(i+k) &= S_y^{-1} \left[r(k+i+1 \mid k) - y(k+i+1 \mid k) \right] \\
e_u(i+k) &= S_u^{-1} \left[u_{target}(k+i \mid k) - u(k+i \mid k) \right] \\
\Delta u(k+i) &= S_u^{-1} \left[u(k+i \mid k) - u(k+i-1 \mid k) \right].
\end{aligned}$$

Also,

- S_y — Diagonal matrix of plant output variable scale factors, in engineering units.
- S_u — Diagonal matrix of MV scale factors in engineering units.
- $r(k+1 \mid k)$ — n_y plant output reference values at the ith prediction horizon step, in engineering units.
- $y(k+1 \mid k)$ — n_y plant outputs at the ith prediction horizon step, in engineering units.
- z_k — QP decision, given by:

$$z_k^T = \begin{bmatrix} u(k \mid k)^T & u(k+1 \mid k)^T & \cdots & u(k+p-1 \mid k)^T & {}_k \end{bmatrix}.$$

- $u_{target}(k+i \mid k)$ — n_u MV target values corresponding to $u(k+i \mid k)$, in engineering units.

Output predictions use the state observer, as in the standard cost function.

The alternative cost function allows off-diagonal weighting, but requires the weights to be identical at each prediction horizon step.

The alternative and standard cost functions are identical if the following conditions hold:

- The standard cost functions employs weights $w_{i,j}^{y}$, $w_{i,j}^{u}$, and $w_{i,j}^{\Delta u}$ that are constant with respect to the index, $i = 1{:}p$.
- The matrices Q, R_u, and $R_{\Delta u}$ are diagonal with the squares of those weights as the diagonal elements.

Constraints

Certain constraints are implicit. For example, a control horizon $m < p$ (or MV blocking) forces some MV increments to be zero, and the state observer used for plant output prediction is a set of implicit equality constraints. Explicit constraints that you can configure are described below.

Bounds on Plant Outputs, MVs, and MV Increments

The most common MPC constraints are bounds, as follows.

$$\frac{y_{j,min}(i)}{s_j^y} - {}_k V_{j,min}^{y}(i) \le \frac{y_j(k+i|k)}{s_j^y} \le \frac{y_{j,max}(i)}{s_j^y} + {}_k V_{j,max}^{y}(i), \quad i = 1:p, \quad j = 1:n_y$$
$$\frac{u_{j,min}(i)}{s_j^u} - {}_k V_{j,min}^{u}(i) \le \frac{u_j(k+i-1|k)}{s_j^u} \le \frac{u_{j,max}(i)}{s_j^u} + {}_k V_{j,max}^{u}(i), \; i = 1:p, \quad j = 1:n_u$$
$$\frac{\Delta u_{j,min}(i)}{s_j^u} - {}_k V_{j,min}^{\Delta u}(i) \le \frac{\Delta u_j(k+i-1|k)}{s_j^u} \le \frac{\Delta u_{j,max}(i)}{s_j^u} + {}_k V_{j,max}^{\Delta u}(i), \; i = 1:p, \quad j = 1:n_u.$$

Here, the V parameters (ECR values) are dimensionless controller constants analogous to the cost function weights but used for constraint softening (see “Constraint Softening” on page 1-12). Also,

- $\in_k$ — Scalar QP slack variable (dimensionless) used for constraint softening.

- s_j^y — Scale factor for jth plant output, in engineering units.
- s_j^u — Scale factor for jth MV, in engineering units.
- $y_{j,\min}(i)$, $y_{j,\max}(i)$ — lower and upper bounds for jth plant output at ith prediction horizon step, in engineering units.
- $u_{j,\min}(i)$, $u_{j,\max}(i)$ — lower and upper bounds for jth MV at ith prediction horizon step, in engineering units.
- $\Delta u_{j,\min}(i)$, $\Delta u_{j,\max}(i)$ — lower and upper bounds for jth MV increment at ith prediction horizon step, in engineering units.

Except for the slack variable non-negativity condition, all of the above constraints are optional and are inactive by default (i.e., initialized with infinite limiting values). To include a bound constraint, you must specify a finite limit when you design the controller.

QP Matrices

This section describes the matrices associated with the model predictive control optimization problem described in "Optimization Problem" on page 2-2.

Prediction

Assume that the disturbance models described in "Input Disturbance Model" is unit gain, for example, $d(k)=n_d(k)$ is a white Gaussian noise). You can denote this problem as

$$x \leftarrow \begin{bmatrix} x \\ x_d \end{bmatrix}, A \leftarrow \begin{bmatrix} A & B_d\bar{C} \\ 0 & \bar{A} \end{bmatrix}, B_u \leftarrow \begin{bmatrix} B_u \\ 0 \end{bmatrix}, B_v \leftarrow \begin{bmatrix} Bv \\ 0 \end{bmatrix}, B_d \leftarrow \begin{bmatrix} B_d\bar{D} \\ \bar{B} \end{bmatrix} C \leftarrow \begin{bmatrix} C & D_d\bar{C} \end{bmatrix}$$

Then, the prediction model is:

$$x(k+1) = Ax(k) + B_u u(k) + B_v v(k) + B_d n_d(k)$$

$$y(k) = Cx(k) + D_v v(k) + D_d n_d(k)$$

Next, consider the problem of predicting the future trajectories of the model performed at time k=0. Set $n_d(i)$=0 for all prediction instants i, and obtain

$$y(i|0) = C\left[A^i x(0) + \sum_{h=0}^{i-1} A^{i-1}\left(B_u\left(u(-1) + \sum_{j=0}^{h} \Delta u(j)\right) + B_v v(h)\right)\right] + D_v v(i)$$

This equation gives the solution

$$\begin{bmatrix} y(1) \\ \cdots \\ y(p) \end{bmatrix} = S_x x(0) + S_{u1} u(-1) + S_u \begin{bmatrix} \Delta u(0) \\ \cdots \\ \Delta u(p-1) \end{bmatrix} + H_v \begin{bmatrix} v(0) \\ \cdots \\ v(p) \end{bmatrix}$$

where

$$S_x = \begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^p \end{bmatrix} \in \Re^{pn_y \times n_x}, S_{u1} = \begin{bmatrix} CB_u \\ CB_u + CAB_u \\ \cdots \\ \sum_{h=0}^{p-1} CA^h B_u \end{bmatrix} \in \Re^{pn_y \times n_u}$$

$$S_u = \begin{bmatrix} CB_u & 0 & \cdots & 0 \\ CB_u + CAB_u & CB_u & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{h=0}^{p-1} CA^h B_u & \sum_{h=0}^{p-2} CA^h B_u & \cdots & CB_u \end{bmatrix} \in \Re^{pn_y \times pn_u}$$

$$H_v = \begin{bmatrix} CB_v & D_v & 0 & \cdots & 0 \\ CAB_v & CB_v & D_v & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ CA^{p-1}B_v & CA^{p-2}B_v & CA^{p-3}B_v & \cdots & D_v \end{bmatrix} \in \Re^{pn_y \times (p+1)n_v}.$$

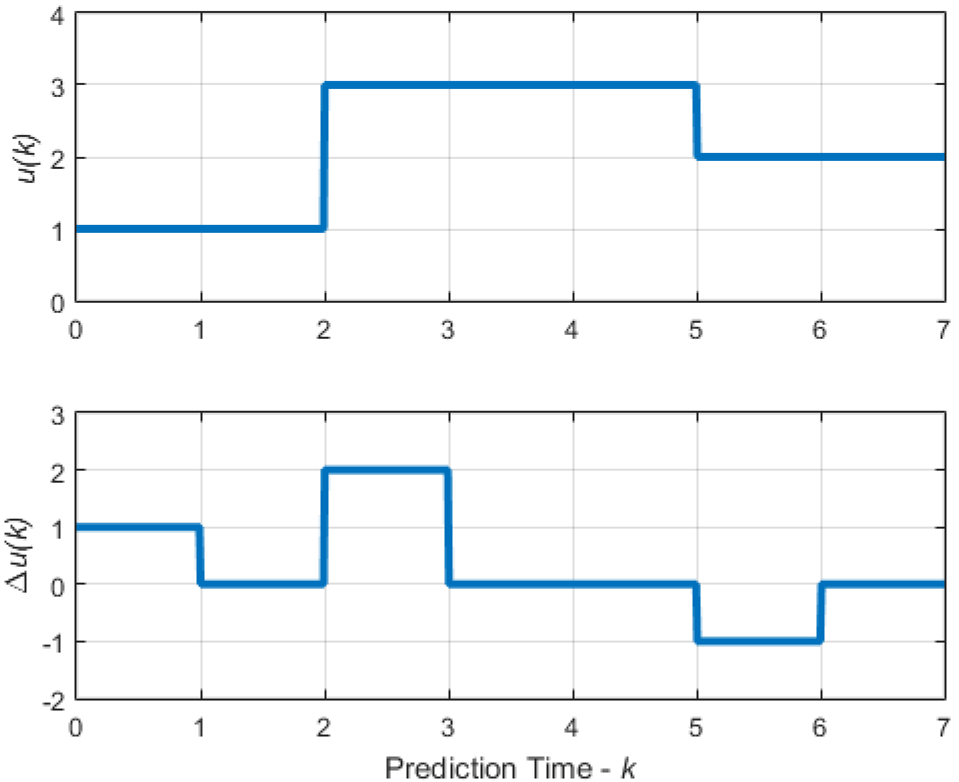
Optimization Variables

Let m be the number of free control moves, and let $z = [z_0; ...; z_{m-1}]$. Then,

$$\begin{bmatrix} \Delta u(0) \\ \cdots \\ \Delta u(p-1) \end{bmatrix} = J_M \begin{bmatrix} z_0 \\ \cdots \\ z_{m-1} \end{bmatrix}$$

where J_M depends on the choice of blocking moves. Together with the slack variable ε, vectors z_0, ..., z_{m-1} constitute the free optimization variables of the optimization problem. In the case of systems with a single manipulated variables, z_0, ..., z_{m-1} are scalars.

Consider the blocking moves depicted in the following graph.



Blocking Moves: Inputs and Input Increments for moves = [2 3 2]

This graph corresponds to the choice `moves=[2 3 2]`, or, equivalently, $u(0)=u(1)$, $u(2)=u(3)=u(4)$, $u(5)=u(6)$, $\Delta u(0)=z0$, $\Delta u(2)=z1$, $\Delta u(5)=z2$, $\Delta u(1)=\Delta u(3)=\Delta u(4)=\Delta u(6)=0$.

Then, the corresponding matrix J_M is

$$J_M = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}$$

Cost Function

- "Standard Form" on page 2-11
- "Alternative Cost Function" on page 2-12

Standard Form

The function to be optimized is

$$J(z,\varepsilon) = \left(\begin{bmatrix} u(0) \\ \cdots \\ u(p-1) \end{bmatrix} - \begin{bmatrix} u_{target}(0) \\ \cdots \\ u_{target}(p-1) \end{bmatrix} \right)^T W_u^2 \left(\begin{bmatrix} u(0) \\ \cdots \\ u(p-1) \end{bmatrix} - \begin{bmatrix} u_{target}(0) \\ \cdots \\ u_{target}(p-1) \end{bmatrix} \right) + \begin{bmatrix} \Delta u(0) \\ \cdots \\ \Delta u(p-1) \end{bmatrix}^T W_{\Delta u}^2 \begin{bmatrix} \Delta u(0) \\ \cdots \\ \Delta u(p-1) \end{bmatrix}$$
$$+ \left(\begin{bmatrix} y(1) \\ \cdots \\ y(p) \end{bmatrix} - \begin{bmatrix} r(1) \\ \cdots \\ r(p) \end{bmatrix} \right)^T W_y^2 \left(\begin{bmatrix} y(1) \\ \cdots \\ y(p) \end{bmatrix} - \begin{bmatrix} r(1) \\ \cdots \\ r(p) \end{bmatrix} \right) + \rho_\varepsilon \varepsilon^2$$

where

$$W_u = \operatorname{diag}\left(w_{0,1}^u, w_{0,2}^u, \ldots, w_{0,n_u}^u, \ldots, w_{p-1,1}^u, w_{p-1,2}^u, \ldots, w_{p-1,n_u}^u \right)$$
$$W_{\Delta u} = \operatorname{diag}\left(w_{0,1}^{\Delta u}, w_{0,2}^{\Delta u}, \ldots, w_{0,n_u}^{\Delta u}, \ldots, w_{p-1,1}^{\Delta u}, w_{p-1,2}^{\Delta u}, \ldots, w_{p-1,n_u}^{\Delta u} \right)$$
$$W_y = \operatorname{diag}\left(w_{1,1}^y, w_{1,2}^y, \ldots, w_{1,n_y}^y, \ldots, w_{p,1}^y, w_{p,2}^y, \ldots, w_{p,n_y}^y \right)$$

Finally, after substituting $u(k)$, $\Delta u(k)$, $y(k)$, $J(z)$ can be rewritten as

$$J(z,\varepsilon) = \rho_\varepsilon \varepsilon^2 + z^T K_{\Delta u} z + 2\left(\begin{bmatrix} r(1) \\ \cdots \\ r(p) \end{bmatrix}^T K_r + \begin{bmatrix} v(0) \\ \cdots \\ v(p) \end{bmatrix}^T K_v + u(-1)^T K_u + \begin{bmatrix} u_{target}(0) \\ \cdots \\ u_{target}(p-1) \end{bmatrix}^T K_{ut} + x(0)^T K_x \right) z$$
$$+ \text{constant}$$

Note You may want the QP problem to remain strictly convex. If the condition number of the Hessian matrix $K_{\Delta U}$ is larger than 10^{12}, add the quantity `10*sqrt(eps)` on each diagonal term. You can use this solution only when all input rates are unpenalized ($W^{\Delta u}$=0) (see "Weights" in the Model Predictive Control Toolbox reference documentation).

Alternative Cost Function

If you are using the alternative cost function shown in "Alternative Cost Function" on page 2-6, Equation 2-3, then Equation 2-2 is replaced by the following:

$$W_u = \text{blkdiag}(R_u, \ldots, R_u)$$
$$W_{\Delta u} = \text{blkdiag}(R_{\Delta u}, \ldots, R_{\Delta u})$$
$$W_y = \text{blkdiag}(Q, \ldots, Q)$$

In this case, the block-diagonal matrices repeat p times, for example, once for each step in the prediction horizon.

You also have the option to use a combination of the standard and alternative forms. See "Weights" in the Model Predictive Control Toolbox reference documentation for more details.

Constraints

Next, consider the limits on inputs, input increments, and outputs along with the constraint $\varepsilon \geq 0$.

$$\begin{bmatrix} y_{\min}(1) - \varepsilon V^{y}_{\min}(1) \\ \cdots \\ y_{\min}(p) - \varepsilon V^{y}_{\min}(p) \\ u_{\min}(0) - \varepsilon V^{u}_{\min}(0) \\ \cdots \\ u_{\min}(p-1) - \varepsilon V^{u}_{\min}(p-1) \\ \Delta u_{\min}(0) - \varepsilon V^{\Delta u}_{\min}(0) \\ \cdots \\ \Delta u_{\min}(p-1) - \varepsilon V^{\Delta u}_{\min}(p-1) \end{bmatrix} \le \begin{bmatrix} y(1) \\ \cdots \\ y(p) \\ u(0) \\ \cdots \\ u(p-1) \\ \Delta u(0) \\ \cdots \\ \Delta u(p-1) \end{bmatrix} \le \begin{bmatrix} y_{\max}(1) + \varepsilon V^{y}_{\max}(1) \\ \cdots \\ y_{\max}(p) + \varepsilon V^{y}_{\max}(p) \\ u_{\max}(0) + \varepsilon V^{u}_{\max}(0) \\ \cdots \\ u_{\max}(p-1) + \varepsilon V^{u}_{\max}(p-1) \\ \Delta u_{\max}(0) + \varepsilon V^{\Delta u}_{\max}(0) \\ \cdots \\ \Delta u_{\max}(p-1) + \varepsilon V^{\Delta u}_{\max}(p-1) \end{bmatrix}$$

Note To reduce computational effort, the controller automatically eliminates extraneous constraints, such as infinite bounds. Thus, the constraint set used in real time may be much smaller than that suggested in this section.

Similar to what you did for the cost function, you can substitute $u(k)$, $\Delta u(k)$, $y(k)$, and obtain

$$M_z z + M_\varepsilon \varepsilon \le M_{\lim} + M_v \begin{bmatrix} v(0) \\ \cdots \\ v(p) \end{bmatrix} + M_u u(-1) + M_x x(0)$$

In this case, matrices $M_z, M_\varepsilon, M_{\lim}, M_v, M_u, M_x$ are obtained from the upper and lower bounds and ECR values.

Unconstrained Model Predictive Control

The optimal solution is computed analytically

$$z^* = -K_{\Delta u}^{-1} \left(\begin{bmatrix} r(1) \\ \cdots \\ r(p) \end{bmatrix}^T K_r + \begin{bmatrix} v(0) \\ \cdots \\ v(p) \end{bmatrix} K_v + u(-1)^T K_u + \begin{bmatrix} u_{target}(0) \\ \cdots \\ u_{target}(p-1) \end{bmatrix}^T K_{ut} + x(0)^T K_x \right)^T$$

and the model predictive controller sets $\Delta u(k)=z^*_0$, $u(k)=u(k-1)+\Delta u(k)$.

More About

- "Adjusting Disturbance and Noise Models" on page 2-15
- "Time-Varying Weights and Constraints" on page 2-26
- "Terminal Weights and Constraints" on page 2-30

Adjusting Disturbance and Noise Models

A model predictive control requires the following to reject unknown disturbances effectively:

- Application-specific disturbance models
- Measurement feedback to update the controller state estimates

You can modify input and output disturbance models, and the measurement noise model using the MPC Designer app and at the command line. You can then adjust controller tuning weights to improve disturbance rejection.

In this section...

Overview

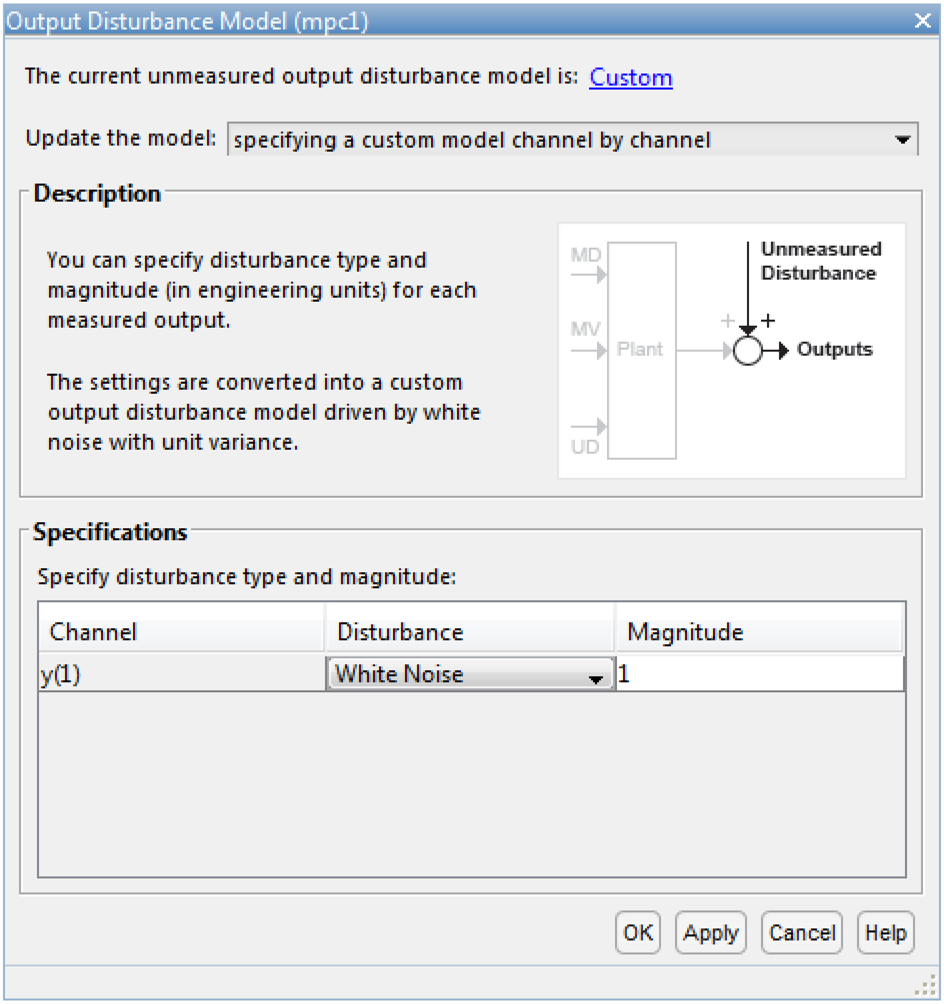
MPC attempts to predict how known and unknown events affect the plant output variables (OVs). Known events are changes in the measured plant input variables (MV and MD inputs). The plant model of the controller predicts the impact of these events, and such predictions can be quite accurate. For more information, see"MPC Modeling".

The impacts of unknown events appear as errors in the predictions of known events. These errors are, by definition, impossible to predict accurately. However, an ability to anticipate trends can improve disturbance rejection For example, suppose that the control system has been operating at a near-steady condition with all measured OVs near their predicted values. There are no known events, but one or more of these OVs suddenly deviates from its prediction. The controller disturbance and measurement models allow you to provide guidance on how to handle such errors.

Output Disturbance Model

Suppose that your plant model includes no unmeasured disturbance inputs. The MPC controller then models unknown events using an *output disturbance model*. As shown in "MPC Modeling", the output disturbance model is independent of the plant, and its output adds directly to that of the plant model.

Using the MPC Designer app, you can specify the type of noise that is expected to affect each plant OV. In the MPC Designer app, on the **Tuning** tab, in the **Design** section, click **Estimation Models** > **Output Disturbance Model**. In the Output Disturbance Model dialog box, in the **Update the model** drop-down list, select **specifying a custom model channel by channel**.



In the **Specifications** section, in the **Disturbance** column, select one of the following disturbance models for each output:

- `White Noise` — Prediction errors are due to random zero-mean white noise. This option implies that the impact of the disturbance is short-lived, and therefore requires a modest, short-term controller response.

- `Random Step-like` — Prediction errors are due to a random step-like disturbance, which lasts indefinitely, maintaining a roughly constant magnitude. Such a disturbance requires a more aggressive, sustained controller response.
- `Random Ramp-like` — Prediction errors are due to a random ramp-like disturbance, which lasts indefinitely and tends to grow with time. Such a disturbance requires an even more aggressive controller response.

Model Predictive Control Toolbox software represents each disturbance type as a model in which white noise, with zero mean and unit variance, enters a SISO dynamic system consisting of one of the following:

- A static gain — For a white noise disturbance
- An integrator in series with a static gain — For a step-like disturbance
- Two integrators in series with a static gain — For a ramp-like disturbance

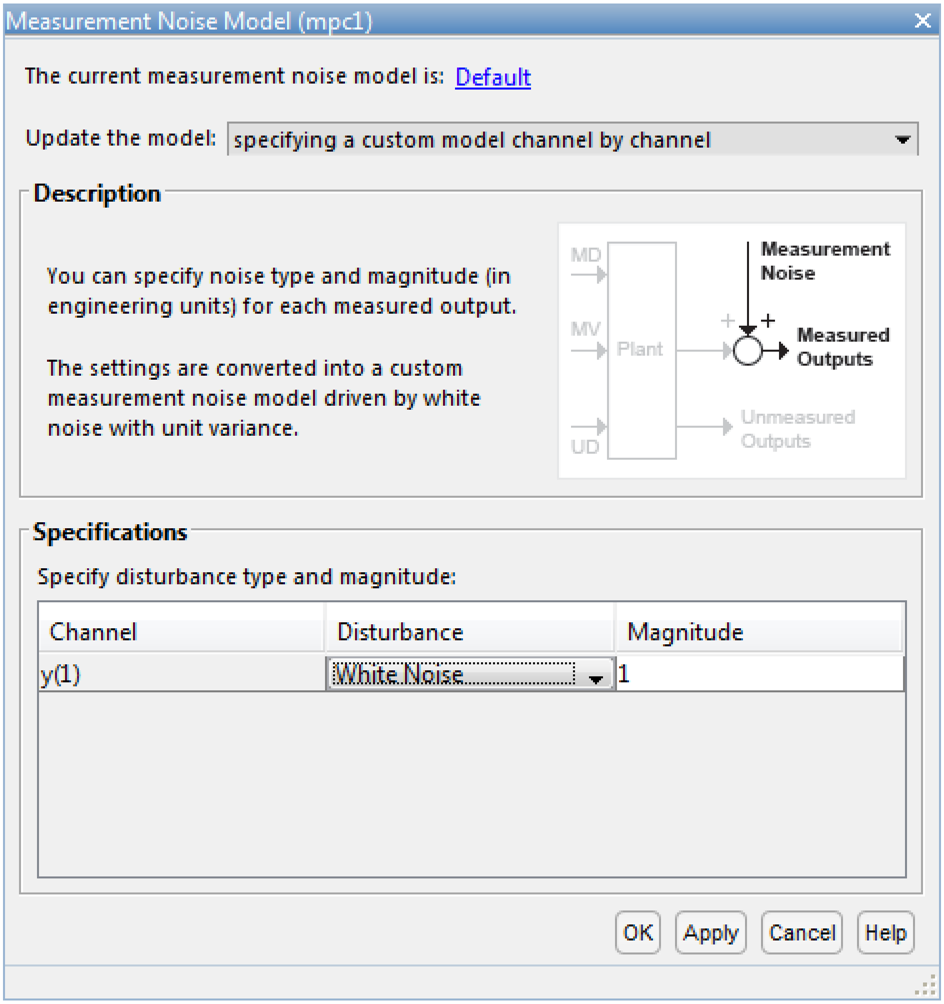
You can also specify the white noise input **Magnitude** for each disturbance model, overriding the assumption of unit variance. As you increase the noise magnitude, the controller responds more aggressively to a given prediction error. The specified noise magnitude corresponds to the static gain in the SISO model for each type of noise.

You can also view or modify the output disturbance model from the command line using `getoutdist` and `setoutdist` respectively.

Measurement Noise Model

MPC also attempts to distinguish disturbances, which require a controller response, from measurement noise, which the controller should ignore. Using the MPC Designer app, you can specify the expected measurement noise magnitude and character. In the MPC Designer app, on the **Tuning** tab, in the **Design** section, click **Estimation Models** > **Measurement Noise Model**. In the Model Noise Model dialog box, in the **Update the model** drop-down list, select **specifying a custom model channel by channel**.

In the **Specifications** section, in the **Disturbance** column, select a noise model for each measured output channel. The noise options are the same as the output disturbance model options.



`White Noise` is the default option and, in nearly all applications, should provide adequate performance.

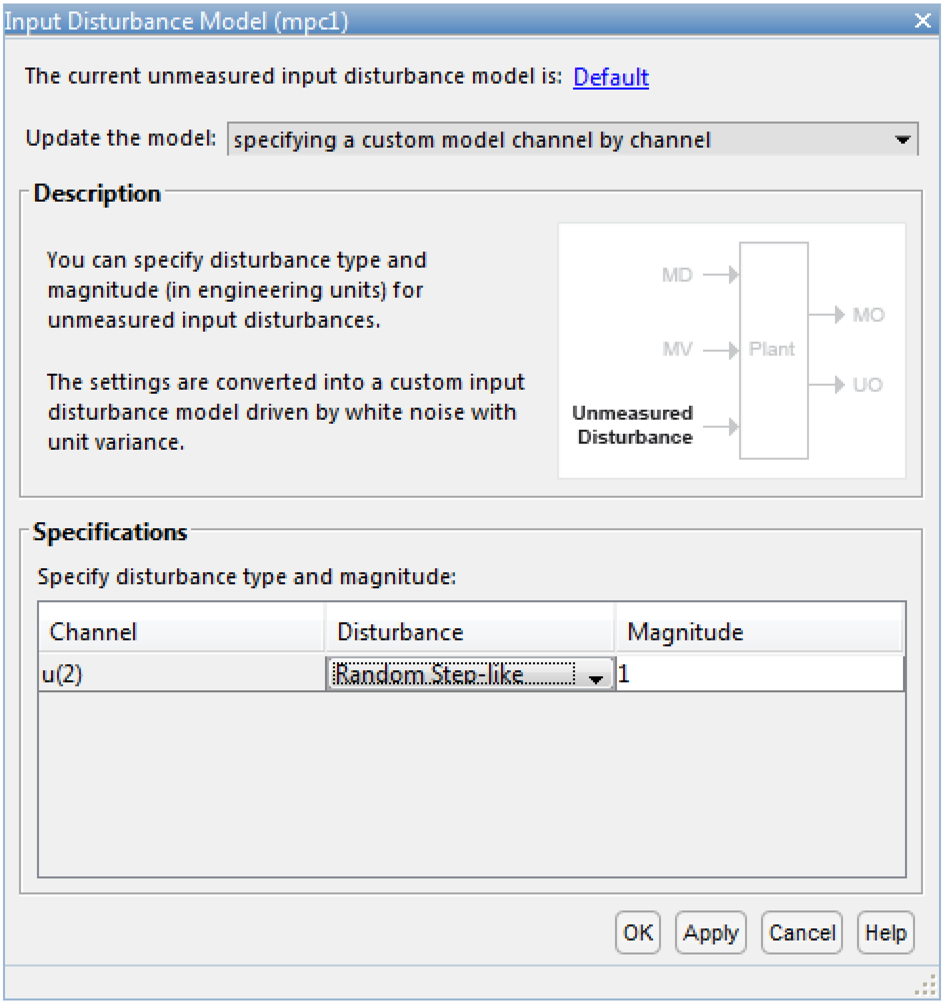
When you include a measurement noise model, the controller considers each prediction error to be a combination of disturbance and noise effects. Qualitatively, as you increase the specified noise **Magnitude**, the controller attributes a larger fraction of each prediction error to noise, and it responds less aggressively. Ultimately, the controller stops responding to prediction errors and only changes its MVs when you change the OV or MV reference signals.

Input Disturbance Model

When your plant model includes unmeasured disturbance (UD) inputs, the controller can use an *input disturbance model* in addition to the standard output disturbance model. The former provides more flexibility and is generated automatically by default. If the chosen input disturbance model does not appear to allow complete elimination of sustained disturbances, an output disturbance model is also added by default.

As shown in “MPC Modeling”, the input disturbance model consists of one or more white noise signals, with unit variance and zero mean, entering a dynamic system. The outputs of this system are the UD inputs to the plant model. In contrast to the output disturbance model, input disturbances affect the plant outputs in a more complex way as they pass through the plant model dynamics.

As with the output disturbance model, you can use the MPC Designer app to specify the type of disturbance you expect for each UD input. In the MPC Designer app, on the **Tuning** tab, in the **Design** section, click **Estimation Models** > **Input Disturbance Model**. In the Input Disturbance Model dialog box, in the **Update the model** drop-down list, select **specifying a custom model channel by channel**.



In the **Specifications** section, in the **Disturbance** column, select a noise model for each measured output channel. The input disturbance model options are the same as the output disturbance model options.

A common approach is to model unknown events as disturbances adding to the plant MVs. These disturbances, termed *load disturbances* in many texts, are realistic in that some unknown events are failures to set the MVs to the values requested by the controller. You can create a load disturbance model as follows:

1 Begin with an LTI plant model, `Plant`, in which all inputs are known (MVs and MDs).
2 Obtain the state-space matrices of `Plant`. For example:

```
[A,B,C,D] = ssdata(Plant);
```

3 Suppose that there are n_u MVs. Set B_u = columns of B corresponding to the MVs. Also, set D_u = columns of D corresponding to the MVs.
4 Redefine the plant model to include n_u additional inputs. For example:

```
Plant.b = [B Bu];
Plant.d = [D Du]);
```

5 To indicate that the new inputs are unmeasured disturbances, use `setmpcsignals`, or set the `Plant.InputGroup` property.

This procedure adds load disturbance inputs without increasing the number of states in the plant model.

By default, given a plant model containing load disturbances, the Model Predictive Control Toolbox software creates an input disturbance model that generates n_{ym} step-like load disturbances. If $n_{ym} > n_u$, it also creates an output disturbance model with integrated white noise adding to $(n_{ym} - n_u)$ measured outputs. If $n_{ym} < n_u$, the last $(n_u - n_{ym})$ load disturbances are zero by default. You can modify these defaults using the MPC Designer app.

You can also view or modify the input disturbance model from the command line using `getindist` and `setindist` respectively.

Restrictions

As discussed in "Controller State Estimation" on page 2-40, the plant, disturbance, and noise models combine to form a state observer, which must be detectable using the measured plant outputs. If not, the software displays a command-window error message when you attempt to use the controller.

This limitation restricts the form of the disturbance and noise models. If any models are defined as anything other than white noise with a static gain, their model states must be detectable. For example, an integrated white noise disturbance adding to an unmeasured OV would be undetectable. The MPC Designer app prevents you from choosing such a model. Similarly, the number of measured disturbances, n_{ym}, limits the number of step-like UD inputs from an input disturbance model.

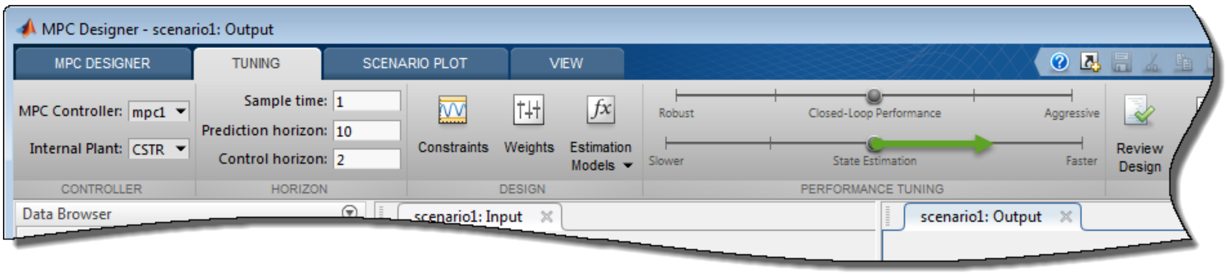
By default, the Model Predictive Control Toolbox software creates detectable models. If you modify the default assumptions (or change n_{ym}) and encounter a detectability error, you can revert to the default case.

Disturbance Rejection Tuning

During the design process, you can tune the disturbance rejection properties of the controller.

1. Before any controller tuning, define scale factors for each plant input and output variable (see "Specifying Scale Factors" on page 1-2). In the context of disturbance and noise modeling, this makes the default assumption of unit-variance white noise inputs more likely to yield good performance.
2. Initially, keep the disturbance models in their default configuration.
3. After tuning the cost function weights (see "Tuning Weights" on page 1-16), test your controller response to an unmeasured disturbance input other than a step disturbance at the plant output. Specifically, if your plant model includes UD inputs, simulate a disturbance using one or more of these. Otherwise, simulate one or more load disturbances, that is, a step disturbance added to a designated MV. Both the MPC Designer app and the `sim` command support such simulations.
4. If the response in the simulations is too sluggish, try one or more of the following to produce more aggressive disturbance rejection:

   - Increase all disturbance model gains by a multiplicative factor. In the MPC Designer app, do this by increasing the magnitude of each disturbance. If this helps but is insufficient, increase the magnitude further.
   - Decrease the measurement noise gains by a multiplicative factor. In the MPC Designer app, do this by increasing the measurement noise magnitude. If this helps but is insufficient, increase the magnitude further.
   - In the MPC Designer app, in the **Tuning** tab, drag the **State Estimation** slider to the right. Moving towards **Faster** state estimation simultaneously increases the gains for disturbance models and decreases the gains for noise models.



If this helps but is insufficient, drag the slider further to the right.

- Change one or more disturbances to model that requires a more aggressive controller response. For example, change the model from white noise disturbance to a step-like disturbance.

Note: Changing the disturbances in this way adds states to disturbance model, which can cause violations of the state observer detectability restriction.

---

5. If the response is too aggressive, and in particular, if the controller is not robust when its prediction of known events is inaccurate, try reversing the previous adjustments.

See Also

`getindist` | `getoutdist` | MPC Designer | `setindist` | `setmpcsignals` | `setoutdist`

Related Examples

- "Design Controller Using MPC Designer"

More About

- "MPC Modeling"
- "Controller State Estimation" on page 2-40

Custom State Estimation

The Model Predictive Control Toolbox software allows the following alternatives to the default state estimation approach:

- You can override the default Kalman gains, L and M. Obtain the default values using `getEstimator`. Then, use `setEstimator` to override those values. These commands assume that the columns of L and M are in the engineering units for the measured plant outputs. Internally, the software converts them to dimensionless form.
- You can use the custom estimation option. This skips all Kalman gain calculations. When the controller operates, at each control interval you must use an external procedure to estimate the controller states, `xc(k|k)`, providing this to the controller.

Note: You cannot use custom state estimation with the MPC Designer app.

Related Examples

- Using Custom State Estimation

More About

- “Controller State Estimation” on page 2-40

Time-Varying Weights and Constraints

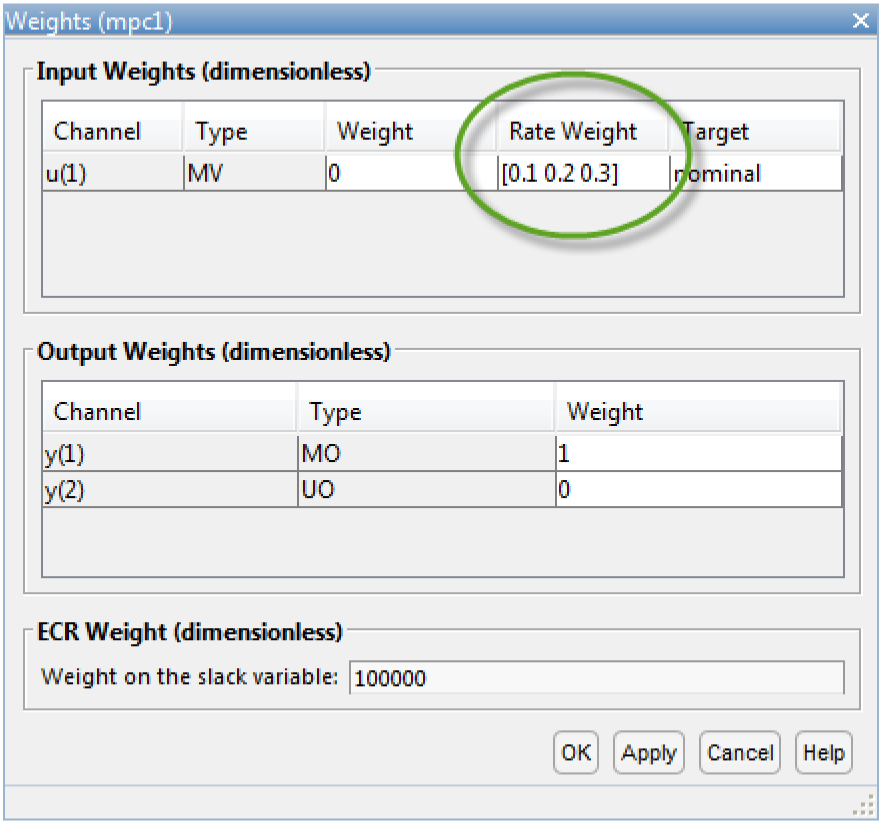
In this section...

Time-Varying Weights

As explained in "Optimization Problem" on page 2-2, the w^y, w^u, and $w^{\Delta u}$ weights can change from one step in the prediction horizon to the next. Such a *time-varying weight* is an array containing p rows, where p is the prediction horizon, and either n_y or n_u columns (number of OVs or MVs).

Using time-varying weights provides additional tuning possibilities. However, it complicates tuning. Recommended practice is to use constant weights unless your application includes unusual characteristics. For example, an application requiring terminal weights must employ time-varying weights. See "Terminal Weights and Constraints" on page 2-30.

You can specify time-varying weights in the MPC Designer app. In the Weights dialog box, specify a time-varying weight as a vector. Each element of the vector corresponds to one step in the prediction horizon. If the length of the vector is less than p, the last weight value applies for the remainder of the prediction horizon.



Note: For any given input channel, you can specify different vector lengths for **Rate Weight** and **Weight**. However, if you specify a time-varying **Weight** for any input channel, you must specify a time-varying **Weight** for all inputs using the same length weight vectors. Similarly, all input **Rate Weight** values must use the same vector length.

Also, if you specify a time-varying **Weight** for any output channel, you must specify a time-varying **Weight** for all output using the same length weight vectors.

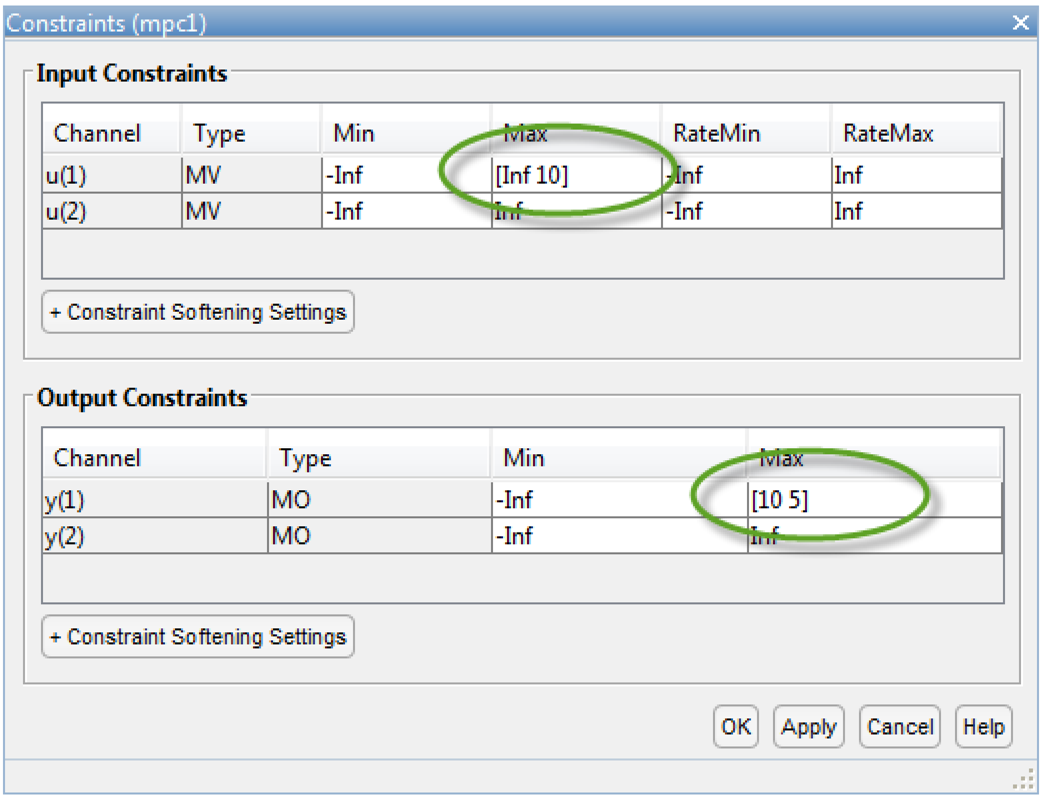
Time-Varying Constraints

When bounding an MV, OV, or MV increment, you can use a different bound value at each prediction-horizon step. To do so, specify the bound as a vector of up to p values, where p is the prediction horizon length (number of control intervals). If you specify $n < p$ values, the nth value applies for the remaining $p - n$ steps.

You can remove constraints at selected steps by specifying `Inf` (or `-Inf`).

If plant delays prevent the MVs from affecting an OV during the first d steps of the prediction horizon and you must include bounds on that OV, leave the OV unconstrained for the first d steps.

You can specify time-varying constraints in the MPC Designer app. In the Constraints dialog box, specify a vector for each time-varying constraint.



Related Examples

- Varying Input and Output Constraints

More About

- “Optimization Problem” on page 2-2
- “Terminal Weights and Constraints” on page 2-30

Terminal Weights and Constraints

Terminal weights are the quadratic weights *Wy* on $y(t+p)$ and *Wu* on $u(t + p - 1)$. The variable p is the prediction horizon. You apply the quadratic weights at time $k + p$ only, such as the prediction horizon's final step. Using terminal weights, you can achieve infinite horizon control that guarantees closed-loop stability. However, before using terminal weights, you must distinguish between problems with and without constraints.

Terminal constraints are the constraints on $y(t + p)$ and $u(t + p - 1)$, where p is the prediction horizon. You can use terminal constraints as an alternative way to achieve closed-loop stability by defining a terminal region.

Note: You can use terminal weights and constraints only at the command line. See `setterminal`.

For the relatively simple unconstrained case, a terminal weight can make the finite-horizon Model Predictive Controller behave as if its prediction horizon were infinite. For example, the MPC controller behavior is identical to a linear-quadratic regulator (LQR). The standard LQR derives from the cost function:

$$J(u) = \sum_{i=1}^{\infty} x(k+i)^T Q x(k+i) + u(k+i-1)^T R u(k+i-1)$$

where x is the vector of plant states in the standard state-space form:

$x(k + 1) = Ax + Bu(k)$

The LQR provides nominal stability provided matrices Q and R meet certain conditions. You can convert the LQR to a finite-horizon form as follows:

$$J(u) = \sum_{i=1}^{p-1} [x(k+i)^T Q x(k+i) + u(k+i-1)^T R u(k+i-1)] + x(k+p)^T Q_p x(k+p)$$

where Q_p , the terminal penalty matrix, is the solution of the Riccati equation:

$$Q_p = A^T Q_p A - A^T Q_p B (B^T Q_p B + R)^{-1} B^T Q_p A + Q$$

You can obtain this solution using the `lqr` command in Control System Toolbox™ software.

In general, Q_p is a full (symmetric) matrix. You cannot use the "Standard Cost Function" on page 2-2 to implement the LQR cost function. The only exception is for the first $p - 1$ steps if Q and R are diagonal matrices. Also, you cannot use the "Alternative Cost Function" on page 2-6 because it employs identical weights at each step in the horizon. Thus, by definition, the terminal weight differs from those in steps 1 to $p - 1$. Instead, use the following steps:

1. Augment the model (Equation 2-7) to include the weighted terminal states as auxiliary outputs:

   $y_{aug}(k) = Q_c x(k)$

   where Q_c is the Cholesky factorization of Q_p such that $Q_p = Q_c^T Q_c$.
2. Define the auxiliary outputs y_{aug} as unmeasured, and specify zero weight to them.
3. Specify unity weight on y_{aug} at the last step in the prediction horizon using `setterminal`.

To make the Model Predictive Controller entirely equivalent to the LQR, use a control horizon equal to the prediction horizon. In an unconstrained application, you can use a short horizon and still achieve nominal stability. Thus, the horizon is no longer a parameter to be tuned.

When the application includes constraints, the horizon selection becomes important. The constraints, which are usually softened, represent factors not considered in the LQR cost function. If a constraint becomes active, the control action deviates from the LQR (state feedback) behavior. If this behavior is not handled correctly in the controller design, the controller may destabilize the plant.

For an in-depth discussion of design issues for constrained systems see [1]. Depending on the situation, you might need to include terminal constraints to force the plant states into a defined region at the end of the horizon, after which the LQR can drive the plant signals to their targets. Use `setterminal` to add such constraints to the controller definition.

The standard (finite-horizon) Model Predictive Controller provides comparable performance, if the prediction horizon is long. You must tune the other controller parameters (weights, constraint softening, and control horizon) to achieve this performance.

Tip Robustness to inaccurate model predictions is usually a more important factor than nominal performance in applications.

References

[1] Rawlings, J. B., and David Q. Mayne "Model Predictive Control: Theory and Design" Nob Hill Publishing, 2010.

See Also

`setterminal`

Related Examples

- "Designing Model Predictive Controller Equivalent to Infinite-Horizon LQR"
- "Providing LQR Performance Using Terminal Penalty" on page 4-83

Constraints on Linear Combinations of Inputs and Outputs

You can constrain linear combinations of plant input and output variables. For example, you can constrain a particular manipulated variable (MV) to be greater than a linear combination of two other MVs. The general form of such constraints is the following:

$$Eu\left(k+i|k\right)+Fy\left(k+i|k\right)+Sv(k+i\mid k)\leq G+\ {}_{k}V.$$

- $\in_k$ — QP slack variable used for constraint softening (See “Constraint Softening” on page 1-12)
- $u(k+i\,|\,k)$ — n_u MV values, in engineering units
- $y(k+i\,|\,k)$ — n_y predicted plant outputs, in engineering units
- $v(k+i\,|\,k)$ — n_v measured plant disturbance inputs, in engineering units
- E, F, S, G, and V are constants

As with the QP cost function, output prediction using the state observer makes these constraints a function of the QP decision.

Custom constraints are dimensional by default.

You can update custom constraints at run time by calling `setconstraint` before calling `mpcmove`. However, updating the custom constraint matrices at run time is not supported in Simulink.

Note: Using custom constraints is not supported in the MPC Designer app.

See Also

`getconstraint` | `setconstraint`

Related Examples

- Using Custom Input and Output Constraints
- Nonlinear Blending Process with Custom Constraints

More About

- “Optimization Problem” on page 2-2

- “Run-Time Constraint Updating”

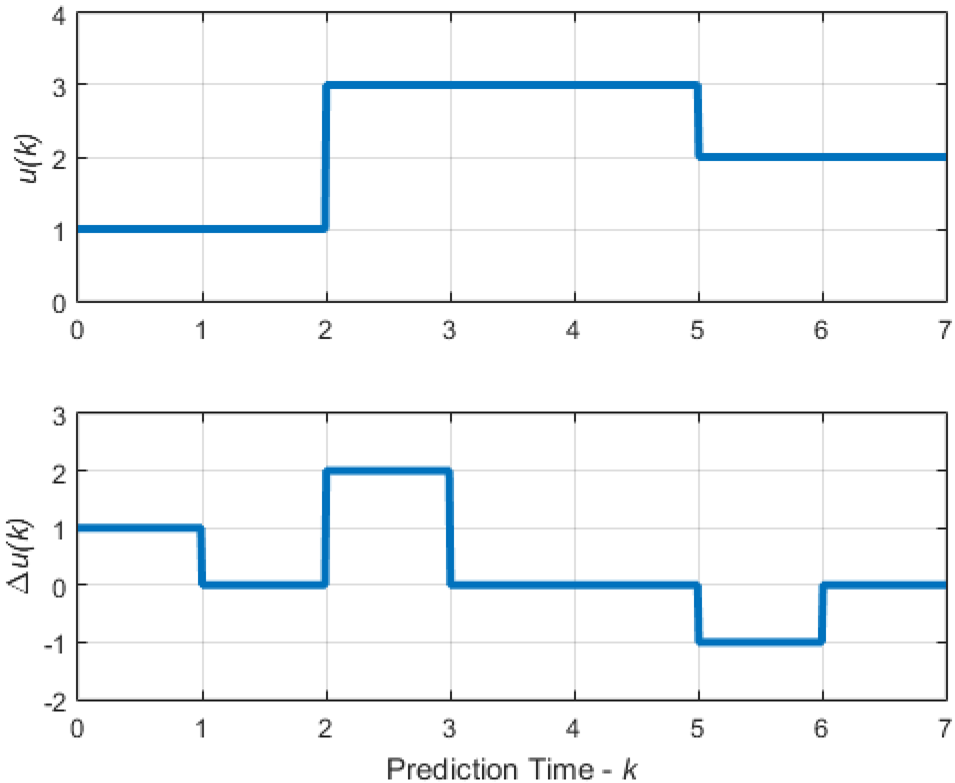
Manipulated Variable Blocking

Blocking is an alternative to the simpler control horizon concept (see "Choosing Sample Time and Horizons" on page 1-6). It has many of the same benefits. It also provides more tuning flexibility and potential to smooth MV adjustments. To manipulated variable blocking, you divide the prediction horizon into a series of blocks. The controller then holds the manipulated variable constant within each block.

A recommended approach to blocking is as follows:

- Divide the prediction horizon into 3-5 blocks.
- Try the following alternatives
  - Equal block sizes (one-fifth to one-third of the prediction horizon, p)
  - Block sizes increasing. Example, with $p = 20$, three blocks of duration 3, 7 and 10 intervals.

To use manipulated variable blocking, specify your control horizon as a vector of block sizes. For example, the following figure represent control moves for a control horizon of `p = [2 3 2]`:



For each block, the manipulated variable, u, is constant, that is:

- $u(0) = u(1)$
- $u(2) = u(3) = u(4)$
- $u(5) = u(6)$

Test the resulting controller in the same way that you test cost function weights. See "Tuning Weights" on page 1-16.

Related Examples

- “Design MPC Controller for Plant with Delays”

More About

- “Optimization Problem” on page 2-2
- “Tuning Weights” on page 1-16

QP Solver

The model predictive controller QP solver converts an MPC optimization problem to the general QP form

$$\underset{x}{Min}\left(\frac{1}{2}x'Hx+f'x\right)$$

such that

$$Ax \le b$$

where

- x is the solution vector.
- H is the Hessian matrix.
- A is a matrix of linear constraint coefficients.
- b and f are vectors.

The H and A matrices are constants. The controller computes these constant matrices during initialization and retrieves them from computer memory when needed. It computes the time-varying b and f vectors at the beginning of each control instant.

The toolbox uses the KWIK algorithm [1] to solve the QP problem, which requires the Hessian to be positive definite. In the first control step, KWIK uses a *cold start*, in which the initial guess is the unconstrained solution described in "Unconstrained Model Predictive Control" on page 2-13. If x satisfies the constraints, it is the optimal QP solution, x^*, and the algorithm terminates. Otherwiseat least one of the linear inequality constraints must be satisfied as an equality. In this case, KWIK uses an efficient, numerically robust strategy to determine the active constraint set satisfying the standard optimization conditions. In the following control steps, KWIK uses a *warm start*. In this case, the active constraint set determined at the previous control step becomes the initial guess for the next.

Although KWIK is robust, consider the following:

- One or more linear constraints can be violated slightly due to numerical round-off errors. The toolbox employs a nonadjustable relative tolerance. This tolerance allows constraint violations of 10^{-6} times the magnitude of each term. Such violations are considered normal and do not generate warning messages.

- The toolbox also uses a nonadjustable tolerance when testing for an optimal solution.
- The search for the active constraint set is an iterative process. If the iterations reach a problem-dependent maximum, the algorithm terminates.
- If your problem includes hard constraints, these constraints can be *infeasible* (impossible to satisfy). If the algorithm detects infeasibility, it terminates immediately.

In the last two situations, with an abnormal outcome to the search, the controller retains the last successful control output. For more information, see, the `mpcmove` command. You can detect an abnormal outcome and override the default behavior as you see fit.

If you have an advanced MPC application that is beyond the scope of Model Predictive Control Toolbox software, you can use `mpcqpsolver` to access the toolbox QP Solver.

References

[1] Schmid, C. and L.T. Biegler, "Quadratic programming methods for reduced Hessian SQP," *Computers & Chemical Engineering*, Vol. 18, Number 9, 1994, pp. 817–832.

See Also

`mpcqpsolver`

More About

- "Optimization Problem" on page 2-2

Controller State Estimation

In this section...

Controller State Variables

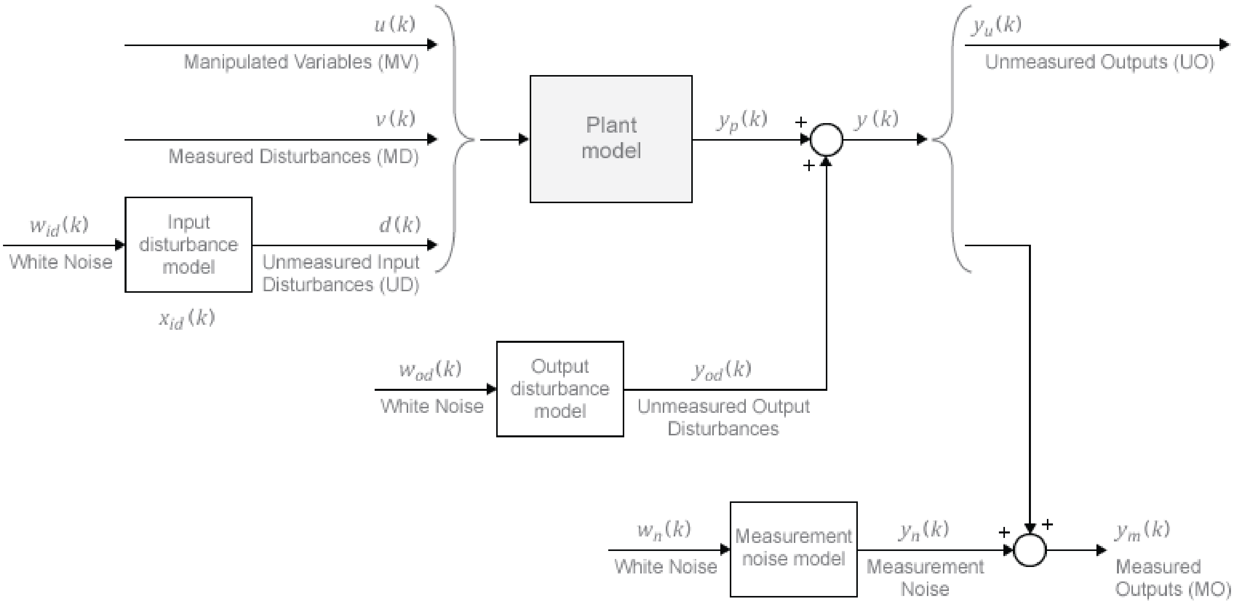
As the controller operates, it uses its current state, x_c, as the basis for predictions. By definition, the state vector is the following:

$$x_c^T(k) = \left[x_p^T(k) \quad x_{id}^T(k) \quad x_{od}^T(k) \quad x_n^T(k) \right].$$

Here,

- x_c is the controller state, comprising $n_{xp} + n_{xid} + n_{xod} + n_{xn}$ state variables.
- x_p is the plant model state vector, of length n_{xp}.
- x_{id} is the input disturbance model state vector, of length n_{xid}.
- x_{od} is the output disturbance model state vector, of length n_{xod}.
- x_n is the measurement noise model state vector, of length n_{xn}.

Thus, the variables comprising x_c represent the models appearing in the following diagram of the MPC system.



Some of the state vectors may be empty. If not, they appear in the sequence defined within each model.

By default, the controller updates its state automatically using the latest plant measurements. See "State Estimation" on page 2-42 for details. Alternatively, the custom state estimation feature allows you to update the controller state using an external procedure, and then supply these values to the controller. See "Custom State Estimation" on page 2-25 for details.

State Observer

Combination of the models shown in the diagram yields the state observer:

$$\begin{aligned} x_c(k+1) &= Ax_c(k) + Bu_o(k) \\ y(k) &= Cx_c(k) + Du_o(k). \end{aligned}$$

MPC controller uses the state observer in the following ways:

- To estimate values of unmeasured states needed as the basis for predictions (see "State Estimation" on page 2-42).

- To predict how the controller's proposed manipulated variable (MV) adjustments will affect future plant output values (see "Output Variable Prediction" on page 2-45).

The observer's input signals are the dimensionless plant manipulated and measured disturbance inputs, and the white noise inputs to the disturbance and noise models:

$$u_o^T(k) = \left[u^T(k) \quad v^T(k) \quad w_{id}^T(k) \quad w_{od}^T(k) \quad w_n^T(k) \right].$$

The observer's outputs are the n_y dimensionless plant outputs.

In terms of the parameters defining the four models shown in the diagram, the observer's parameters are:

$$A = \begin{bmatrix} A_p & B_{pd}C_{id} & 0 & 0 \\ 0 & A_{id} & 0 & 0 \\ 0 & 0 & A_{od} & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix}, \quad B = \begin{bmatrix} B_{pu} & B_{pv} & B_{pd}D_{id} & 0 & 0 \\ 0 & 0 & B_{id} & 0 & 0 \\ 0 & 0 & 0 & B_{od} & 0 \\ 0 & 0 & 0 & 0 & B_n \end{bmatrix},$$

$$C = \left[C_p \quad D_{pd}C_{id} \quad C_{od} \quad \begin{bmatrix} C_n \\ 0 \end{bmatrix} \right], \quad D = \left[0 \quad D_{pv} \quad D_{pd}D_{id} \quad D_{od} \quad \begin{bmatrix} D_n \\ 0 \end{bmatrix} \right].$$

Here, the plant and output disturbance models are resequenced so that the measured outputs precede the unmeasured outputs.

State Estimation

In general, the controller states are unmeasured and must be estimated. By default, the controller uses a steady state Kalman filter that derives from the state observer. (See "State Observer" on page 2-41.)

At the beginning of the kth control interval, the controller state is estimated with the following steps:

1. Obtain the following data:
   - $x_c(k|k–1)$ — Controller state estimate from previous control interval, k–1
   - $u^{act}(k–1)$ — Manipulated variable (MV) actually used in the plant from k–1 to k (assumed constant)

- $u^{opt}(k–1)$ — Optimal MV recommended by MPC and assumed to be used in the plant from $k–1$ to k
- $v(k)$ — Current measured disturbances
- $y_m(k)$ — Current measured plant outputs
- B_u, B_v — Columns of observer parameter B corresponding to $u(k)$ and $v(k)$ inputs
- C_m — Rows of observer parameter C corresponding to measured plant outputs
- D_{mv} — Rows and columns of observer parameter D corresponding to measured plant outputs and measured disturbance inputs
- L, M — Constant Kalman gain matrices

Plant input and output signals are scaled to be dimensionless prior to use in calculations.

2 Revise $x_c(k\,|\,k–1)$ when $u^{act}(k–1)$ and $u^{opt}(k–1)$ are different:

$$x_c^{rev}(k|k-1) = x_c(k|k-1) + B_u\left[u^{act}(k-1) - u^{opt}(k-1)\right].$$

3 Compute the innovation:

$$e(k) = y_m(k) - \left[C_m x_c^{rev}(k|k-1) + D_{mv}v(k)\right].$$

4 Update the controller state estimate to account for the latest measurements.

$$x_c(k|k) = x_c^{rev}(k|k-1) + Me(k).$$

Then, the software uses the current state estimate $x_c(k\,|\,k)$ to solve the quadratic program at interval k. The solution is $u^{opt}(k)$, the MPC-recommended manipulated-variable value to be used between control intervals k and k+1.

Finally, the software prepares for the next control interval assuming that the unknown inputs, $w_{id}(k)$, $w_{od}(k)$, and $w_n(k)$ assume their mean value (zero) between times k and k+1. The software predicts the impact of the known inputs and the innovation as follows:

$$x_c(k+1\,|\,k) = Ax_c^{rev}(k\,|\,k-1) + B_u u^{opt}(k) + B_v v(k) + Le(k).$$

Built-in Steady-State Kalman Gains Calculation

Model Predictive Control Toolbox software uses the `kalman` command to calculate Kalman estimator gains L and M. The following assumptions apply:

- State observer parameters A, B, C, D are time-invariant.
- Controller states, x_c, are detectable. (If not, or if the observer is numerically close to undetectability, the Kalman gain calculation fails, generating an error message.)
- Stochastic inputs $w_{id}(k)$, $w_{od}(k)$, and $w_n(k)$ are independent white noise, each with zero mean and identity covariance.
- Additional white noise $w_u(k)$ and $w_v(k)$ with the same characteristics adds to the dimensionless $u(k)$ and $v(k)$ inputs respectively. This improves estimator performance in certain cases, such as when the plant model is open-loop unstable.

Without loss of generality, set the $u(k)$ and $v(k)$ inputs to zero. The effect of the stochastic inputs on the controller states and measured plant outputs is:

$$\begin{aligned} x_c(k+1) &= Ax_c(k) + Bw(k) \\ y_m(k) &= C_m x_c(k) + D_m w(k). \end{aligned}$$

Here,

$$w^T(k) = \left[w_u^T(k) \quad w_v^T(k) \quad w_{id}^T(k) \quad w_{od}^T(k) \quad w_n^T(k) \right].$$

Inputs to the `kalman` command are the state observer parameters A, C_m, and the following covariance matrices:

$$\begin{aligned} Q &= E\left\{ Bww^T B^T \right\} = BB^T \\ R &= E\left\{ D_m ww^T D_m^T \right\} = D_m D_m^T \\ N &= E\left\{ Bww^T D_m^T \right\} = BD_m^T. \end{aligned}$$

Here, $E\{...\}$ denotes the expectation.

Output Variable Prediction

Model Predictive Control requires prediction of noise-free future plant outputs used in optimization. This is a key application of the state observer (see “State Observer” on page 2-41).

In control interval k, the required data are as follows:

- p — Prediction horizon (number of control intervals, which is greater than or equal to 1)
- $x_c(k|k)$ — Controller state estimates (see “State Estimation” on page 2-42)
- $v(k)$ — Current measured disturbance inputs (MDs)
- $v(k+i|k)$ — Projected future MDs, where i=1:p–1. If you are not using MD previewing, then $v(k+i|k) = v(k)$.
- A, B_u, B_v, C, D_v — State observer constants, where B_u, B_v, and D_v denote columns of the B and D matrices corresponding to inputs u and v. D_u is a zero matrix because of no direct feedthrough

Predictions assume that unknown white noise inputs are zero (their expectation). Also, the predicted plant outputs are to be noise-free. Thus, all terms involving the measurement noise states disappear from the state observer equations. This is equivalent to zeroing the last nxn elements of $x_c(k|k)$.

Given the above data and simplifications, for the first step the state observer predicts:

$$x_c(k+1|k) = Ax_c(k|k) + B_u u(k|k) + B_v v(k).$$

Continuing for successive steps, i = 2:p, the state observer predicts:

$$x_c(k+i|k) = Ax_c(k+i-1|k) + B_u u(k+i-1|k) + B_v v(k+i-1|k).$$

At any step, i = 1:p, the predicted noise-free plant outputs are:

$$y(k+i|k) = Cx_c(k+i|k) + D_v v(k+i|k).$$

All of these equations employ dimensionless plant input and output variables. See “Specifying Scale Factors” on page 1-2. The equations also assume zero offsets. Inclusion of nonzero offsets is straightforward.

For faster computations, the MPC controller uses an alternative form of the above equations in which constant terms are computed and stored during controller initialization. See “QP Matrices” on page 2-8.

More About

- “MPC Modeling”
- “Optimization Problem” on page 2-2

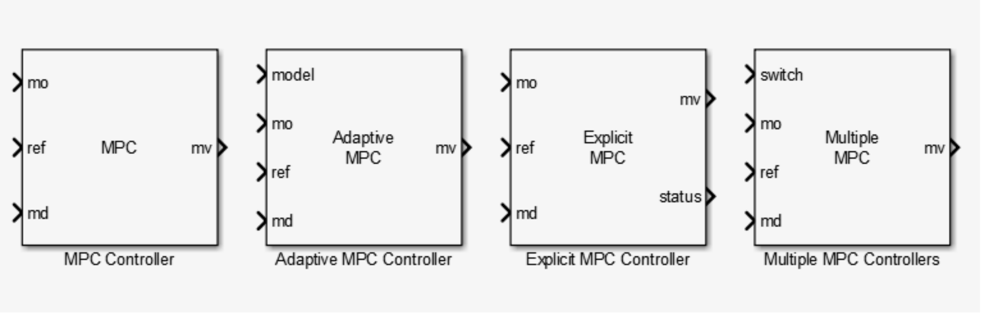
3

Model Predictive Control Simulink Library

MPC Library

The MPC Simulink Library provides two blocks you can use to implement MPC control in Simulink, MPC Controller, and Multiple MPC Controllers.

Access the library using the Simulink Library Browser or by typing `mpclib` at the command prompt. The library contains the following blocks:



MPC Simulink Library

For more information on each block, see their respective block reference pages:

- `MPC Controller`
- `Adaptive MPC Controller`
- `Explicit MPC Controller`
- `Multiple MPC Controllers`

Once you have access to the library, you can add one of its blocks to your Simulink model by clicking-and-dragging or copying-and-pasting.

Relationship of Multiple MPC Controllers to MPC Controller Block

The key difference between the Multiple MPC Controllers and the MPC Controller blocks is the way in which you designate the controllers to be used.

Listing the controllers

You must provide an ordered list containing N names, where N is the number of controllers and each name designates a valid MPC object in your base workspace. Each named controller must use the identical set of plant signals (for example, the same measured outputs and manipulated variables). See the `Multiple MPC Controllers` reference for more information on creating lists.

Designing the controllers

Use your knowledge of the process to identify distinct operating regions and design a controller for each. One convenient approach is to use the Simulink Control Design™ software to calculate each nominal operating point (typically a steady-state condition). Then, obtain a linear prediction model at this condition. To learn more, see the Simulink Control Design documentation. You must have a Simulink Control Design license to use this approach.

After the prediction models have been defined for each operating region, design each corresponding MPC Controller and give it a unique name in your base workspace.

Defining controller switching

Next, define the switching mechanism that will select among the controllers in real time. Add this mechanism to your Simulink model. For example, you could use one or more selected plant measurements to determine when each controller becomes active.

Your mechanism must define a scalar switching signal in the range 1 to N, where N is the number of controllers in your list. Connect this signal to the block's switch inport. Set it to `1` when you want the first controller in your list to become active, to `2` when the second is to become active, and so on.

Note: The Multiple MPC Controllers block automatically rounds the switching signal to the nearest integer. If the signal is outside the range 1 to N, none of the controllers activate and the block output is zero.

Improving prediction accuracy

During operation, all inactive controllers receive the current manipulated variable and measured output signals so they can update their state estimates. These updates minimize bumps during controller transitions. See "Bumpless Transfer Between Manual and Automatic Operations" on page 4-50 for more information. It is good practice to enable the **Externally supplied MV signal** option and feedback the actual manipulated variables measured in the plant to the `ext.mv` inport. This signal is provided to all the controllers in the block's list.

Generate Code and Deploy Controller to Real-Time Targets

After designing a controller in Simulink software using the MPC Controller block, you can generate code and deploy it for real-time control. You can deploy the controller to all targets supported by the following products:

- Simulink Coder™
- Embedded Coder®
- Simulink PLC Coder™
- Simulink Real-Time™

The sampling rate that a controller can achieve in real-time environment is system dependent. For example, for a typical small MIMO control application running on Simulink Real-Time, the sampling rate may go as low as 1–10ms. To determine the sampling rate, first test a less aggressive controller whose sampling rate produces acceptable performance on the target. Next, increase the sampling rate and monitor the execution time used by the controller. You can further decrease the sampling rate as long as the optimization safely completes within each sampling period under the normal plant operations.

Note: The MPC Controller block is implemented using the MATLAB Function block. To see the structure, right-click the block and select **Mask** > **Look Under Mask**. Open the `MPC` subsystem underneath.

See Also

`MPC Controller` | `Multiple MPC Controllers` | `review`

Related Examples

- "Simulation and Code Generation Using Simulink Coder" on page 4-94
- "Simulation and Structured Text Generation Using PLC Coder" on page 4-101

4

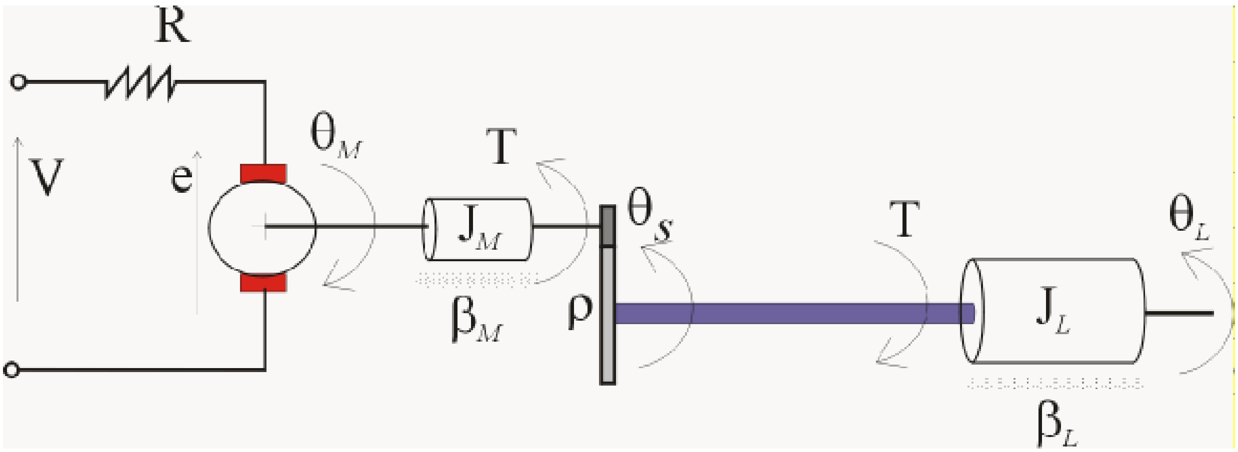
Case-Study Examples

Design MPC Controller for Position Servomechanism

This example shows how to design a model predictive controller for a position servomechanism using the MPC Designer app.

System Model

A position servomechanism consists of a DC motor, gearbox, elastic shaft, and load.



The differential equations representing this system are

$$\dot{\omega}_L = -\frac{k_T}{J_L}\left(\theta_L - \frac{\theta_M}{\rho}\right) - \frac{\beta_L}{J_L}\omega_L$$

$$\dot{\omega}_M = \frac{k_M}{J_M}\left(\frac{V - k_M\omega_M}{R}\right) - \frac{\beta_M\omega_M}{J_M} + \frac{k_T}{\rho J_M}\left(\theta_L - \frac{\theta_M}{\rho}\right)$$

where,

- V is the applied voltage.
- T is the torque acting on the load.
- $\omega_L = \dot{\theta}_L$ is the load angular velocity.

- $\omega_M = \dot{\theta}_M$ is the motor shaft angular velocity.

The remaining terms are constant parameters.

Constant Parameters for Servomechanism Model

Symbol	Value (SI Units)	Definition
k_T	1280.2	Torsional rigidity
k_M	10	Motor constant
J_M	0.5	Motor inertia
J_L	$50J_M$	Load inertia
ρ	20	Gear ratio
β_M	0.1	Motor viscous friction coefficient
β_L	25	Load viscous friction coefficient
R	20	Armature resistance

If you define the state variables as

$$x_p = [\theta_L \quad \omega_L \quad \theta_M \quad \omega_M]^T,$$

then you can model the servomechanism as an LTI state-space system.

$$\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_T}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_T}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_T}{\rho J_M} & 0 & -\frac{k_T}{\rho^2 J_M} & -\frac{\beta_M + \frac{k_M^2}{R}}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_M}{R J_M} \end{bmatrix} V$$

$$\theta_L = [1 \quad 0 \quad 0 \quad 0] x_p$$

$$T = \begin{bmatrix} k_T & 0 & -\frac{k_T}{\rho} & 0 \end{bmatrix} x_p$$

The controller must set the angular position of the load, θ_L, at a desired value by adjusting the applied voltage, V.

However, since the elastic shaft has a finite shear strength, the torque, T, must stay within the range $|T| \leq 78.5$ Nm. Also, the voltage source physically limits the applied voltage to the range $|V| \leq 220$ V.

Construct Plant Model

Specify the model constants.

```
Kt = 1280.2;  % Torsional rigidity
Km = 10;      % Motor constant
Jm = 0.5;     % Motor inertia
Jl = 50*Jm;   % Load inertia
N = 20;       % Gear ratio
Bm = 0.1;     % Rotor viscous friction
Bl = 25;      % Load viscous friction
R = 20;       % Armature resistance
```

Define the state-space matrices derived from the model equations.

```
A = [       0       1             0                0;
        -Kt/Jl  -Bl/Jl     Kt/(N*Jl)                0;
            0       0             0                1;
     Kt/(Jm*N)       0  -Kt/(Jm*N^2)  -(Bm+Km^2/R)/Jm];
B = [0; 0; 0; Km/(R*Jm)];
C = [  1  0       0  0;
      Kt  0   -Kt/N  0];
D = [0; 0];
```

Create a state-space model.

```
plant = ss(A,B,C,D);
```

Open MPC Designer App

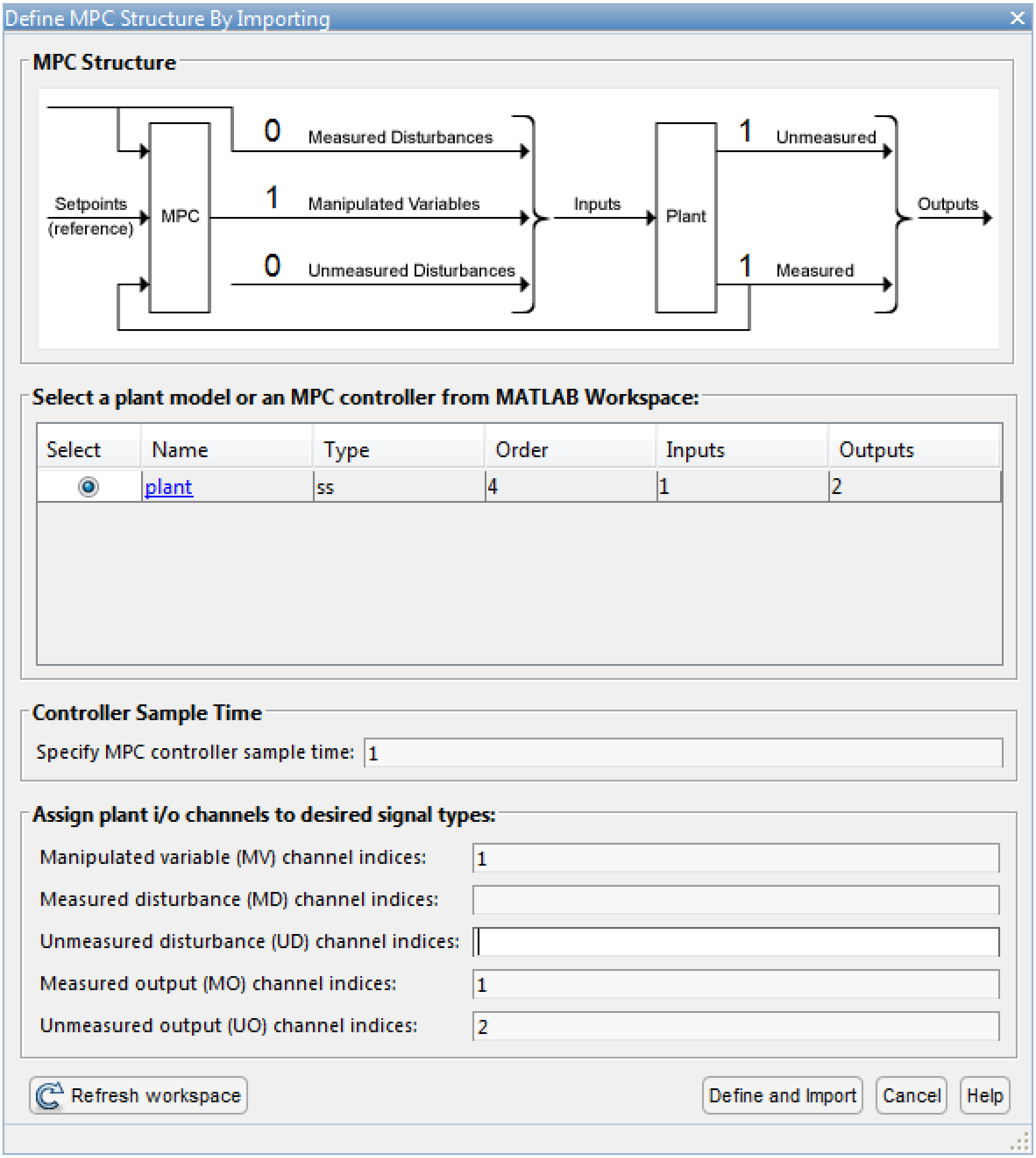
```
mpcDesigner
```

Import Plant and Define Signal Configuration

In the MPC Designer app, in the **MPC Designer** tab, select **MPC Structure**.

In the Define MPC Structure By Importing dialog box, select the `plant` plant model, and assign the plant I/O channels to the following signal types:

- Manipulated variable — Voltage, V
- Measured output — Load angular position, θ_L
- Unmeasured output — Torque, T


Define MPC Structure By Importing
MPC Structure
0 Measured Disturbances
1 Manipulated Variables
0 Unmeasured Disturbances
Setpoints
(reference)
MPC
Inputs
Plant
1 Unmeasured
1 Measured
Outputs
Select a plant model or an MPC controller from MATLAB Workspace:
Select
Name
Type
Order
Inputs
Outputs
plant
ss
4
1
2
Controller Sample Time
Specify MPC controller sample time: 1
Assign plant i/o channels to desired signal types:
Manipulated variable (MV) channel indices: 1
Measured disturbance (MD) channel indices:
Unmeasured disturbance (UD) channel indices:
Measured output (MO) channel indices: 1
Unmeasured output (UO) channel indices: 2
Refresh workspace
Define and Import
Cancel
Help

Click **Define and Import**.

The MPC Designer app imports the specified plant to the **Data Browser**. The following are also added to the **Data Browser**:

- `mpc1` — Default MPC controller created using `plant` as its internal model.
- `scenario1` — Default simulation scenario. The results of this simulation are displayed in the **Input Response** and **Output Response** plots.

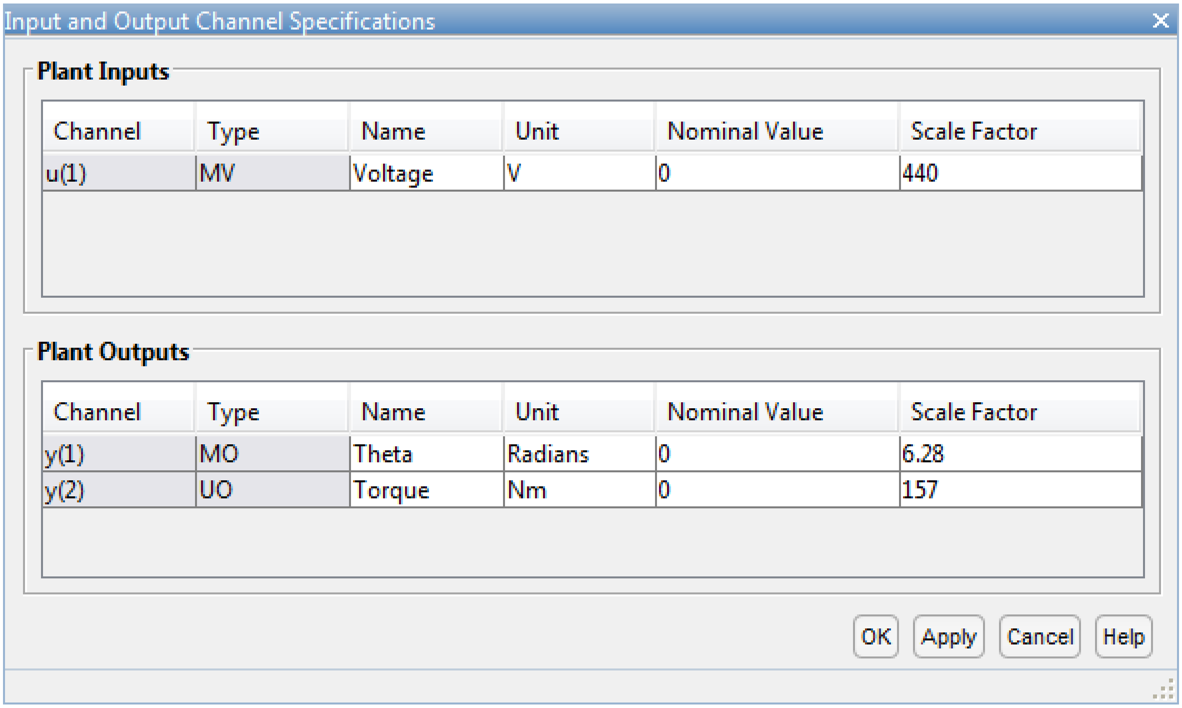
Define Input and Ouput Channel Attributes

On the **MPC Designer** tab, in the **Structure** section, click **I/O Attributes**.

In the Input and Output Channel Specifications dialog box, for each input and output channel:

- Specify a meaningful **Name** and **Unit**.
- Keep the **Nominal Value** at its default value of `0`.
- Specify a **Scale Factor** for normalizing the signal. Select a value that approximates the predicted operating range of the signal:

Channel Name	Minimum Value	Maximum Value	Scale Factor
`Voltage`	–220 V	220 V	`440`
`Theta`	–π radians	π radians	`6.28`
`Torque`	–78.5 Nm	78.5 Nm	`157`



Click **OK** to update the channel attributes and close the dialog box.

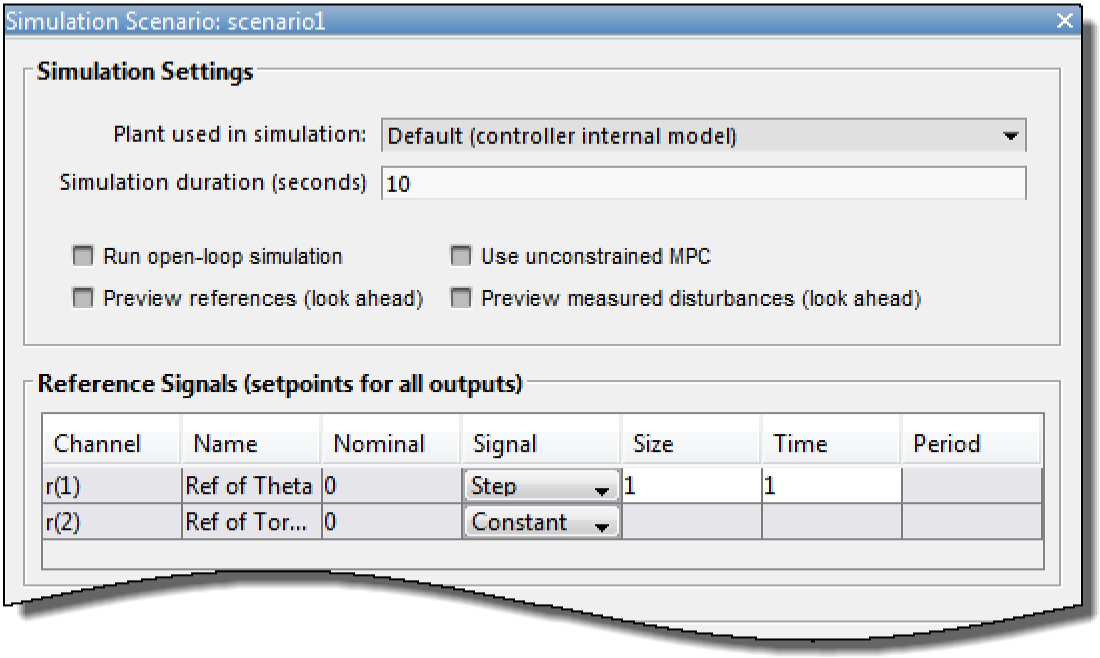
Modify Scenario To Simulate Angular Position Step Response

In the **Scenario** section, **Edit Scenario** drop-down list, select `scenario1` to modify the default simulation scenario.

In the Simulation Scenario dialog box, specify a **Simulation duration** of `10` seconds.

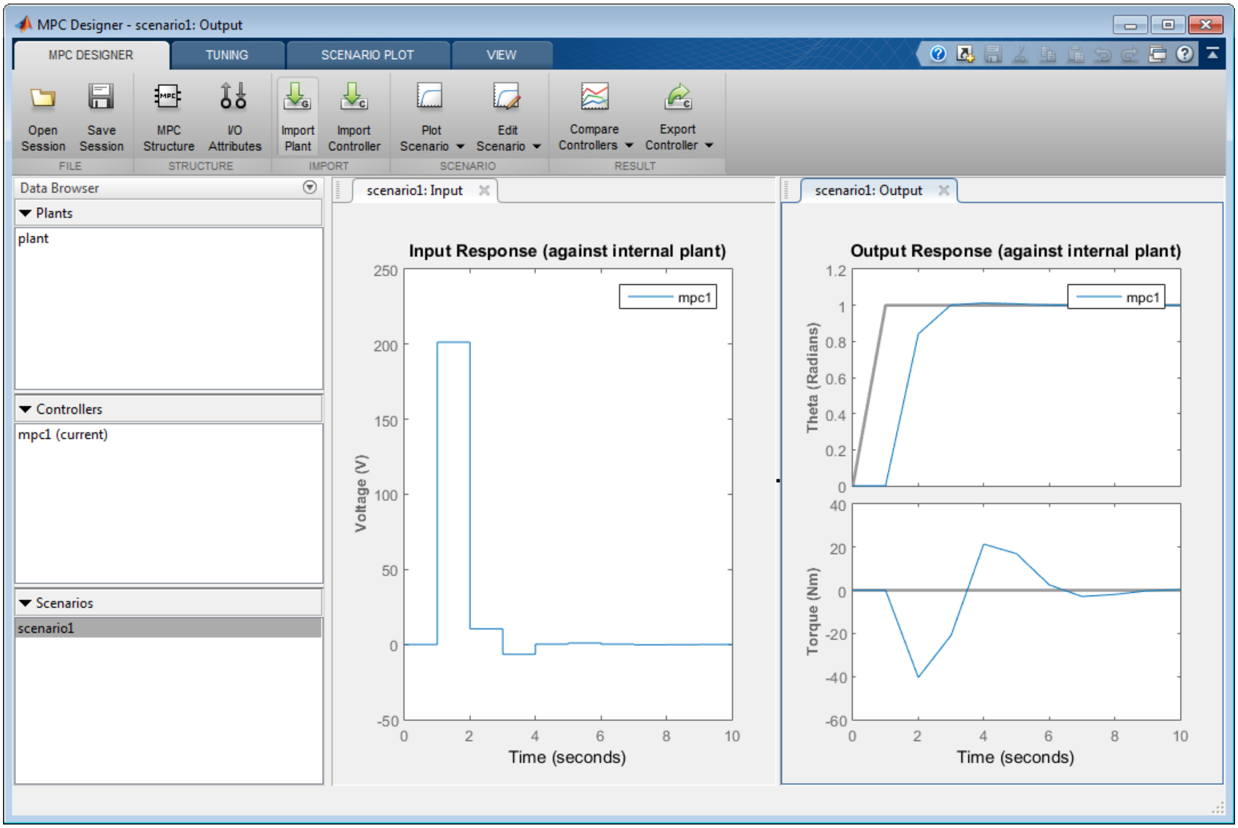
In the **Reference Signals** table, keep the default configuration for the first channel. These settings create a `Step` change of `1` radian in the angular position setpoint at a **Time** of `1` second.

For the second output, in the **Signal** drop-down list, select `Constant` to keep the torque setpoint at its nominal value.



Click **OK**.

The app runs the simulation with the new scenario settings and updates the **Input Response** and **Output Response** plots.



Specify Controller Sample Time and Horizons

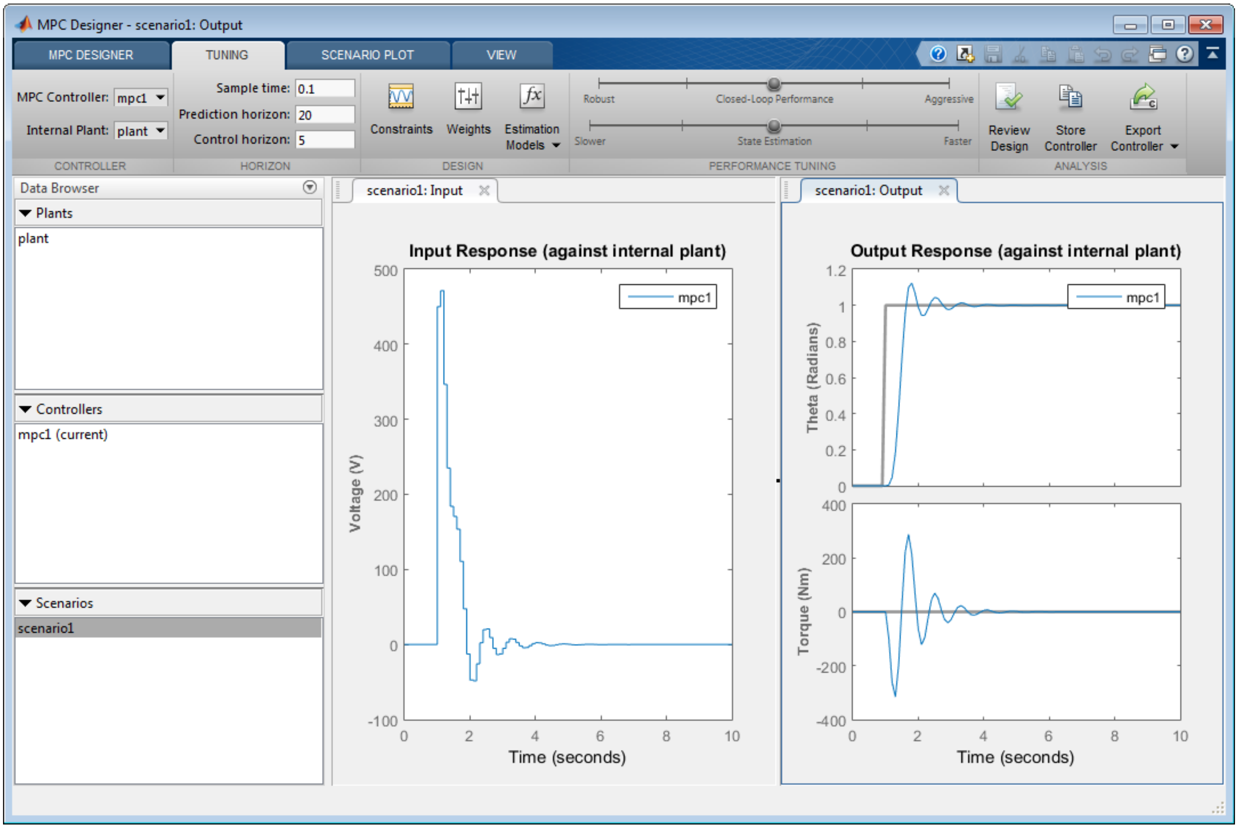
On the **Tuning** tab, in the **Horizon** section, specify a **Sample time** of `0.1` seconds.

For the specified sample time, T_s, and a desired response time of $T_r = 2$ seconds, select a prediction horizon, p, such that:

$$T_r \approx pT_s.$$

Therefore, specify a **Prediction horizon** of `20`.

Specify a **Control horizon** of `5`.



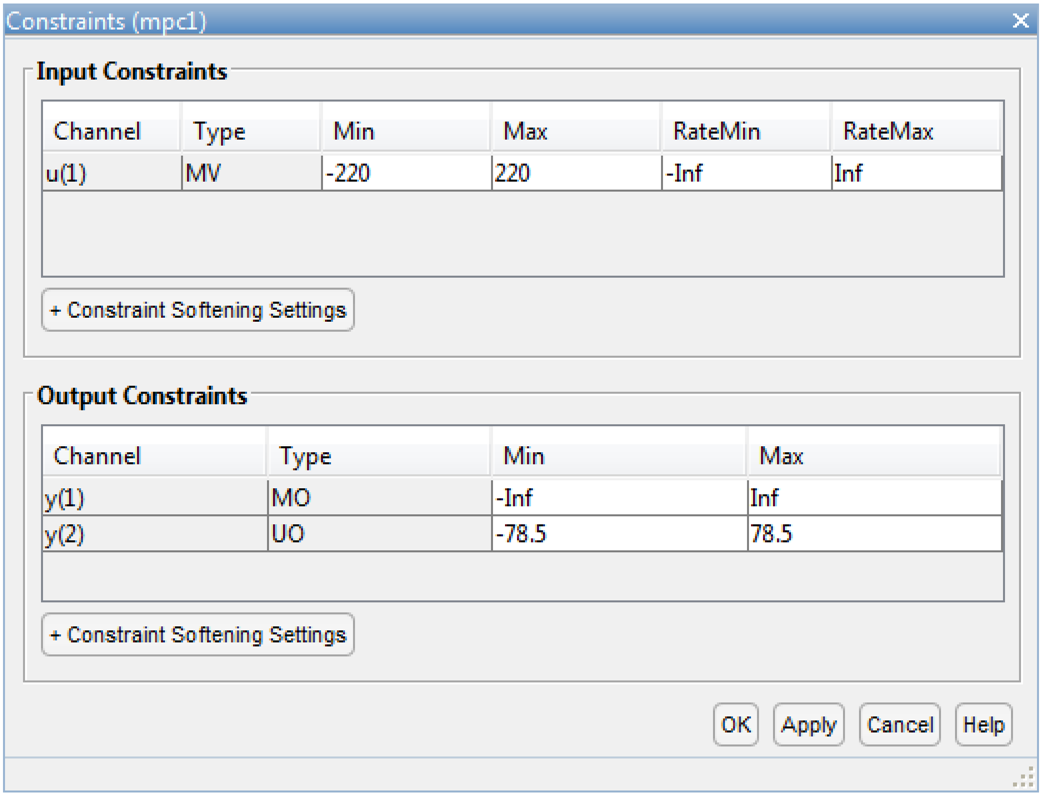
As you update the sample time and horizon values, the **Input Response** and **Output Response** plots update automatically. Both the input voltage and torque values exceed the constraints defined in the system model specifications.

Specify Constraints

In the **Design** section, select **Constraints**.

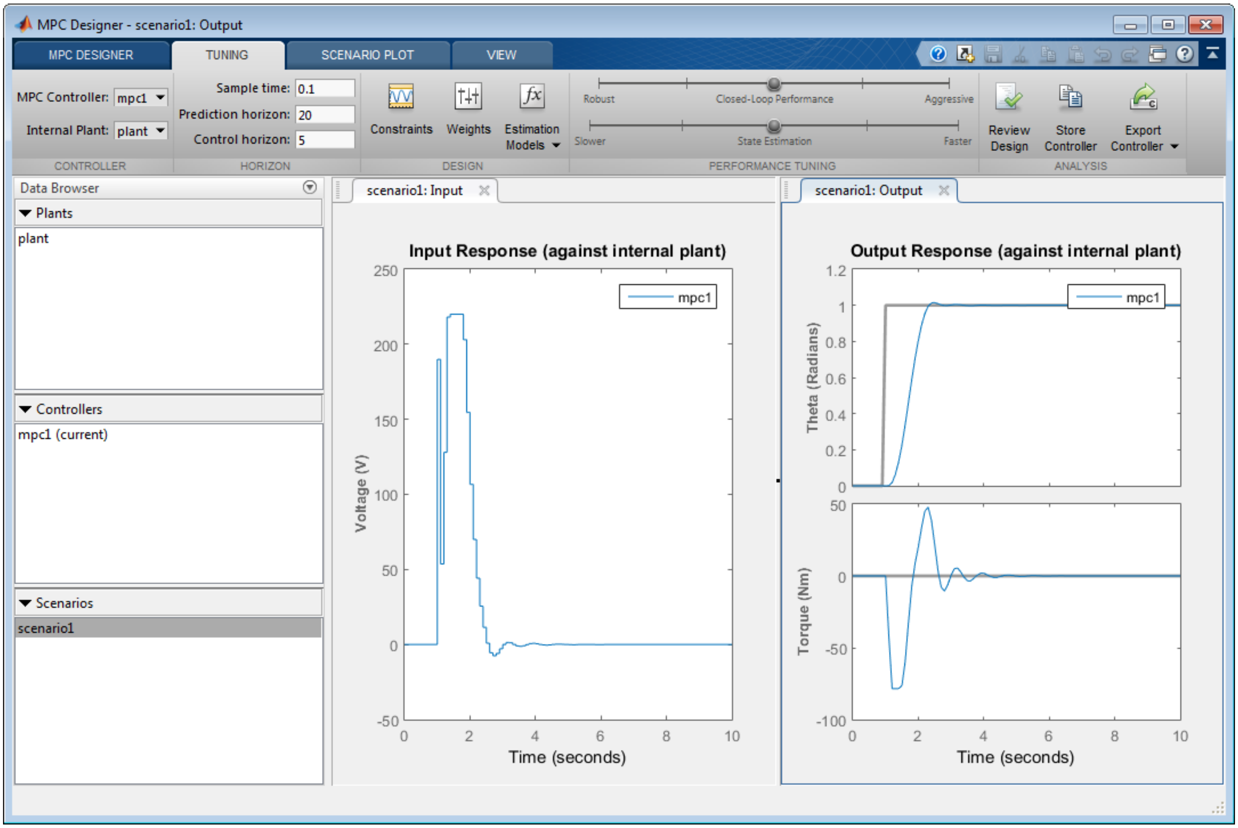
In the Constraints dialog box, in the **Input Constraints** section, specify the **Min** and **Max** voltage values for the manipulated variable (MV).

In the **Output Constraints** section, specify **Min** and **Max** torque values for the unmeasured output (UO).



There are no additional constraints, that is the other constraints remain at their default maximum and minimum values, `–Inf` and `Inf` respectively

Click **OK**.

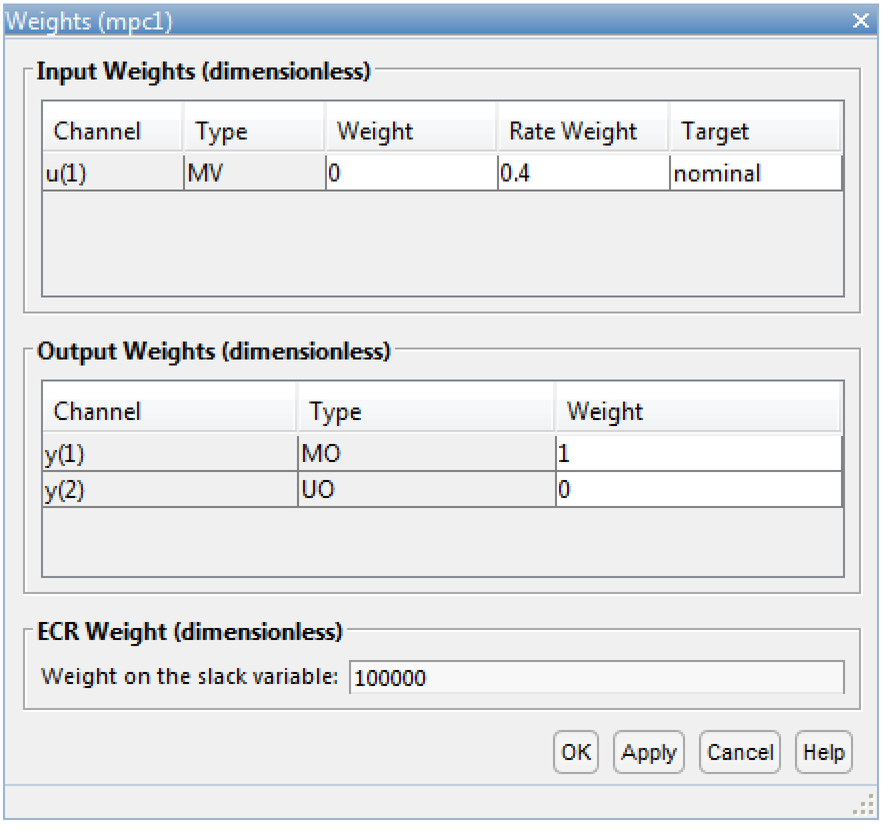


The response plots update to reflect the new constraints. In the **Input Response** plot, there are undesirable large changes in the input voltage.

Specify Tuning Weights

In the **Design** section, select **Weights**.

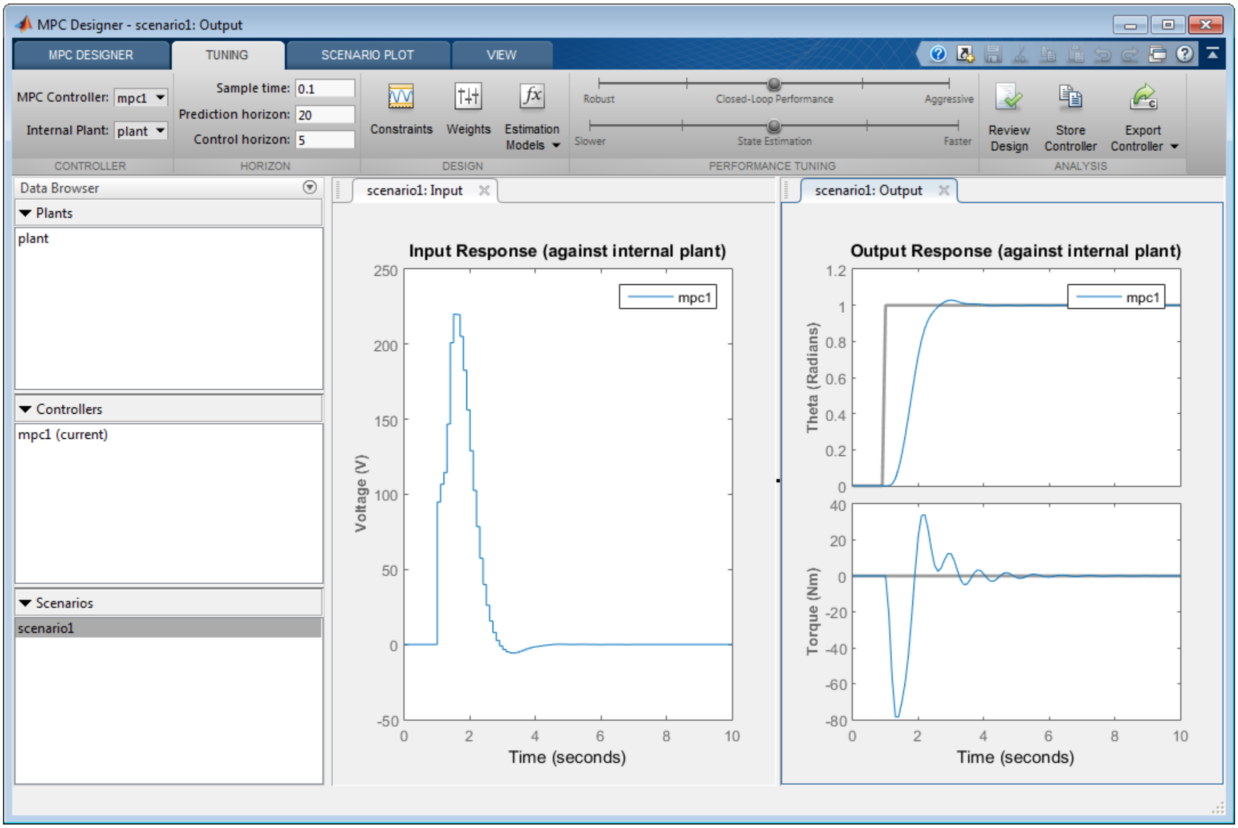
In the Weights dialog box, in the **Input Weights** table, increase the manipulated variable **Rate Weight**.



The tuning **Weight** for the manipulated variable (MV) is 0. This weight indicates that the controller can allow the input voltage to vary within its constrained range. The increased **Rate Weight** limits the size of manipulated variable changes.

Since the control objective is for the angular position of the load to track its setpoint, the tuning **Weight** on the measured output is 1. There is no setpoint for the applied torque, so the controller can allow the second output to vary within its constraints. Therefore, the **Weight** on the unmeasured output (UO) is 0, which enables the controller to ignore the torque setpoint.

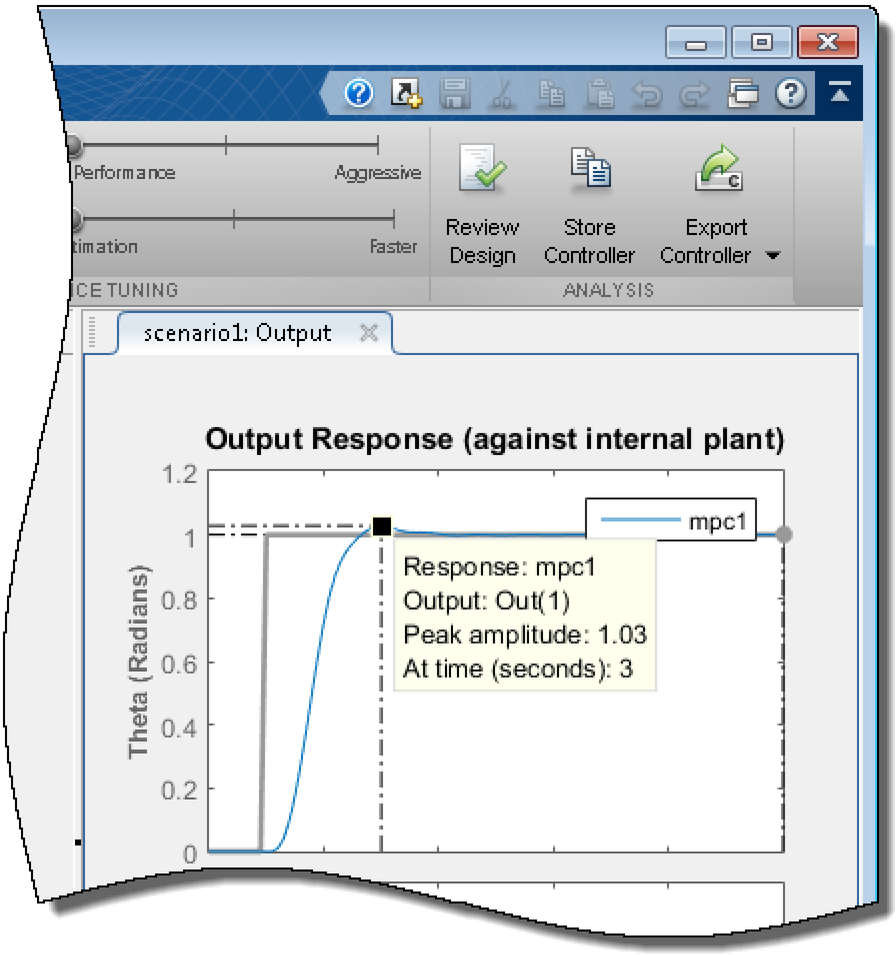
Click **OK**.



The response plots update to reflect the increased rate weight. The **Input Response** is smoother with smaller voltage changes.

Examine Output Response

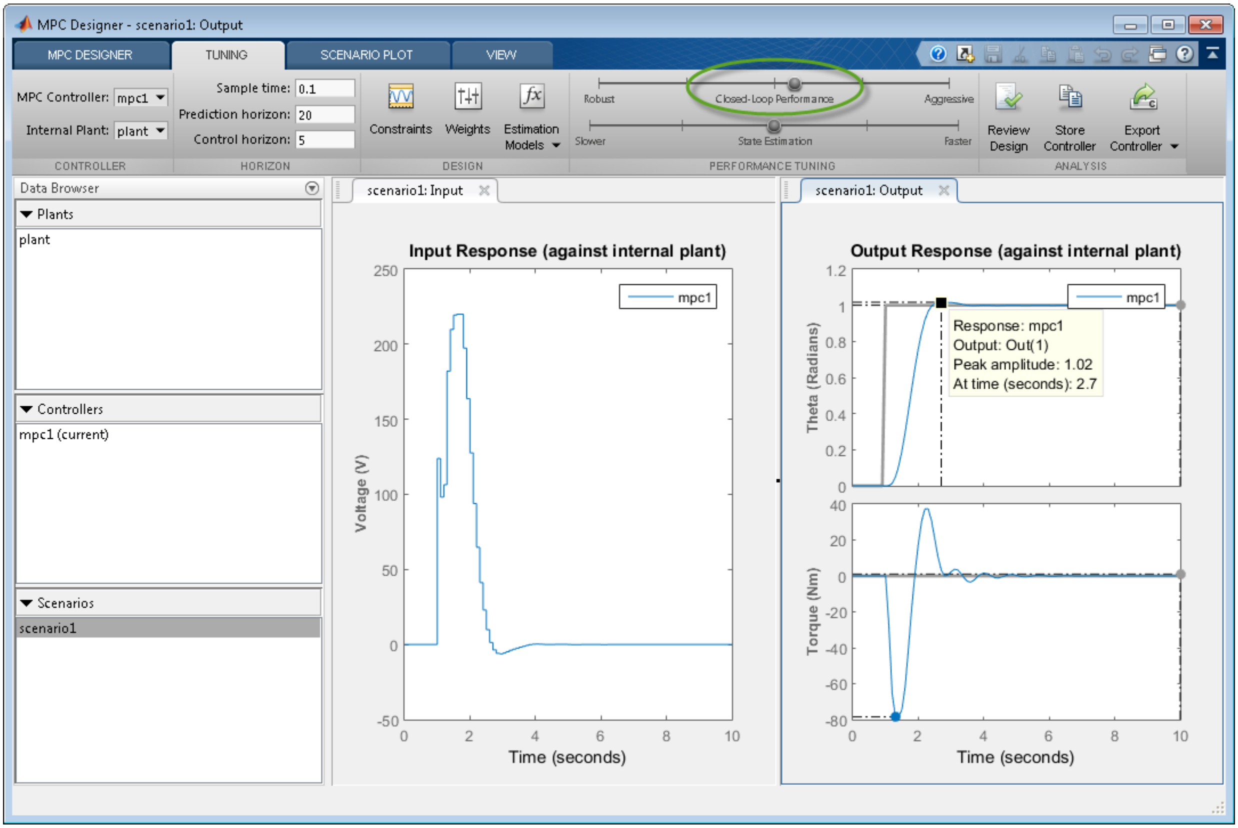
In the **Output Response** plot, right-click the **Theta** plot area, and select **Characteristics > Peak Response**.



The peak output response occurs at time of 3 seconds with a maximum overshoot of 3%. Since the reference signal step change is at 1 second, the controller has a peak time of 2 seconds.

Improve Controller Response Time

Click and drag the **Closed-Loop Performance** slider to the right to produce a more **Aggressive** response. The further you drag the slider to the right, the faster the controller responds. Select a slider position such that the peak response occurs at 2.7 seconds.



The final controller peak time is 1.7 seconds. Reducing the response time further results in overly-aggressive input voltage changes.

Generate and Run MATLAB Script

In the **Analysis** section, click the **Export Controller** arrow .

Under **Export Controller**, click `Generate Script`.

In the Generate MATLAB® Script dialog box, check the box next to `scenario1`.

Click **Generate Script**.

The app exports a copy of the plant model, `plant_C`, to the MATLAB workspace, along with simulation input and reference signals.

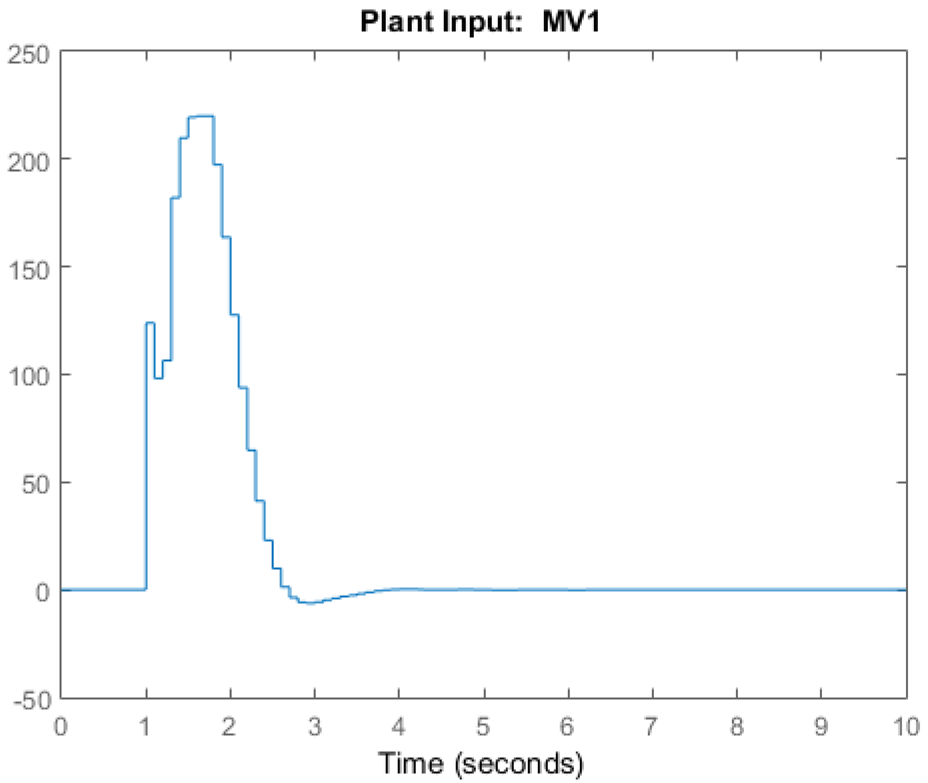
Additionally, the app generates the following code in the MATLAB Editor.

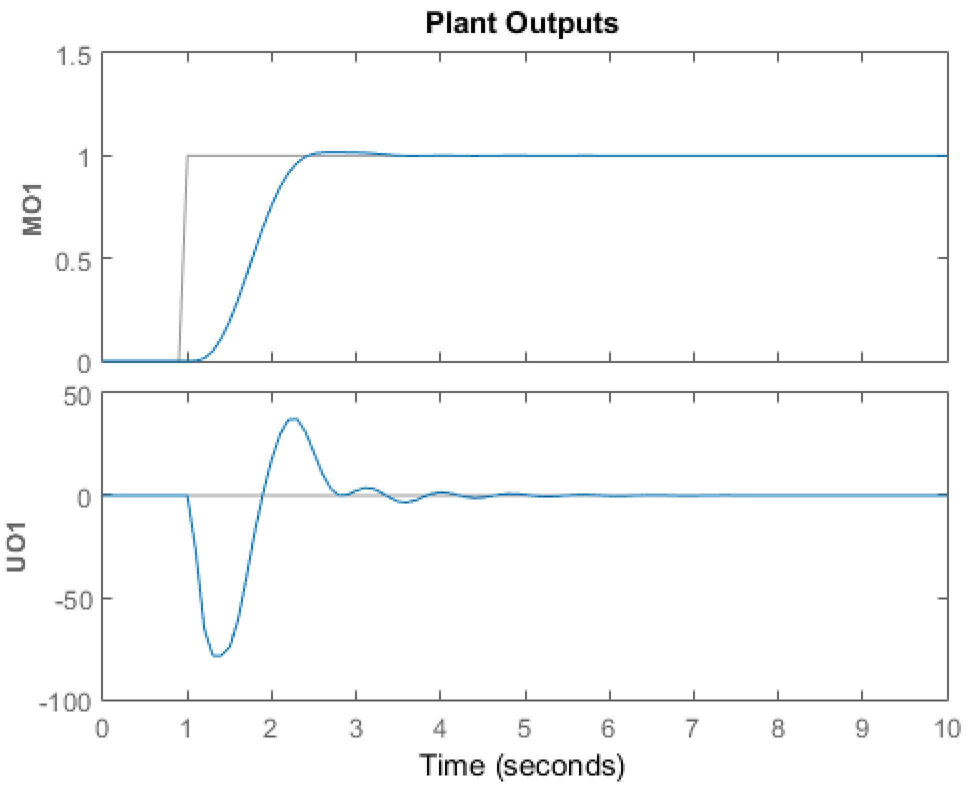
```
%% create MPC controller object with sample time
mpc1 = mpc(plant_C, 0.1);
%% specify prediction horizon
mpc1.PredictionHorizon = 20;
%% specify control horizon
mpc1.ControlHorizon = 5;
%% specify nominal values for inputs and outputs
mpc1.Model.Nominal.U = 0;
mpc1.Model.Nominal.Y = [0;0];
%% specify scale factors for inputs and outputs
mpc1.MV(1).ScaleFactor = 440;
mpc1.OV(1).ScaleFactor = 6.28;
mpc1.OV(2).ScaleFactor = 157;
%% specify constraints for MV and MV Rate
mpc1.MV(1).Min = -220;
mpc1.MV(1).Max = 220;
%% specify constraints for OV
mpc1.OV(2).Min = -78.5;
mpc1.OV(2).Max = 78.5;
%% specify overall adjustment factor applied to weights
beta = 1.2712;
%% specify weights
mpc1.Weights.MV = 0*beta;
mpc1.Weights.MVRate = 0.4/beta;
mpc1.Weights.OV = [1 0]*beta;
mpc1.Weights.ECR = 100000;
%% specify simulation options
options = mpcsimopt();
options.RefLookAhead = 'off';
options.MDLookAhead = 'off';
options.Constraints = 'on';
options.OpenLoop = 'off';
%% run simulation
sim(mpc1, 101, mpc1_RefSignal, mpc1_MDSignal, options);
```

In the MATLAB Window, in the **Editor** tab, select **Save**.

Complete the Save dialog box and then click **Save**.

In the **Editor** tab, click **Run**.



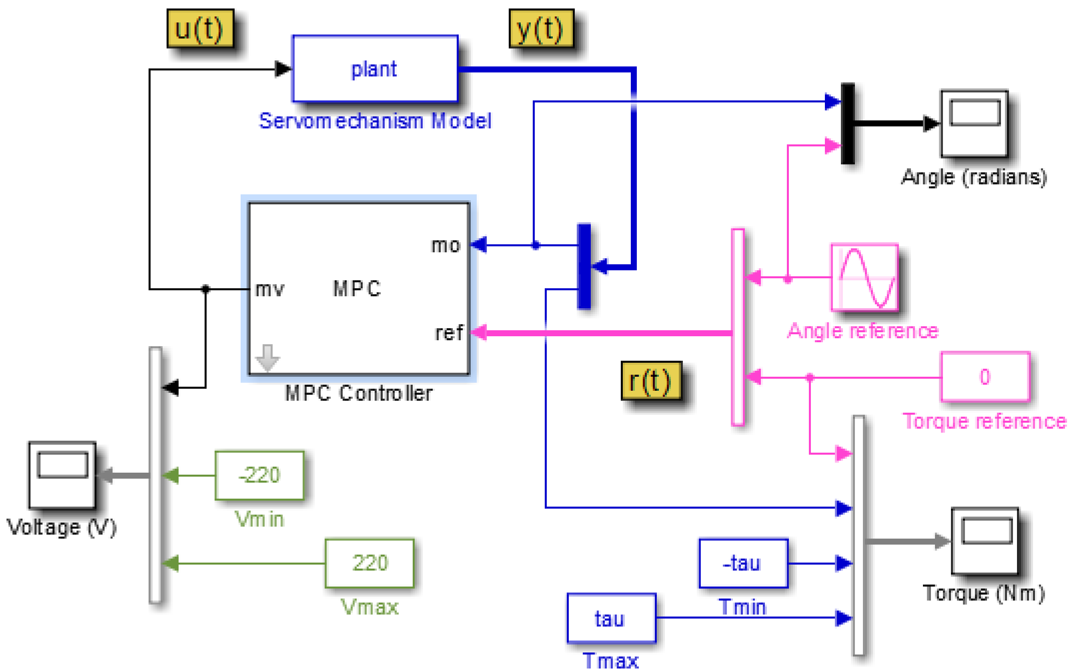


The script creates the controller, `mpc1`, and runs the simulation scenario. The input and output responses match the simulation results from the app.

Validate Controller Performance In Simulink

Open the servomechanism Simulink model.

```
open_system('mpc_motor');
```

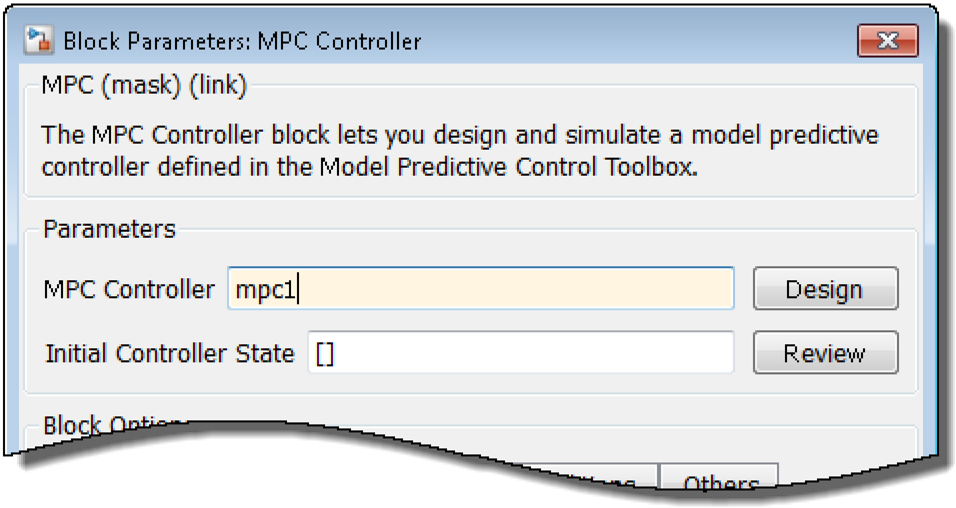


This model uses an MPC Controller block to control a servomechanism plant. The Servomechanism Model block is already configured to use the `plant` model from the MATLAB workspace.

The Angle reference source block creates a sinusoidal reference signal with a frequency of `0.4` rad/sec and an amplitude of π.

Double-click the MPC Controller block.

In the MPC Controller Block Parameters dialog box, specify an **MPC Controller** from the MATLAB workspace. Use the `mpc1` controller created using the generated script.



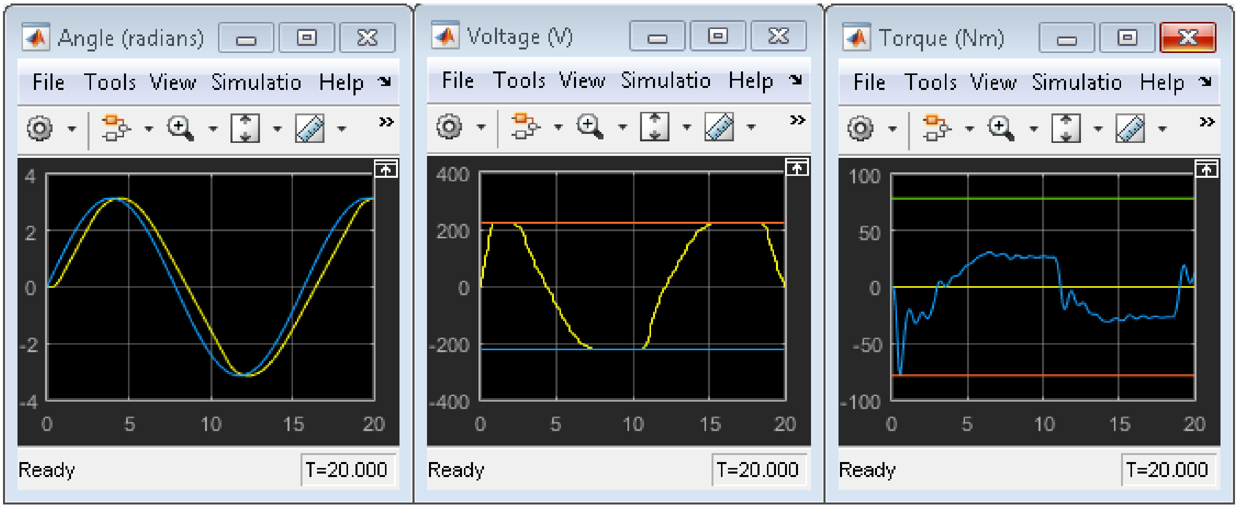
Click **OK**.

At the MATLAB command line, specify a torque magnitude constraint variable.

```
tau = 78.5;
```

The model uses this value to plot the constraint limits on the torque output scope.

In the Simulink model window, click **Run** to simulate the model.



In the **Angle** scope, the output response, yellow, tracks the angular position setpoint, blue, closely.

See Also

`mpc` | `MPC Controller` | MPC Designer

Related Examples

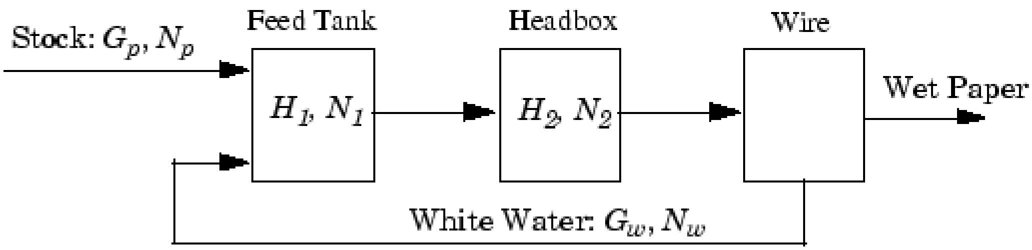
- "Design Controller Using MPC Designer"
- "Design MPC Controller at the Command Line"

Design MPC Controller for Paper Machine Process

This example shows how to design a model predictive controller for a nonlinear paper machine process using the MPC Designer app.

System Model

Ying *et al.* studied the control of consistency (percentage of pulp fibers in aqueous suspension) and liquid level in a paper machine headbox.



The process is nonlinear and has three outputs, two manipulated inputs, and two disturbance inputs, one of which is measured for feedforward control.

The process model is a set of ordinary differential equations (ODEs) in bilinear form. The states are

$$x = [H_1 \quad H_2 \quad N_1 \quad N_2]^T$$

- H_1 — Feed tank liquid level
- H_2 — Headbox liquid level
- N_1 — Feed tank consistency
- N_2 — Headbox consistency

The primary control objective is to hold H_2 and N_2 at their setpoints by adjusting the manipulated variables:

- G_p — Flow rate of stock entering the feed tank
- G_w — Flow rate of recycled white water

The consistency of stock entering the feed tank, N_p, is a measured disturbance, and the white water consistency, N_w, is an unmeasured disturbance.

All signals are normalized with zero nominal steady-state values and comparable numerical ranges. The process is open-loop stable.

The measured outputs are H_2, N_1, and N_2.

The Simulink S-function, `mpc_pmmodel` implements the nonlinear model equations. To view this S-function, enter the following.

```
edit mpc_pmmodel
```

Construct Plant Model

To design a controller for a nonlinear plant using MPC Designer, you must first obtain a linear model of the plant. The paper machine headbox model can be linearized analytically.

At the MATLAB command line, enter the state-space matrices for the linearized model.

```
A = [-1.9300         0         0         0
      0.3940   -0.4260         0         0
           0         0   -0.6300         0
      0.8200   -0.7840    0.4130   -0.4260];
B = [1.2740    1.2740         0         0
          0         0         0         0
     1.3400   -0.6500    0.2030    0.4060
          0         0         0         0];
C = [0    1.0000         0         0
     0         0    1.0000         0
     0         0         0    1.0000];
D = zeros(3,4);
```

Create a continuous-time LTI state-space model.

```
PaperMach = ss(A,B,C,D);
```

Specify the names of the input and output channels of the model.

```
PaperMach.InputName = {'G_p','G_w','N_p','N_w'};
PaperMach.OutputName = {'H_2','N_1','N_2'};
```

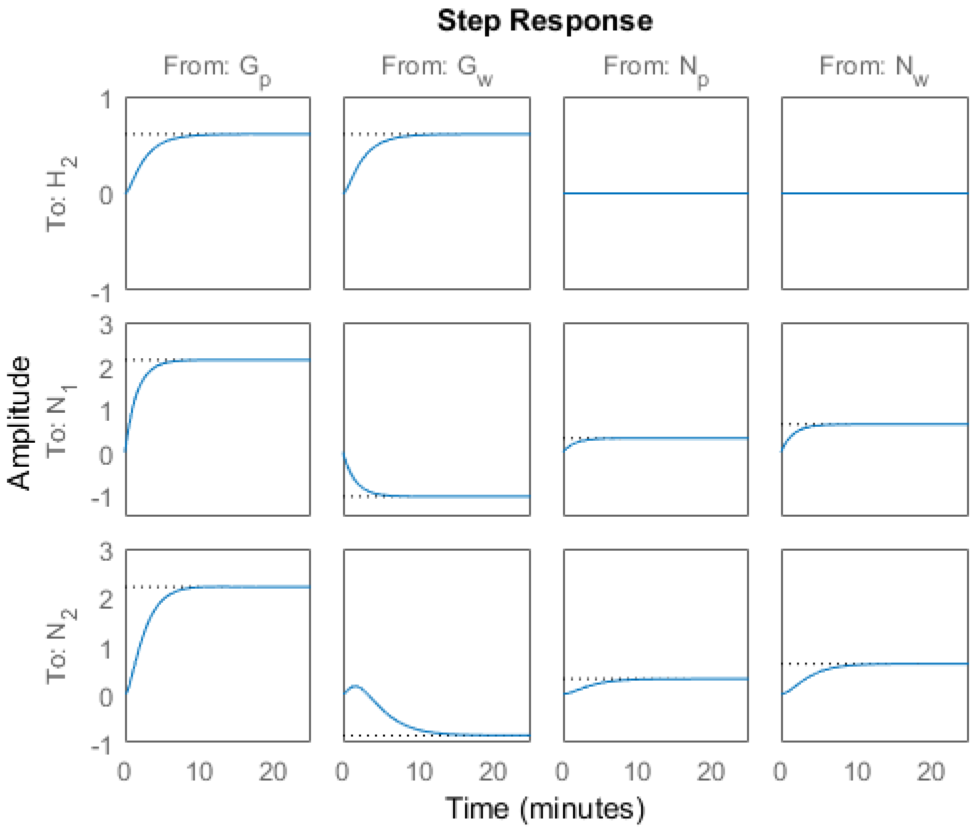
Specify the model time units.

```
PaperMach.TimeUnit = 'minutes';
```

Plot Linear Model Step Response

Examine the open-loop response of the plant.

```
step(PaperMach);
```



The step response shows that:

- Both manipulated variables, G_p and G_w, affect all three outputs.
- The manipulated variables have nearly identical effects on H_2.
- The response from G_w to N_2 is an inverse response.

These features make it difficult to achieve accurate, independent control of H_2 and N_2.

Open MPC Designer App

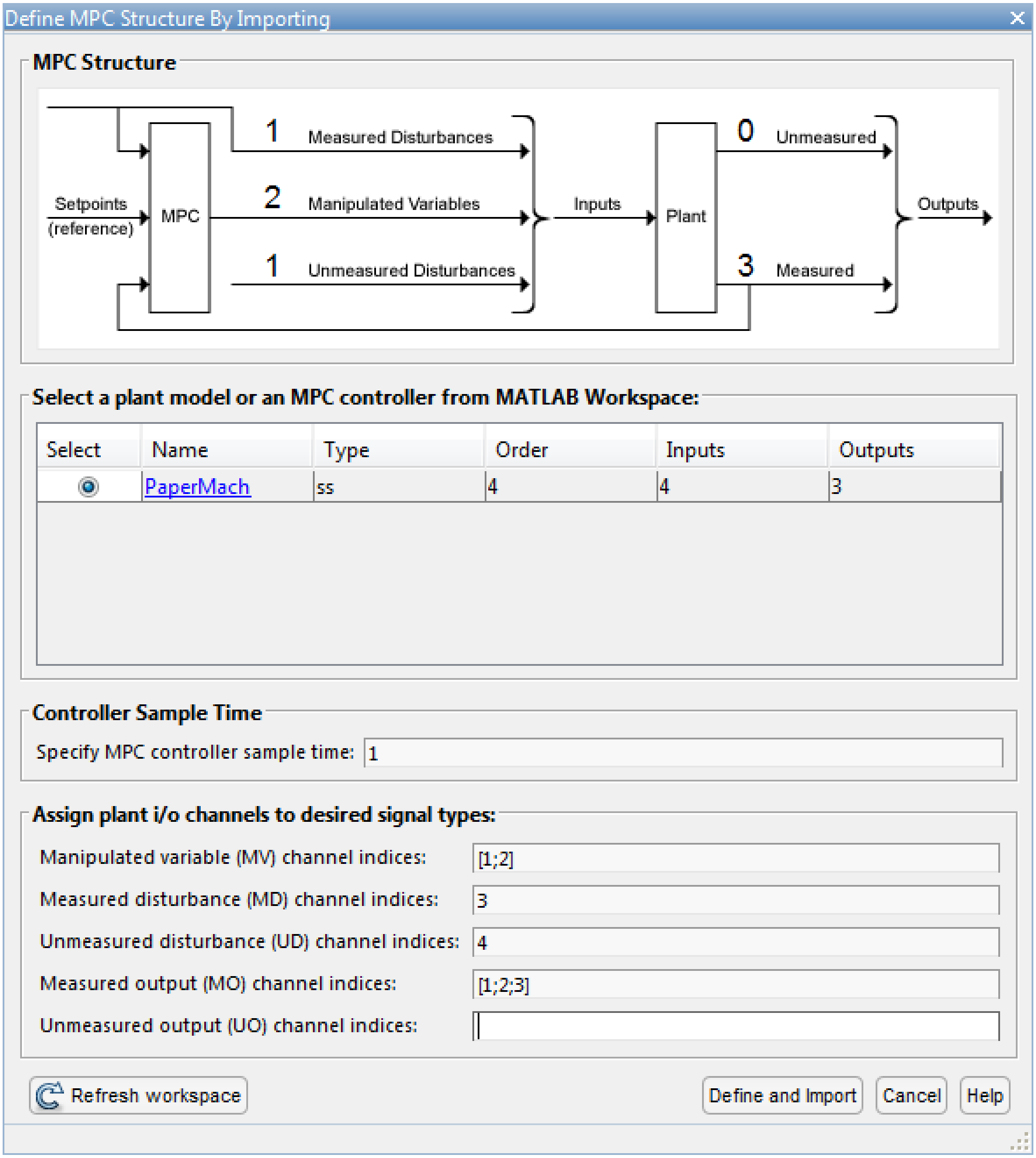
```
mpcDesigner
```

Import Plant Model and Define Signal Configuration

In the MPC Designer app, on the **MPC Designer** tab, in the **Structure** section, click **MPC Structure**.

In the Define MPC Structure By Importing dialog box, select the `PaperMach` plant model and assign the plant I/O channels to the following signal types:

- Manipulated variables — G_p and G_w
- Measured disturbance — N_p
- Unmeasured disturbance — N_w
- Measured outputs — H_2, N_2, and H_2


Define MPC Structure By Importing
MPC Structure
1 Measured Disturbances
2 Manipulated Variables
1 Unmeasured Disturbances
Setpoints
(reference)
MPC
Inputs
Plant
0 Unmeasured
3 Measured
Outputs
Select a plant model or an MPC controller from MATLAB Workspace:
Select
Name
Type
Order
Inputs
Outputs
PaperMach
ss
4
4
3
Controller Sample Time
Specify MPC controller sample time:
1
Assign plant i/o channels to desired signal types:
Manipulated variable (MV) channel indices:
[1;2]
Measured disturbance (MD) channel indices:
3
Unmeasured disturbance (UD) channel indices:
4
Measured output (MO) channel indices:
[1;2;3]
Unmeasured output (UO) channel indices:
Refresh workspace
Define and Import
Cancel
Help

Tip To find the correct channel indices, click the `PaperMach` model **Name** to view additional model details.

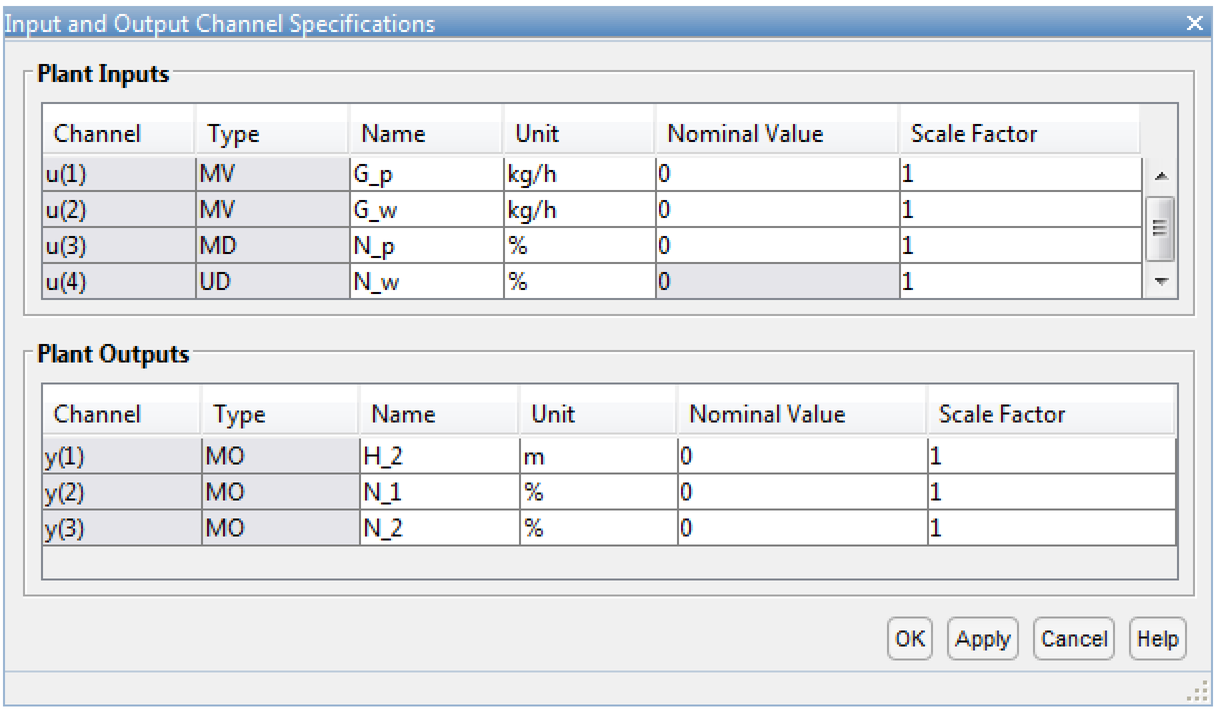
Click **Define and Import**.

The app imports the plant to the **Data Browser** and creates a default MPC controller using the imported plant.

Define Input and Output Channel Attributes

In the **Structure** section, select **I/O Attributes**.

In the Input and Output Channel Specifications dialog box, in the **Unit** column, define the units for each channel. Since all the signals are normalized with zero nominal steady-state values, keep the **Nominal Value** and **Scale Factor** for each channel at their default values.



Click **OK** to update the channel attributes and close the dialog box.

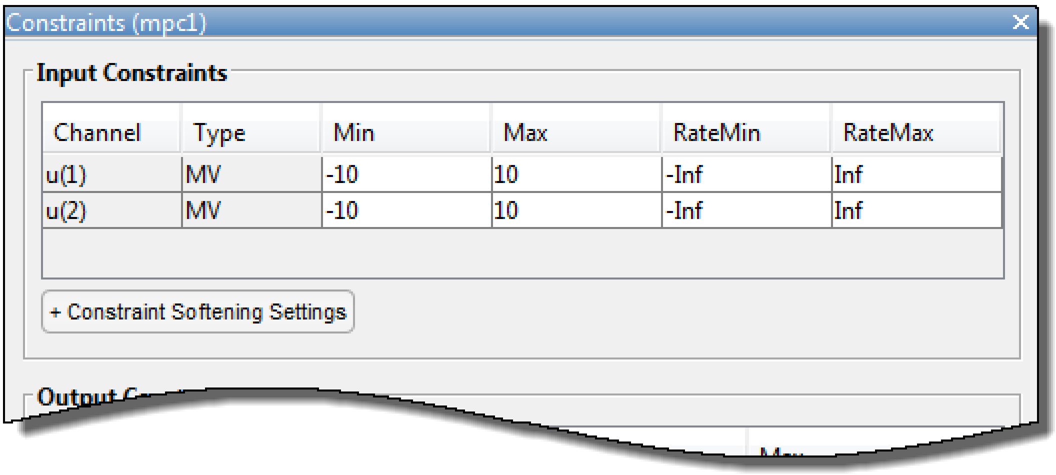
Specify Controller Sample Time and Horizons

On the **Tuning** tab, in the **Horizon** section, keep the **Sample time**, **Prediction Horizon**, and **Control Horizon** at their default values.

Specify Manipulated Variable Constraints

In the **Design** section, click **Constraints**.

In the Constraints dialog box, in the **Input Constraints** section, specify value constraints, **Min** and **Max**, for both manipulated variables.



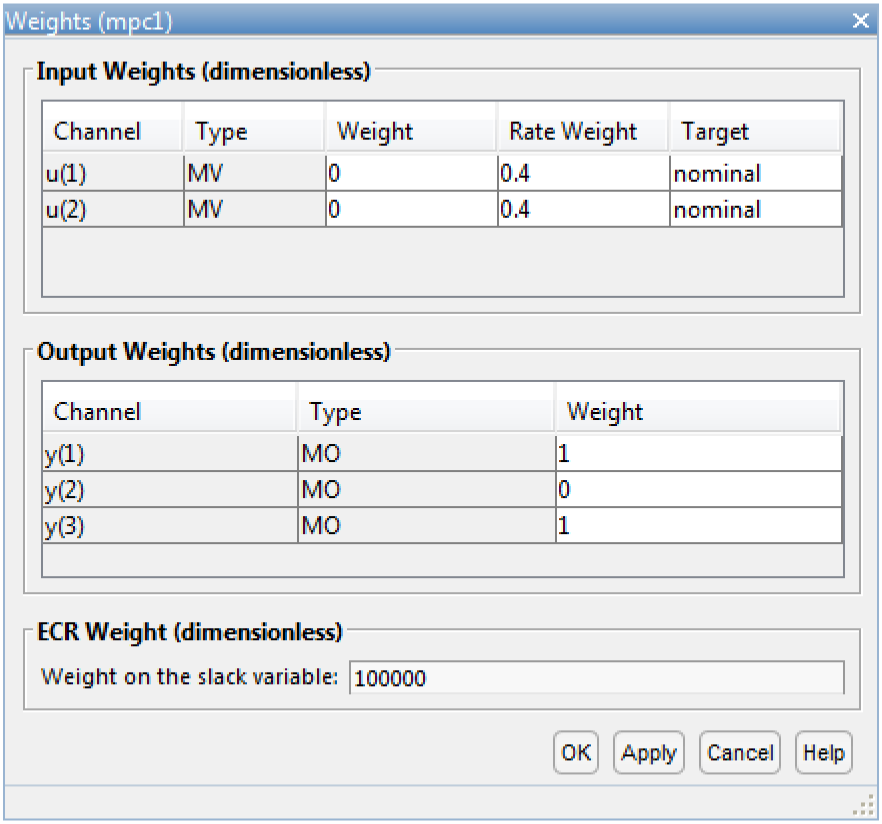
Click **OK**.

Specify Initial Tuning Weights

In the **Design** section, click **Weights**.

In the Weights dialog box, in the **Input Weights** section, increase the **Rate Weight** to `0.4` for both manipulated variables.

In the **Output Weights** section, specify a **Weight** of `0` for the second output, N_1, and a **Weight** of `1` for the other outputs.



Increasing the rate weight for manipulated variables prevents overly-aggressive control actions resulting in a more conservative controller response.

Since there are two manipulated variables, the controller cannot control all three outputs completely. A weight of zero indicates that there is no setpoint for N_1. As a result, the controller can hold H_2 and N_2 at their respective setpoints.

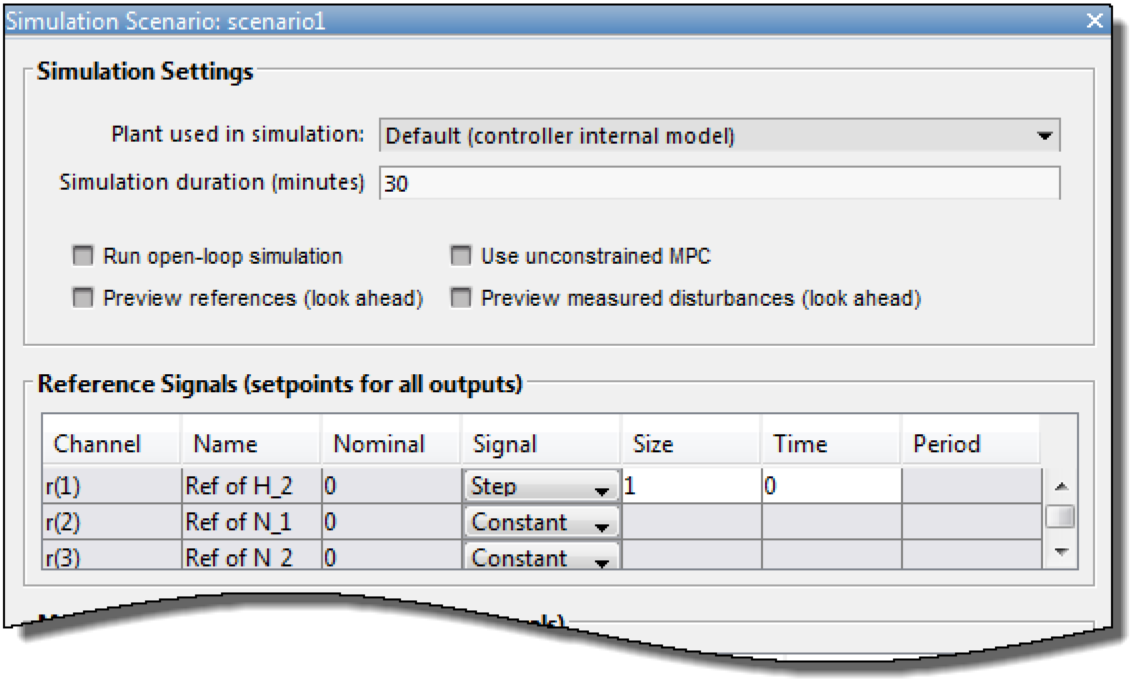
Simulate H_2 Setpoint Step Response

On the **MPC Designer** tab, in the **Scenario** section, click **Edit Scenario** > **scenario1**.

In the Simulation Scenario dialog box, specify a **Simulation duration** of `30` minutes.

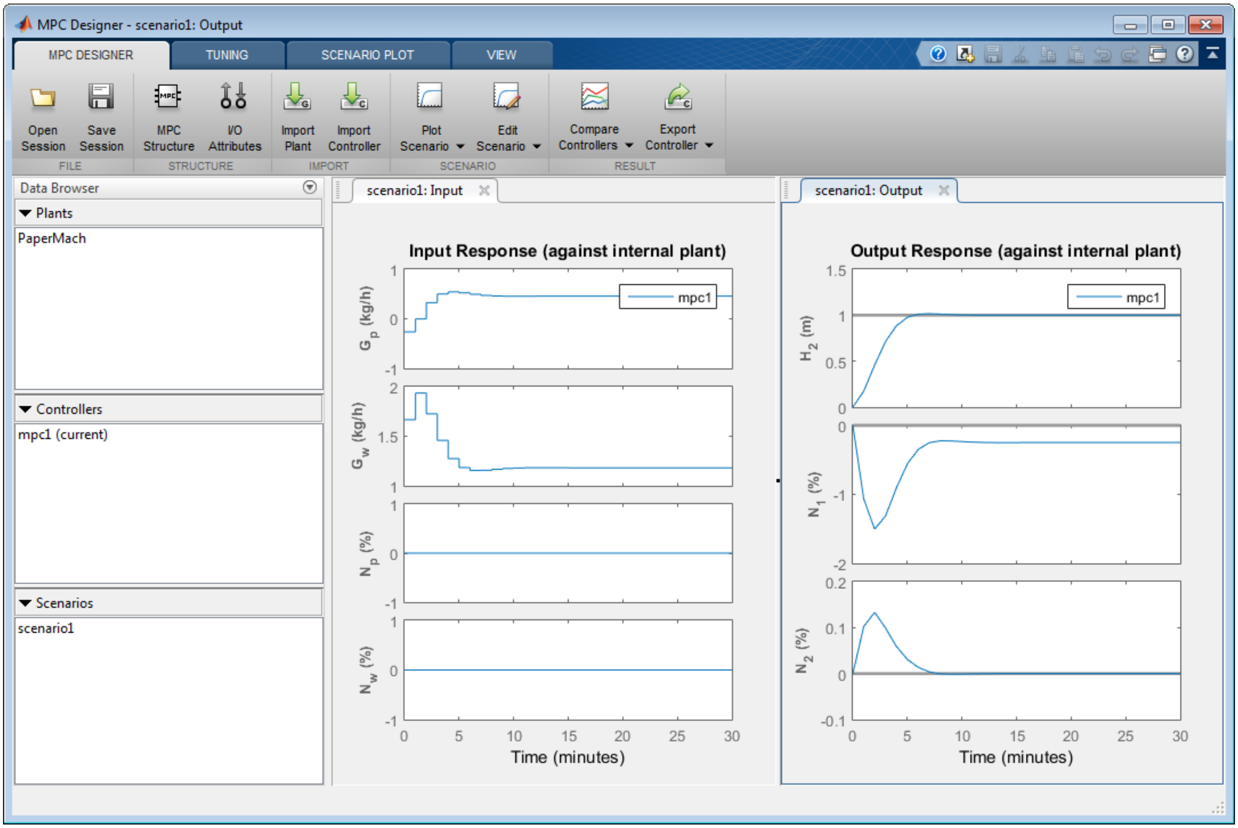
In the **Reference Signals** table, in the **Signal** drop-down list, select `Step` for the first output. Keep the step **Size** at `1` and specify a step **Time** of `0`.

In the **Signal** drop-down lists for the other output reference signals, select `Constant` to hold the values at their respective nominal values. The controller ignores the setpoint for the second output since the corresponding tuning weight is zero.



Click **OK**.

The app runs the simulation with the new scenario settings and updates the **Input Response** and **Output Response** plots.



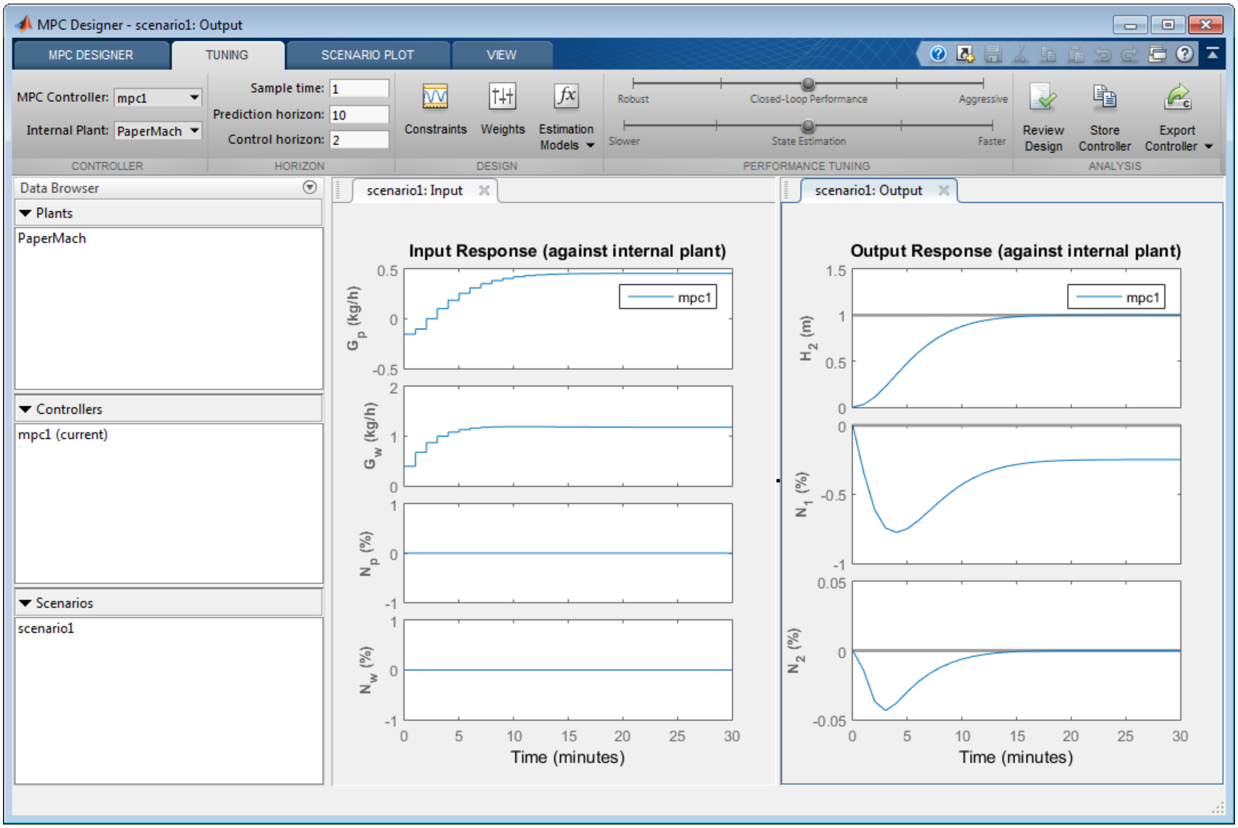
The initial design uses a conservative control effort to produce a robust controller. The response time for output H_2 is about 7 minutes. To reduce this response time, you can decrease the sample time, reduce the manipulated variable rate weights, or reduce the manipulated variable rate constraints.

Since the tuning weight for output N_1 is zero, its output response shows a steady-state error of about –`0.25`.

Adjust Weights to Emphasize Feed Tank Consistency Control

On the **Tuning** tab, in the **Design** section, select **Weights**.

In the Weights dialog box, in the **Output Weights** section, specify a **Weight** of `0.2` for the first output, H_2.



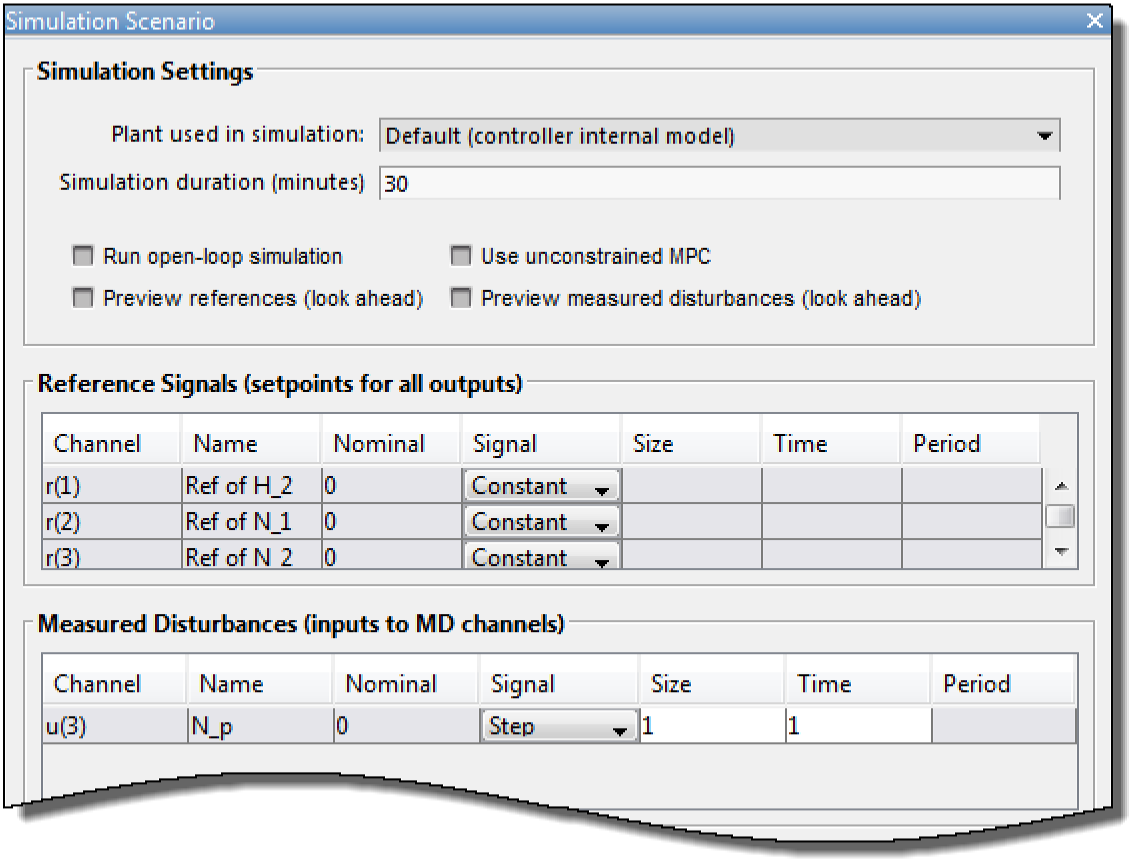
The controller places more emphasis on eliminating errors in feed tank consistency, N_2, which significantly decreases the peak absolute error. The trade-off is a longer response time of about 17 minutes for the feed tank level, H_2.

Test Controller Feedforward Response to Measured Disturbances

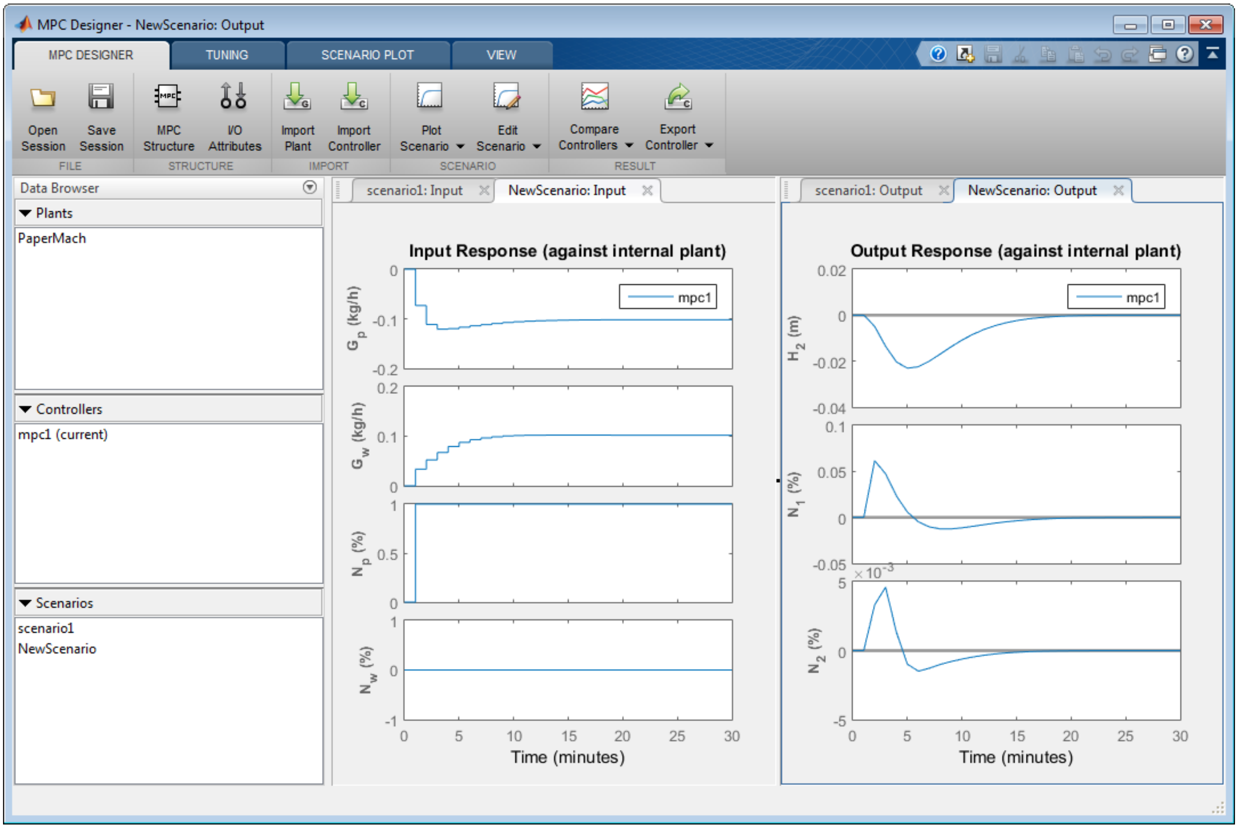
On the **MPC Designer** tab, in the **Scenario** section, click **Plot Scenario** > **New Scenario**.

In the Simulation Scenario dialog box, specify a **Simulation duration** of 30 minutes.

In the **Measured Disturbances** table, specify a step change in measured disturbance, N_p, with a **Size** of 1 and a step **Time** of 1. Keep all output setpoints constant at their nominal values.



Click **OK** to run the simulation and display the input and output response plots.



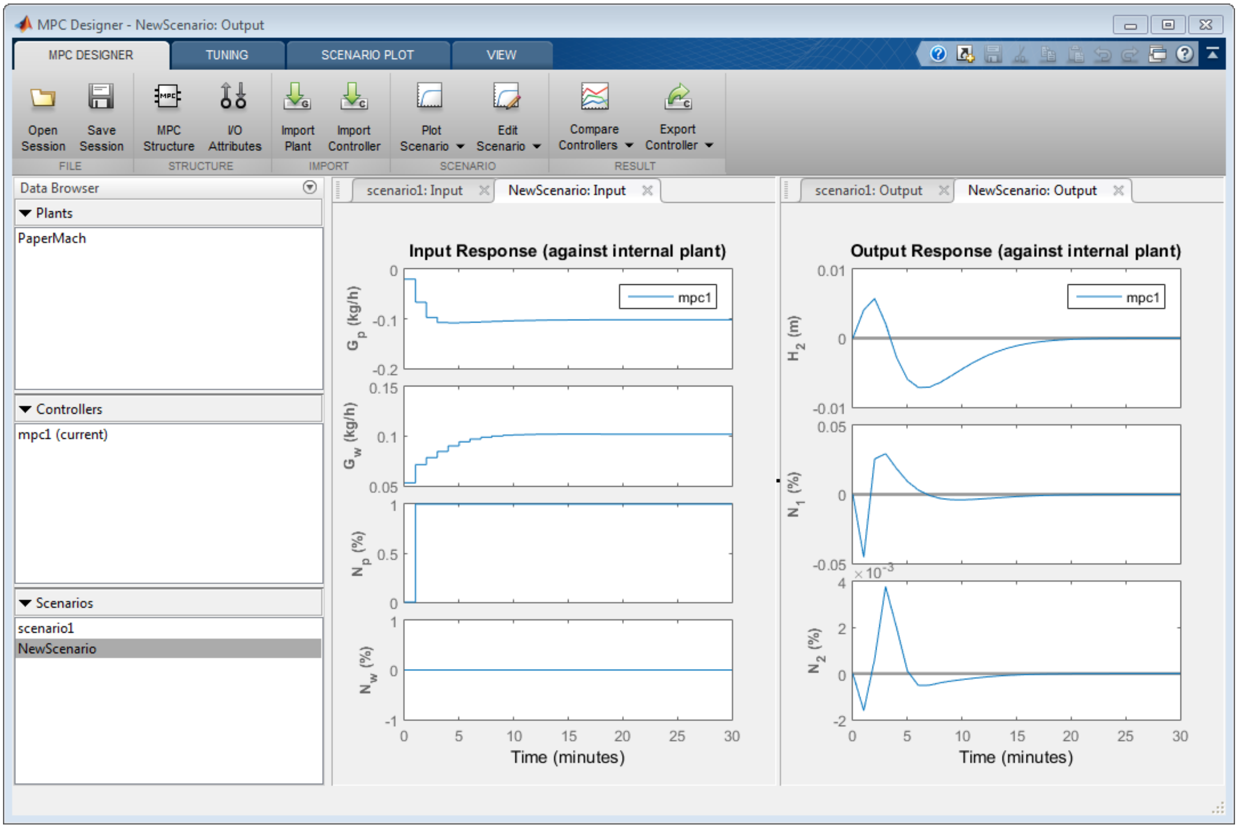
As shown in the **NewScenario: Output** plot, both H_2 and N_2 deviate little from their setpoints.

Experiment with Signal Previewing

In the **Data Browser**, in the **Scenarios** section, right-click `NewScenario`, and select **Edit**.

In the Simulation Scenario dialog box, in the **Simulation Settings** section, check the **Preview measured disturbances** option.

Click **Apply**.



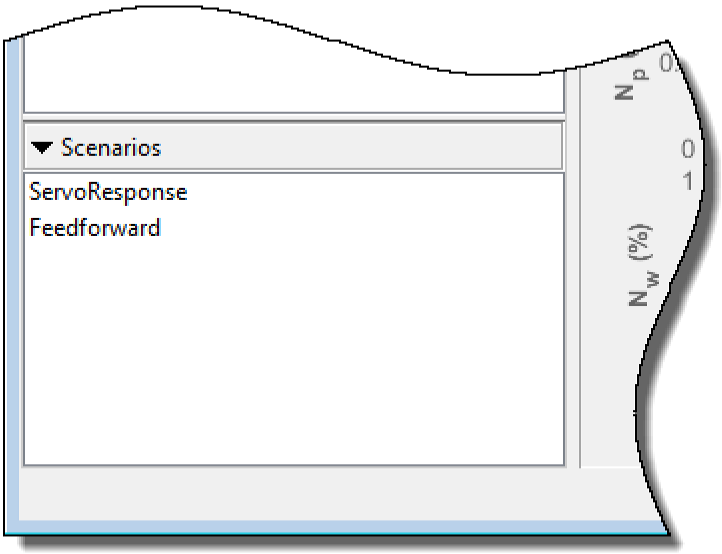
The manipulated variables begin changing before the measured disturbance occurs because the controller uses the known future disturbance value when computing its control action. The output disturbance values also begin changing before the disturbance occurs, which reduces the magnitude of the output errors. However, there is no significant improvement over the previous simulation result.

In the Simulation Scenario dialog box, clear the **Preview measured disturbances** option.

Click **OK**.

Rename Scenarios

With multiple scenarios, it is helpful to provide them with meaningful names. In the **Data Browser**, in the **Scenarios** section, double-click each scenario to rename them as shown:



Test Controller Feedback Response to Unmeasured Disturbances

In the **Data Browser**, in the **Scenarios** section, right-click `Feedforward`,and select **Copy**.

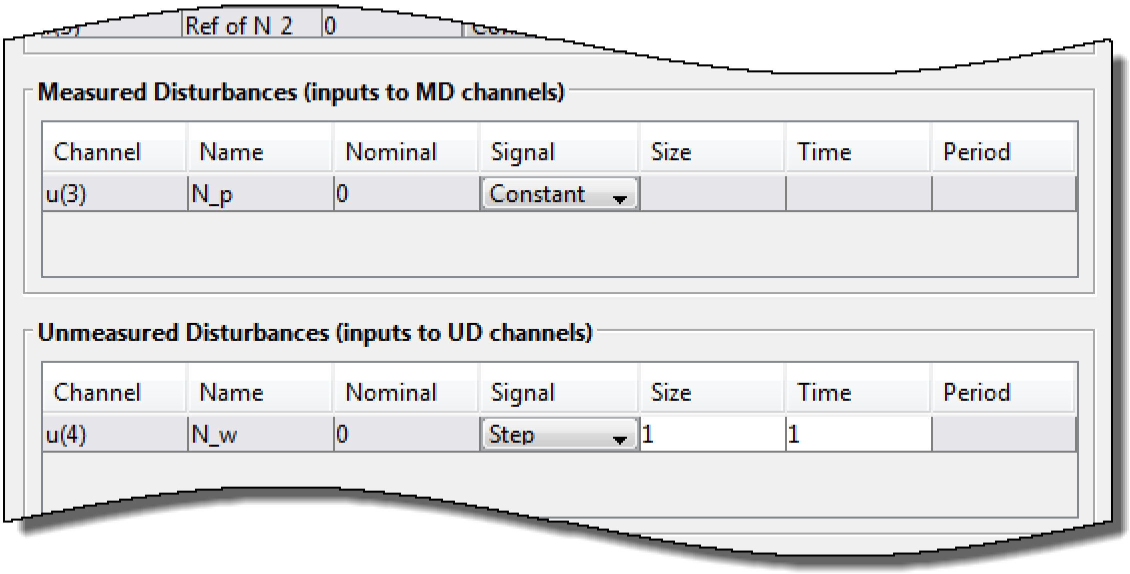
Double-click the new scenario, and rename it `Feedback`.

Right-click the `Feedback` scenario, and select **Edit**.

In the Simulation Scenario dialog box, in the **Measured Disturbances** table, in the **Signal** drop-down list, select `Constant` to remove the measured disturbance.

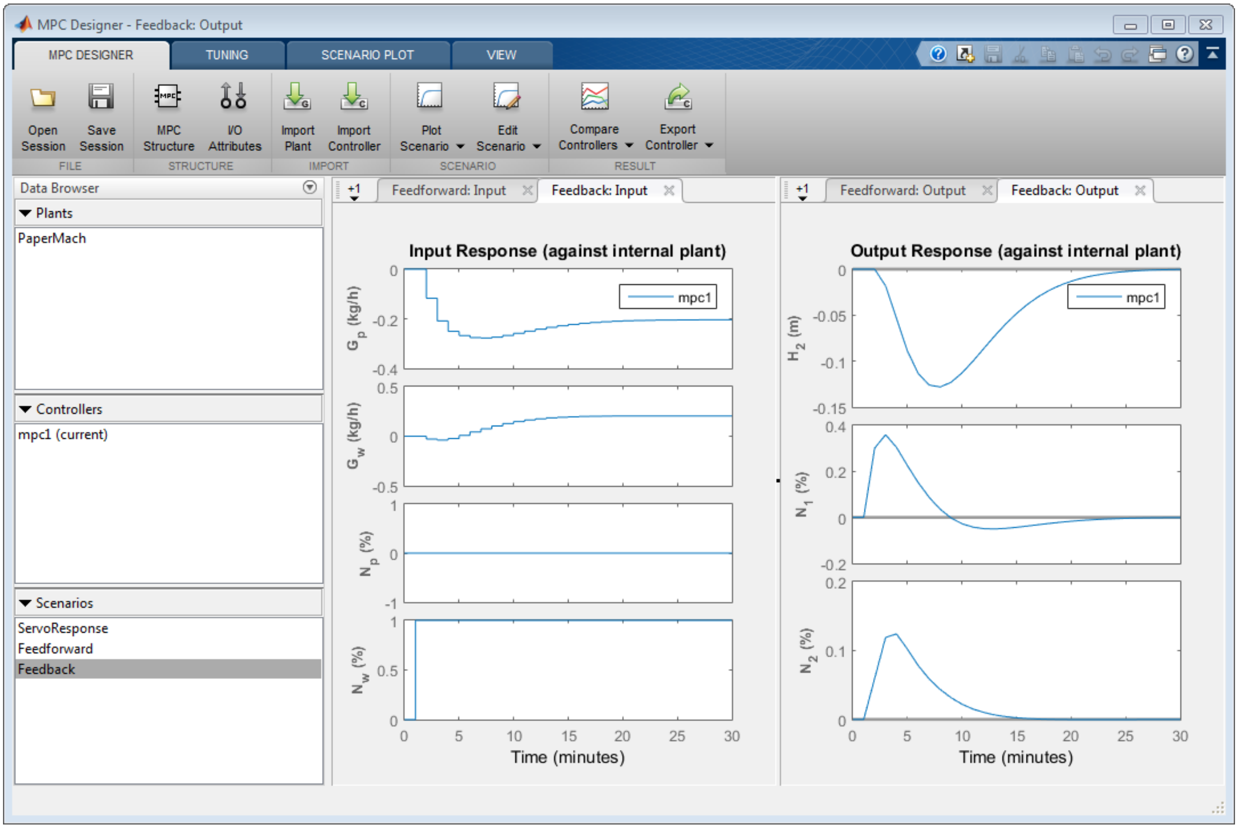
In the **Unmeasured Disturbances** table, in the **Signal** drop-down list, select `Step` to simulate a sudden, sustained unmeasured input disturbance.

Set the step **Size** to `1` and the step **Time** to `1`.



Click **OK** to update the scenario settings, and run the simulation.

In the **Data Browser**, in the **Scenarios** section, right-click `Feedback`, and select **Plot**.



The controlled outputs, H_2 and N_2, both exhibit relatively small deviations from their setpoints. The settling time is longer than for the original servo response, which is typical.

On the **Tuning** tab, in the **Analysis** section, click **Review Design** to check the controller for potential run-time stability or numerical problems.

The review report opens in a new window.

Test	Status
MPC Object Creation	Pass
QP Hessian Matrix Validity	Warning
Controller Internal Stability	Pass
Closed-Loop Nominal Stability	Pass
Closed-Loop Steady-State Gains	Warning
Hard MV Constraints	Pass
Other Hard Constraints	Pass
Soft Constraints	Pass
Memory Size for MPC Data	Pass

The review flags two warnings about the controller design. Click the warning names to determine whether they indicate problems with the controller design.

The **Closed-Loop Steady-State Gains** warning indicates that the plant has more controlled outputs than manipulated variables. This input/output imbalance means that the controller cannot eliminate steady-state error for all of the outputs simultaneously. To meet the control objective of tracking the setpoints of H_2 and N_2, you previously set the output weight for N_1 to zero. This setting causes the **QP Hessian Matrix Validity** warning, which indicates that one of the output weights is zero.

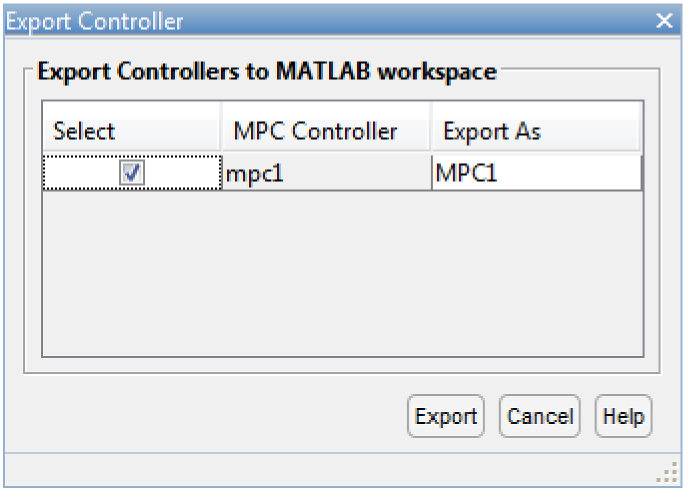
Since the input/output imbalance is a known feature of the paper machine plant model, and you intentionally set one of the output weights to zero to correct for the imbalance, neither warning indicates an issue with the controller design.

Export Controller to MATLAB Workspace

On the **MPC Designer** tab, in the **Result** section, click **Export Controller** .

In the Export Controller dialog box, check the box in the **Select** column.

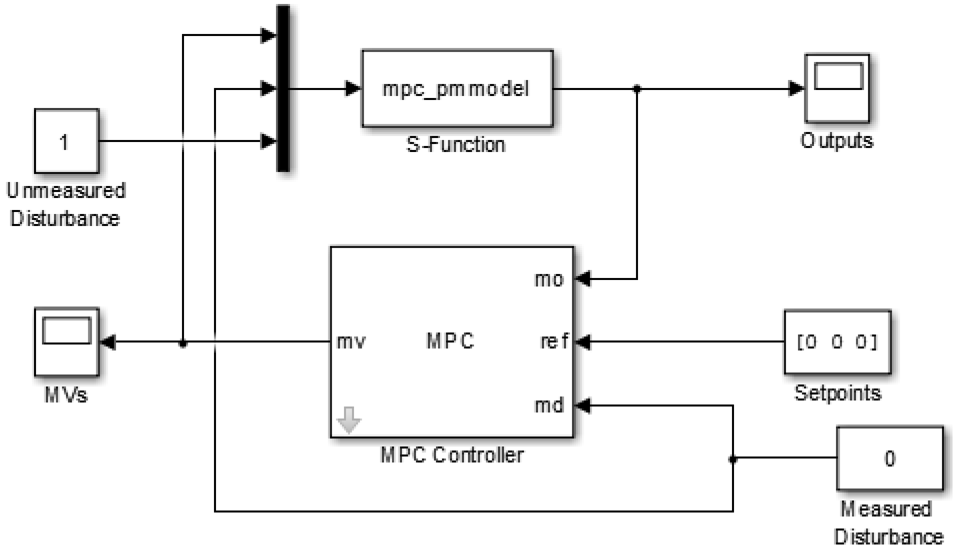
In the **Export As** column, specify `MPC1` as the controller name.



Click **Export** to save a copy of the controller to the MATLAB workspace.

Open Simulink Model

```
open_system('mpc_papermachine')
```

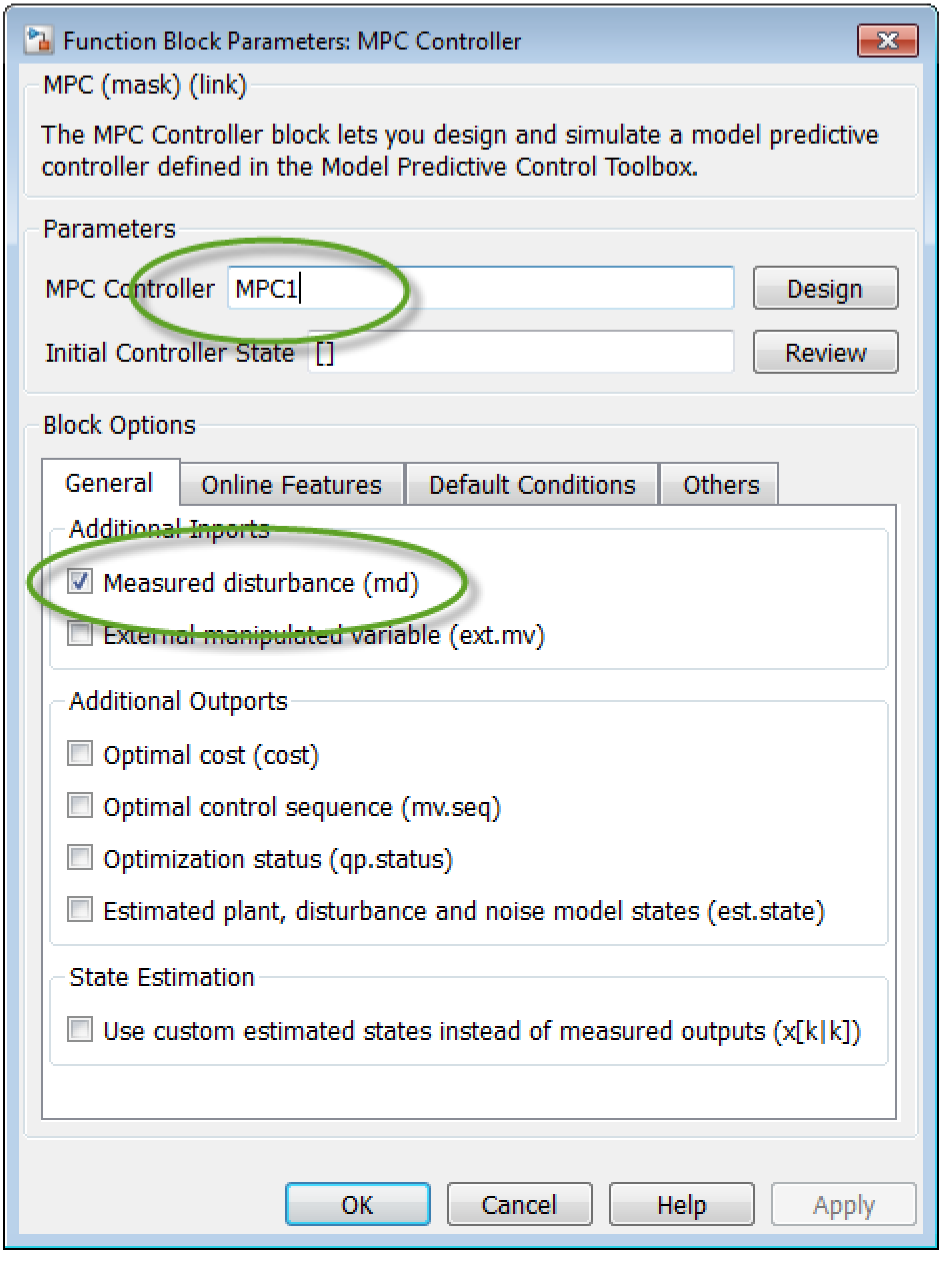


TheMPC Controller block controls the nonlinear paper machine plant model, which is defined using the S-Function `mpc_pmmodel`.

The model is configured to simulate a sustained unmeasured disturbance of size `1`.

Configure MPC Controller Block

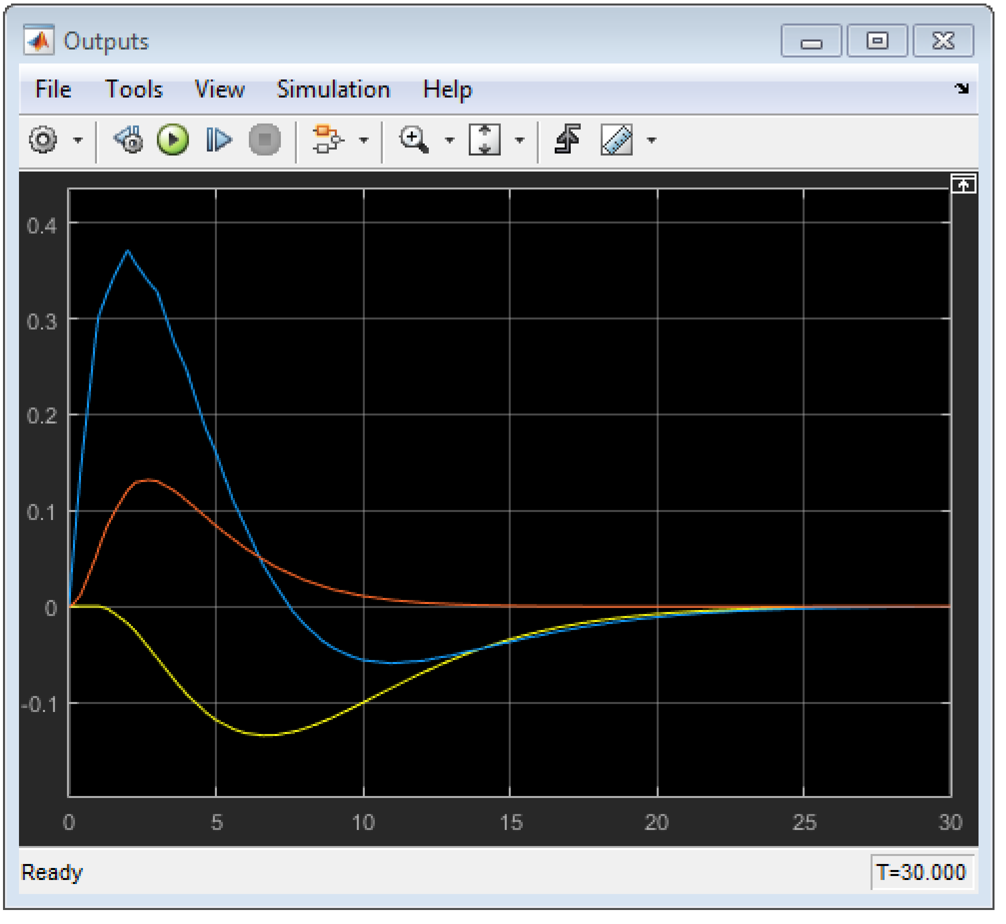
Double-click the MPC Controller block.


Function Block Parameters: MPC Controller
MPC (mask) (link)
The MPC Controller block lets you design and simulate a model predictive controller defined in the Model Predictive Control Toolbox.
Parameters
MPC Controller
MPC1
Design
Initial Controller State
[]
Review
Block Options
General
Online Features
Default Conditions
Others
Additional Inports
Measured disturbance (md)
External manipulated variable (ext.mv)
Additional Outports
Optimal cost (cost)
Optimal control sequence (mv.seq)
Optimization status (qp.status)
Estimated plant, disturbance and noise model states (est.state)
State Estimation
Use custom estimated states instead of measured outputs (x[k|k])
OK
Cancel
Help
Apply

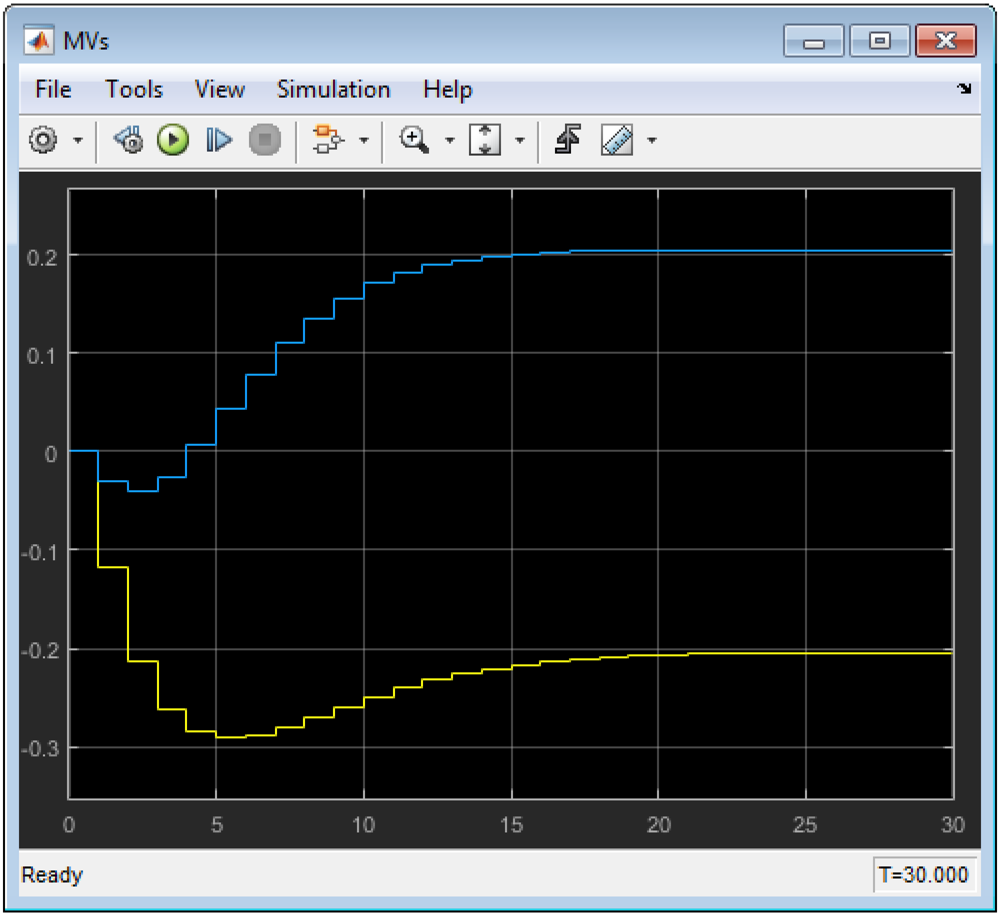
The MPC Controller block is already configured to use the `MPC1` controller that was previously exported to the MATLAB workspace.

Also, the **Measured disturbance** option is selected to add the `md` inport to the controller block.

Simulate the model



In the **Outputs** plot, the responses are almost identical to the responses from the corresponding simulation in MPC Designer. The yellow curve is H_2, the blue is N_1, and the red is N_2.



Similarly, in the **MVs** scope, the manipulated variable moves are almost identical to the moves from corresponding simulation in MPC Designer. The yellow curve is G_p and the blue is G_w.

These results show that there are no significant prediction errors due to the mismatch between the linear prediction model of the controller and the nonlinear plant. Even increasing the unmeasured disturbance magnitude by a factor of four produces similarly shaped response curves. However, as the disturbance size increases further, the effects of nonlinearities become more pronounced.

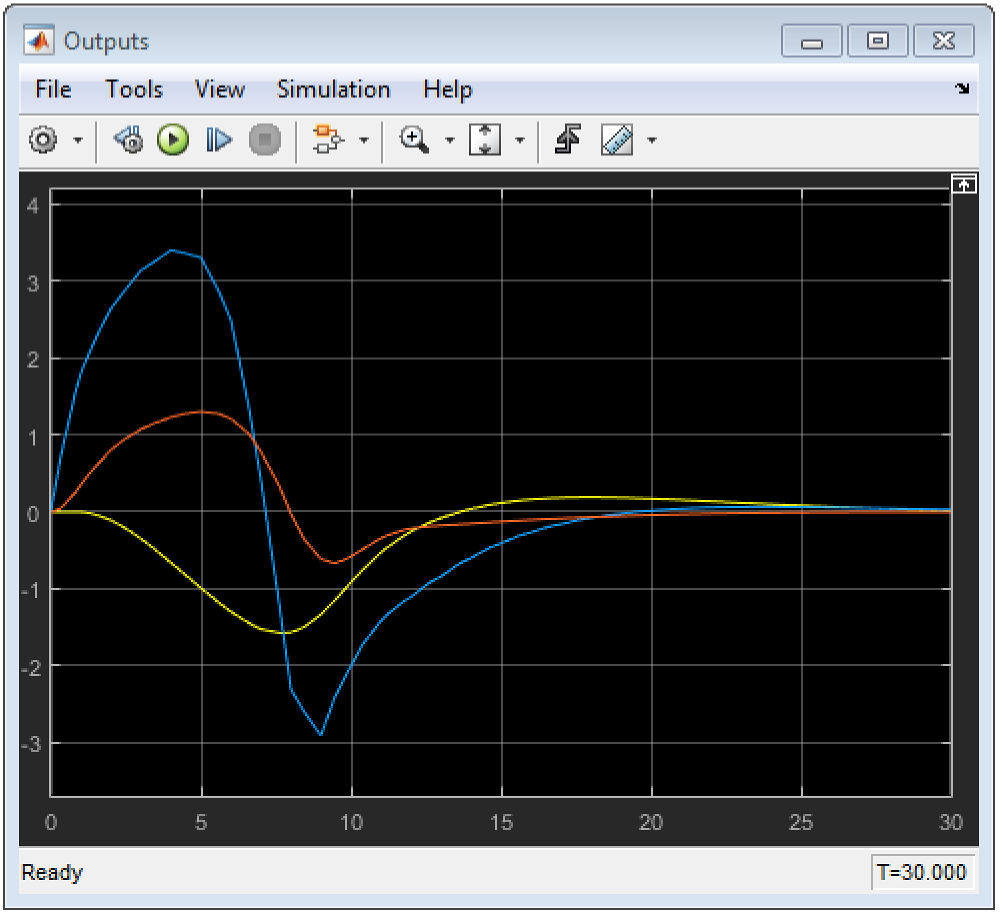
Increase Unmeasured Disturbance Magnitude

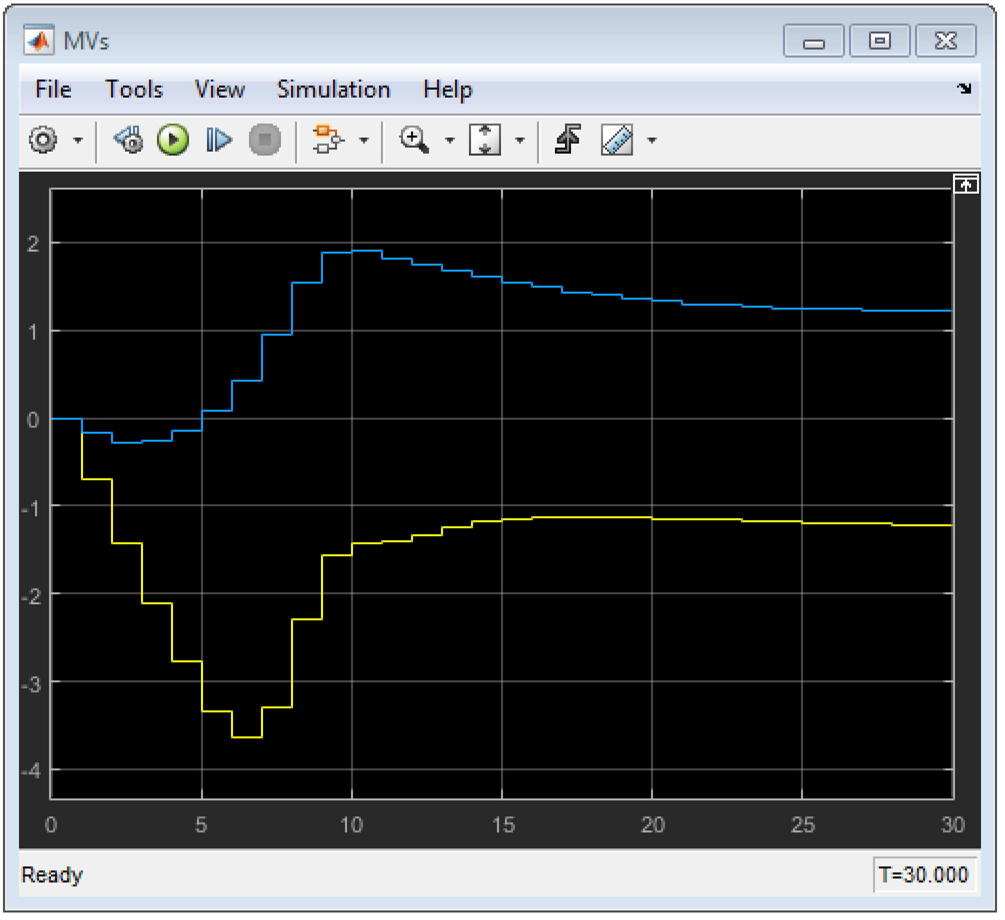
In the Simulink model window, double-click the Unmeasured Disturbance block.

In the Unmeasured Disturbance properties dialog box, specify a **Constant value** of `6.5`.

Click **OK**.

Simulate the model.





The mismatch between the prediction model and the plant now produces output responses with significant differences. Increasing the disturbance magnitude further results in large setpoint deviations and saturated manipulated variables.

References

[1] Ying, Y., M. Rao, and Y. Sun "Bilinear control strategy for paper making process," *Chemical Engineering Communications* (1992), Vol. 111, pp. 13–28.

See Also

`MPC Controller` | MPC Designer

Related Examples

- "Design Controller Using MPC Designer"

Bumpless Transfer Between Manual and Automatic Operations

This example shows how to bumplessly transfer between manual and automatic operations of a plant.

During startup of a manufacturing process, operators adjust key actuators manually until the plant is near the desired operating point before switching to automatic control. If not done correctly, the transfer can cause a *bump*, that is, large actuator movement.

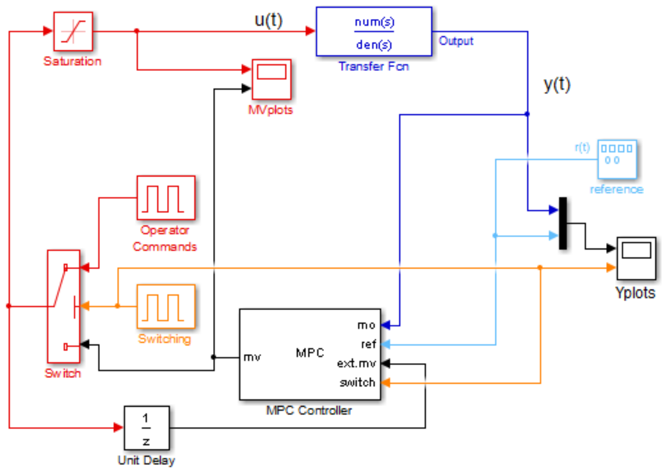
In this example, you simulate a Simulink model that contains a single-input single-output LTI plant and an MPC Controller block.

A model predictive controller monitors all known plant signals, even when it is not in control of the actuators. This monitoring improves its state estimates and allows a bumpless transfer to automatic operation.

Open Simulink Model

Open the Simulink model.

```
open_system('mpc_bumpless')
```



To simulate switching between manual and automatic operation, the Switching block sends either 1 or 0 to control a switch. When it sends 0, the system is in automatic mode, and the output from the MPC Controller block goes to the plant. Otherwise, the system is in manual mode, and the signal from the Operator Commands block goes to the plant.

In both cases, the actual plant input feeds back to the controller `ext.mv` inport, unless the plant input saturates at –1 or 1. The controller constantly monitors the plant output and updates its estimate of the plant state, even when in manual operation.

This model also shows the optimization switching option. When the system switches to manual operation, a nonzero signal enters the `switch` inport of the controller block. The signal turns off the optimization calculations, which reduces computational effort.

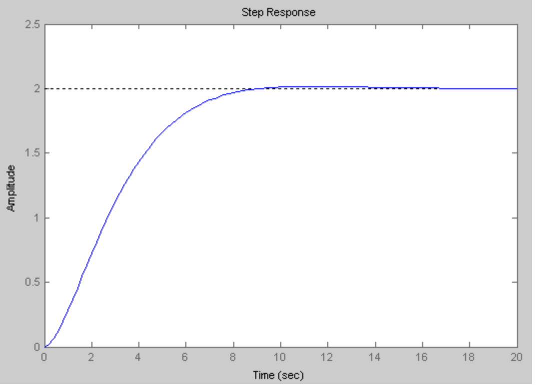
Define Plant and MPC Controller

Create the plant model.

```
num = [1 1];
den = [1 3 2 0.5];
sys = tf(num,den);
```

The plant is a stable single-input single-output system as seen in its step response.

```
step(sys)
```



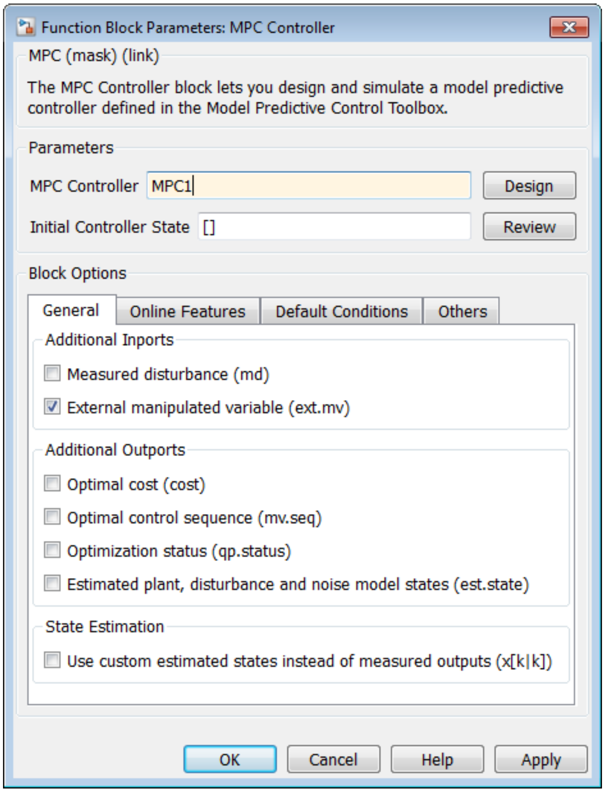
Create an MPC controller.

```
Ts = 0.5;      % sampling time (seconds)
p = 15;        % prediction horizon
m = 2;         % control horizon
MPC1 = mpc(sys,Ts,p,m);
MPC1.Weights.Output = 0.01;
MPC1.MV = struct('Min',-1,'Max',1);
Tstop = 250;
```

Configure MPC Block Settings

Open the Function Block Parameters: MPC Controller dialog box.

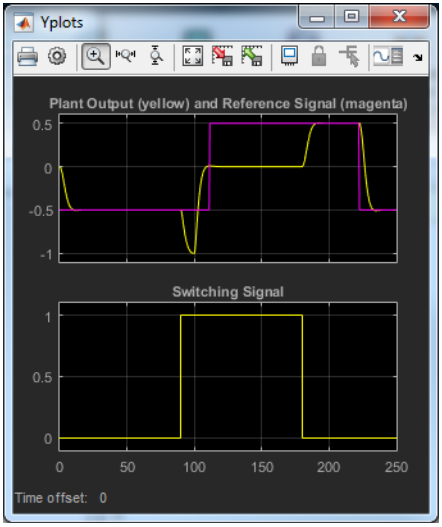
- Specify `MPC1` in the **MPC Controller** box.
- Verify that the **External Manipulated Variable (ext.mv)** option in the **General** tab is selected. This option adds the `ext.mv` inport to the block to enable the use of external manipulated variables.
- Verify that the **Use external signal to enable or disable optimization (switch)** option in the **Others** tab is selected. This option adds the `switch` inport to the controller block to enable switching off the optimization calculations.

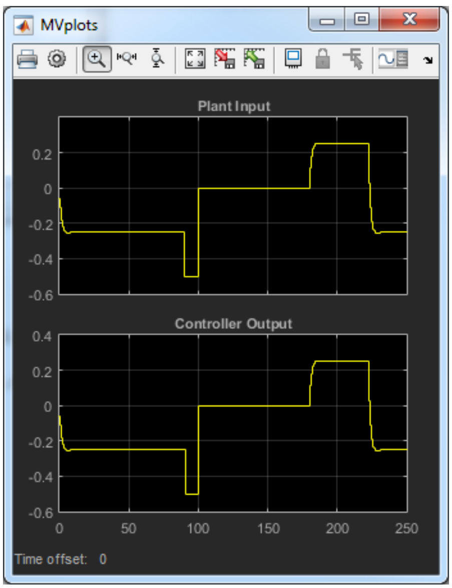


Click **OK**.

Examine Switching Between Manual and Automatic Operation

Click **Run** in the Simulink model window to simulate the model.





For the first 90 time units, the `Switching Signal` is 0, which makes the system operate in automatic mode. During this time, the controller smoothly drives the controlled plant output from its initial value, 0, to the desired reference value, –0.5.

The controller state estimator has zero initial conditions as a default, which is appropriate when this simulation begins. Thus, there is no bump at startup. In general, start the system running in manual mode long enough for the controller to acquire an accurate state estimate before switching to automatic mode.

At time 90, the `Switching Signal` changes to 1. This change switches the system to manual operation and sends the operator commands to the plant. Simultaneously, the nonzero signal entering the `switch` inport of the controller turns off the optimization calculations. While the optimization is turned off, the MPC Controller block passes the current `ext.mv` signal to the `Controller Output`.

Once in manual mode, the operator commands set the manipulated variable to –0.5 for 10 time units, and then to 0. The `Plant Output` plot shows the open-loop response between times 90 and 180 when the controller is deactivated.

At time 180, the system switches back to automatic mode. As a result, the plant output returns to the reference value smoothly, and a similar smooth adjustment occurs in the controller output.

Turn off Manipulated Variable Feedback

Delete the signals entering the `ext.mv` and `switch` inports of the controller block.

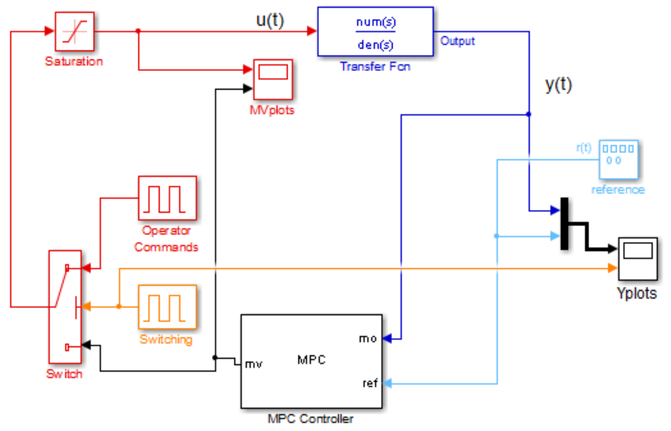
Delete the Unit Delay block and the signal line entering its inport.

Open the Function Block Parameters: MPC Controller dialog box.

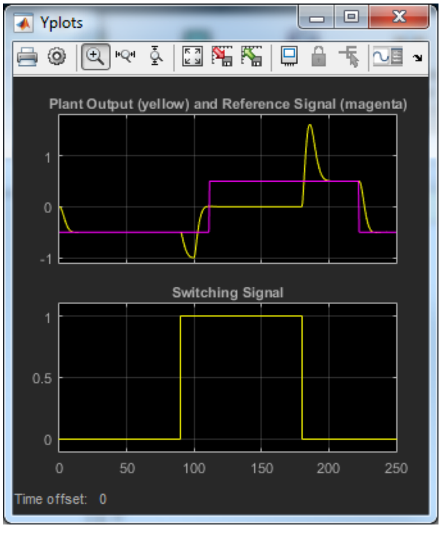
Deselect the **External Manipulated Variable (ext.mv)** option in the **General** tab to remove the `ext.mv` inport from the controller block.

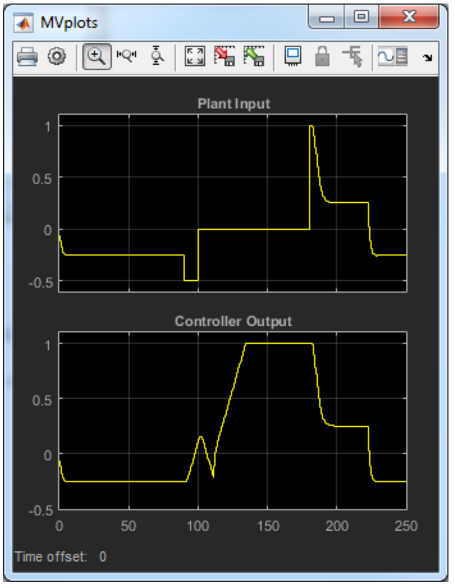
Deselect the **Use external signal to enable or disable optimization (switch)** option in the **Others** tab to remove the `switch` inport from the controller block.

Click **OK**. The Simulink model now resembles the following figure.



Click **Run** to simulate the model.





The behavior is identical to the original case for the first 90 time units.

When the system switches to manual mode at time 90, the plant behavior is the same as before. However, the controller tries to hold the plant at the setpoint. So, its output increases and eventually saturates, as seen in `Controller Output`. Since the controller assumes that this output is going to the plant, its state estimates become inaccurate. Therefore, when the system switches back to automatic mode at time 180, there is a large bump in the `Plant Output`.

Such a bump creates large actuator movements within the plant. By smoothly transferring from manual to automatic operation, a model predictive controller eliminates such undesired movements.

Related Examples

- "Switching Controller Online and Offline with Bumpless Transfer" on page 4-58

Switching Controller Online and Offline with Bumpless Transfer

This example shows how to obtain bumpless transfer when switching model predictive controller from manual to automatic operation or vice versa.

In particular, it shows how the EXT.MV input signal to the MPC block can be used to keep the internal MPC state up to date when the operator or another controller is in control.

Define Plant Model

The linear open-loop dynamic plant model is as follows:

```
num = [1 1];
den = [1 3 2 0.5];
sys = tf(num,den);
```

Design MPC Controller

Construct MPC controller

Create an MPC controller with plant model, sample time and horizons.

```
Ts = 0.5;            % Sampling time
p = 15;              % Prediction horizon
m = 2;               % Control horizon
mpcobj = mpc(sys,Ts,p,m);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

MV Constraints

Define constraints on the manipulated variable.

```
mpcobj.MV=struct('Min',-1,'Max',1);
```

Weights

Change the output weight.

```
mpcobj.Weights.Output=0.01;
```

Simulate Using Simulink®

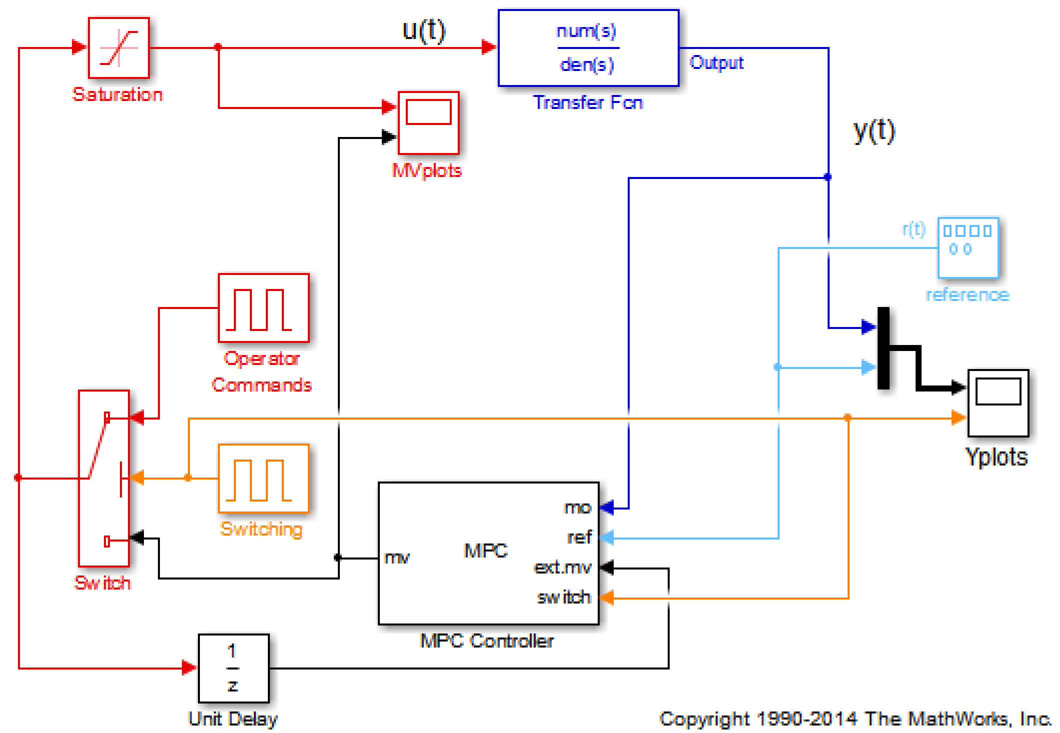
To run this example, Simulink® is required.

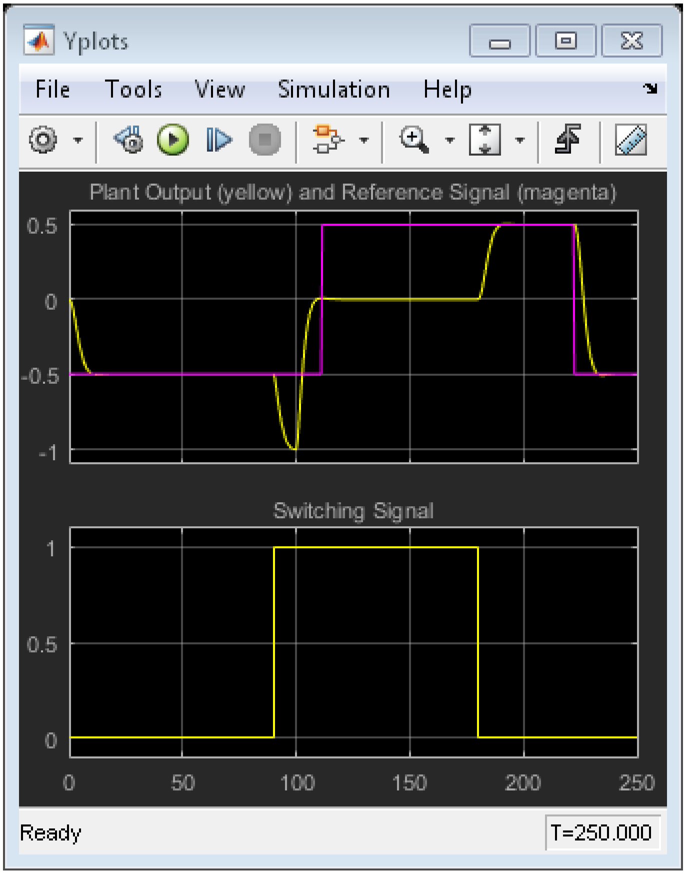
```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
```

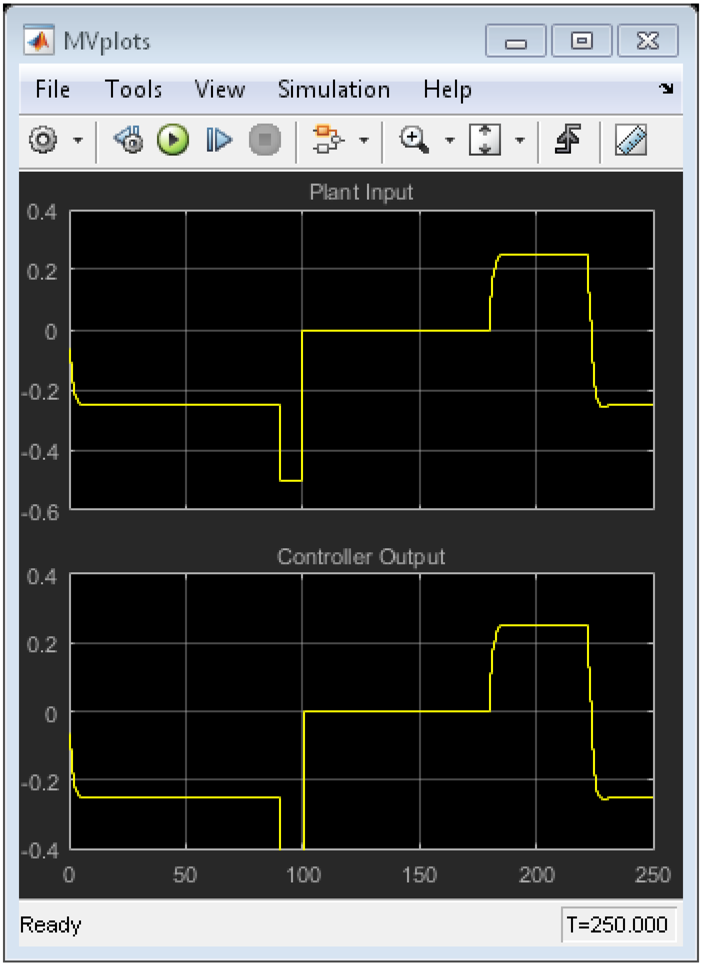
Simulate closed-loop control of the linear plant model in Simulink. Controller "mpcobj" is specified in the block dialog.

```
mdl = 'mpc_bumpless';
open_system(mdl)
sim(mdl)
```

```
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```




Yplots
File Tools View Simulation Help
Plant Output (yellow) and Reference Signal (magenta)
0.5
0
-0.5
-1
Switching Signal
1
0.5
0
0 50 100 150 200 250
Ready
T=250.000

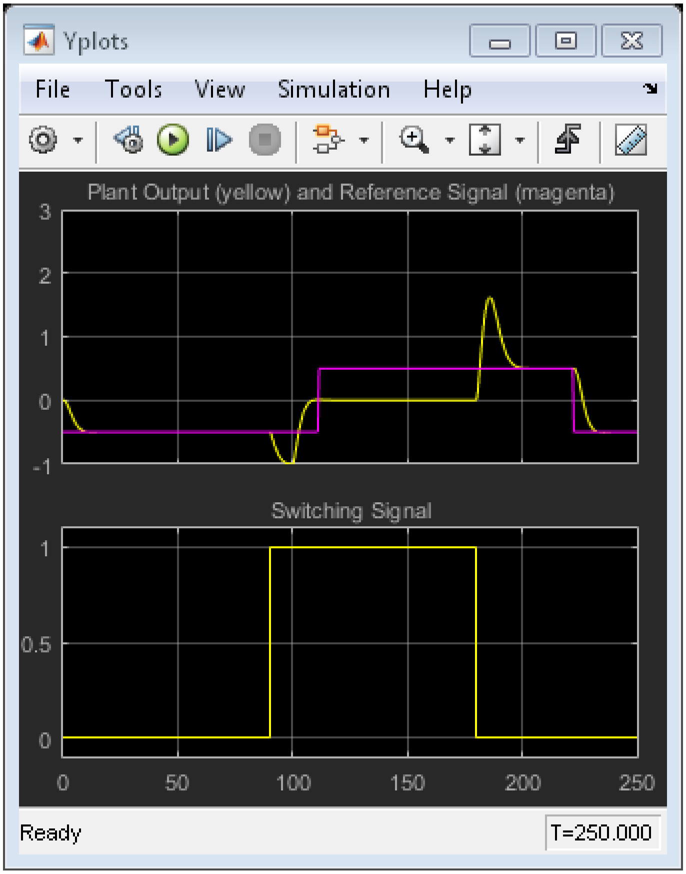


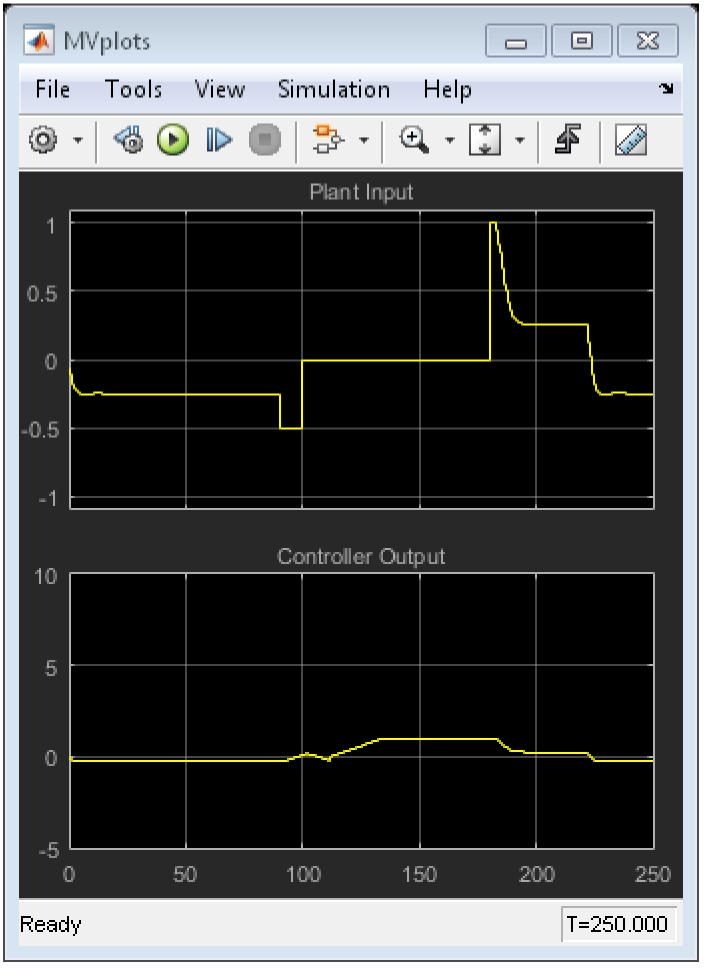
Simulate without Using External MV Signal

Without using the external MV signal, MPC controller is no longer able to provide bumpless transfer because the internal controller states are not estimated correctly.

```
delete_line(mdl,'Switch/1','Unit Delay/1');
delete_line(mdl,'Unit Delay/1','MPC Controller/3');
delete_block([mdl '/Unit Delay']);
delete_line(mdl,'Switching/1','MPC Controller/4');
```

```
set_param([mdl '/MPC Controller'],'mv_inport','off');
set_param([mdl '/MPC Controller'],'switch_inport','off');
set_param([mdl '/Yplots'],'Ymin','-1~-0.1')
set_param([mdl '/Yplots'],'Ymax','3~1.1')
set_param([mdl '/MVplots'],'Ymin','-1.1~-5')
set_param([mdl '/MVplots'],'Ymax','1.1~10')
sim(mdl);
```





Now the transition from manual to automatic control is much less smooth. Note the large "bump" between time = 180 and 200.

```
bdclose(mdl)
```

Related Examples

- "Bumpless Transfer Between Manual and Automatic Operations" on page 4-50

Coordinate Multiple Controllers at Different Operating Points

Chemical reactors can exhibit strongly nonlinear behavior due to the exponential effect of temperature on reaction rate. If the primary reaction is exothermic, an increase in reaction rate causes an increase in reactor temperature. This positive feedback can lead to open-loop unstable behavior.

Reactors operate in either a continuous or a batch mode. In batch mode, operating conditions can change dramatically during a batch as the reactants disappear. Although continuous reactors typically operate at steady state, they must often move to a new steady state. In other words, both batch and continuous reactors need to operate safely and efficiently over a range of conditions.

If the reactor behaves nonlinearly, a single linear controller might not be able to manage such transitions. One approach is to develop linear models that cover the anticipated operating range, design a controller based on each model, and then define a criterion by which the control system switches from one such controller to another. Gain scheduling is an established technique. The challenge is to move the reactor operating conditions from an initial steady-state point to a much different condition. The transition passes through a region in which the plant is open-loop unstable. This example illustrates an alternative — coordination of multiple MPC controllers. The solution uses the Simulink Multiple MPC Controller block to coordinate the use of three controllers, each of which has been designed for a particular operating region.

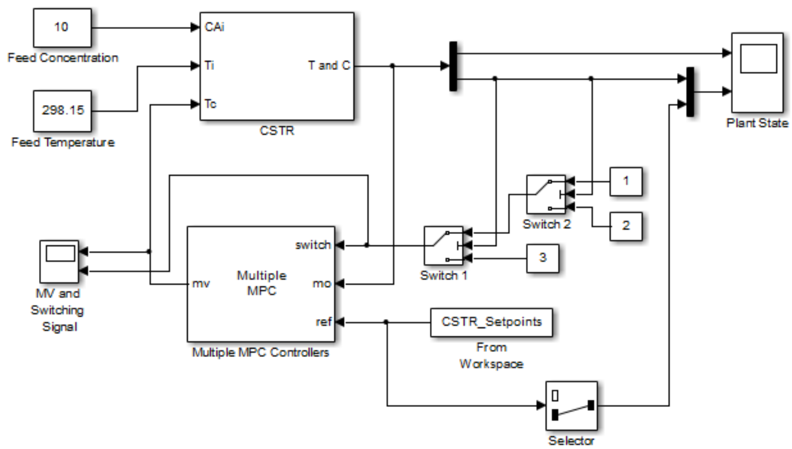
The subject process is a constant-volume continuous stirred-tank reactor (CSTR). The model consists of two nonlinear ordinary differential equations (see [1]). The model states are the reactor temperature and the rate-limiting reactant concentration. For the purposes of this example, both are assumed to be measured plant outputs.

There are three inputs:

- Concentration of the limiting reactant in the reactor feed stream, kmol/m^3
- The reactor feed temperature, K
- The coolant temperature, K

The control system can adjust the coolant temperature in order to regulate the reactor state and the rate of the exothermic main reaction. The other two inputs are independent unmeasured disturbances.

The Simulink diagram for this example appears below. The CSTR model is a masked subsystem. The feed temperature and composition are constants. As discussed above, the control system adjusts the coolant temperature (the `Tc` input on the CSTR block).



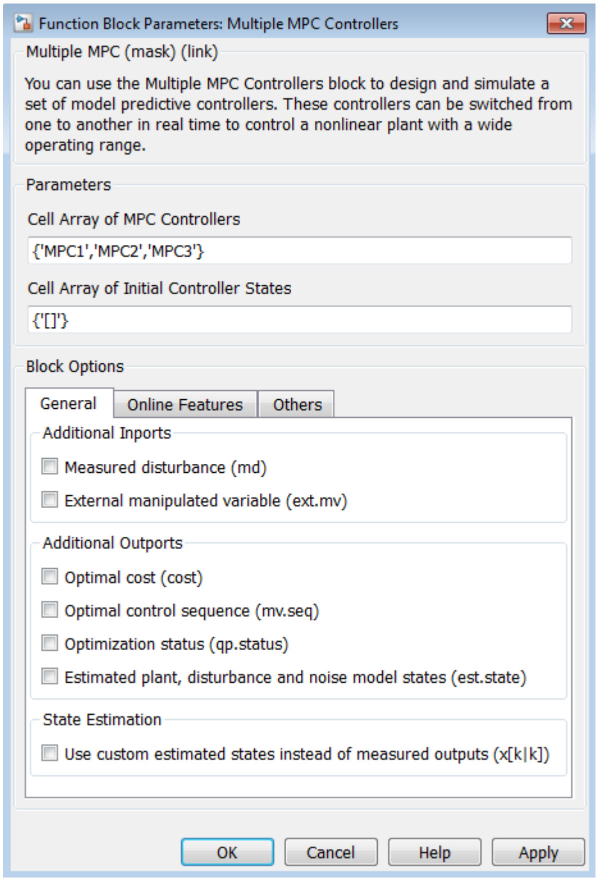
The two CSTR outputs are the reactor temperature and composition respectively. These are being sent to a scope display and to the control system as feedback.

The reference signal (i.e. setpoint) is coming from variable `CSTR_Setpoints`, which is in the base workspace. As there is only one manipulated variable (the coolant temperature) the control objective is to force the *reactor concentration* to track a specified trajectory. The concentration setpoint also goes to the `Plant State` scope for plotting. The control system receives a setpoint for the reactor temperature too but the controller design ignores it.

In that case why supply the temperature measurement to the controller? The main reason is to improve state estimation. If this were not done, the control system would have to infer the temperature value from the concentration measurement, which would introduce an estimation error and degrade the model's predictive accuracy.

The rationale for the `Switch 1` and `Switch 2` blocks appears below.

The figure below shows the `Multi MPC Controller` mask. The block is coordinating three controllers (`MPC1`, `MPC2` and `MPC3` in that sequence). It is also receiving the setpoint signal from the workspace, and the **Look ahead** option is active. This allows the controller to anticipate future setpoint values and usually improves setpoint tracking.



In order to designate which one of the three controllers is active at each time instant, we send the `Multi MPC Controllers` block a switching signal (connected to its `switch` input port). If it is 1, `MPC1` is active. If it is 2, `MPC2` is active, and so on.

In the diagram, `Switch 1` and `Switch 2` perform the controller selection function as follows:

- If the reactor concentration is 8 kmol/m^3 or greater, `Switch 1` sends the constant 1 to its output. Otherwise it sends the constant 2.
- If the reactor concentration is 3 kmol/m^3 or greater, `Switch 2` passes through the signal coming from `Switch 1` (either 1 or 2). Otherwise is sends the constant 3.

Thus, each controller handles a particular composition range. The simulation begins with the reactor at an initial steady state of 311`K and 8.57` kmol/m^3`. The feed concentration is 10` kmol/m^3 so this is a conversion of about 15%, which is low. The control objective is to transition smoothly to 80% conversion with the reactor concentration at 2 kmol/m^3. The simulation will start with `MPC1` active, transition to `MPC2`, and end with `MPC3`.

We decide to design the controllers around linear models derived at the following three reactor compositions (and the corresponding steady-state temperature): 8.5, 5.5, and 2 kmol/m^3.

In practice, you would probably obtain the three models from data. This example linearizes the nonlinear model at the above three conditions (for details see “Using Simulink to Develop LTI Models” in the Getting Started Guide).

Note As shown later, we need to retain at the unmeasured plant inputs in the model. This prevents us from using the Model Predictive Control Toolbox automatic linearization feature. In the current toolbox, the automatic linearization feature can linearize with respect to manipulated variable and measured disturbance inputs only.

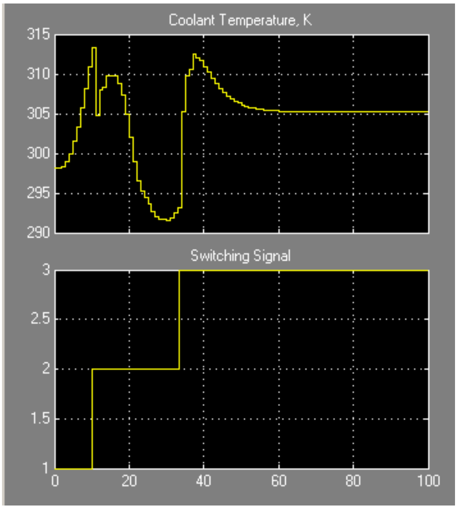
The following code obtains the linear models and designs the three controllers

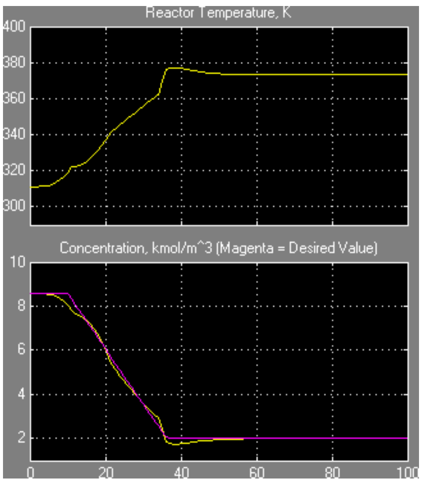
```
[sys, xp] = CSTR_INOUT([],[],[],'sizes');
up = [10 298.15 298.15]';
yp = xp;
Ts = 1;
Nc = 3;
Controllers = cell(1,3);
Concentrations = [8.5 5.5 2];
Y = yp;
for i = 1:Nc
    clear Model
    Y(2) = Concentrations(i);
    [X,U,Y,DX] = trim('CSTR_INOUT',xp(:),up(:),Y(:),[],[1,2]',2)
    [a,b,c,d] = linmod('CSTR_INOUT', X, U );
    Plant = ss(a,b,c,d);
    Plant.InputGroup.MV = 3;
    Plant.InputGroup.UD = [1,2];
    Model.Plant = Plant;
    Model.Nominal.U = [0; 0; up(3)];
```

```
    Model.Nominal.X = xp;
    Model.Nominal.Y = yp;
    MPCobj = mpc(Model, Ts);
    MPCobj.Weight.OV = [0 1];
    D = ss(getindist(MPCobj));
    D.b = D.b*10;
    set(D,'InputName',[],'OutputName',[],'InputGroup',[], ...
     'OutputGroup',[])
    setindist(MPCobj, 'model', D)
    Controllers{i} = MPCobj;
end
MPC1 = Controllers{1};
MPC2 = Controllers{2};
MPC3 = Controllers{3}
```

The key points regarding the designs are as follows:

- All three controllers use the same nominal condition, the values of the plant inputs and outputs at the initial steady-state. Exception: all unmeasured disturbance inputs must have zero nominal values.
- Each controller employs a different prediction model. The model structure is the same in each case (input and outputs are identical in number and type) but each model represents a particular steady-state reactor composition.
- It turns out that the MPC2 plant model obtained at 5 kmol/m^3 is open-loop unstable. We must use a model structure that promotes a stable Kalman state estimator. If we include the unmeasured disturbance inputs in the prediction model, the default estimator assumes integrated white noise at each such input, which produces a stable estimator in this case.
- The default estimator signal-to-noise settings are inappropriate, however. If you use them and monitor the state estimates (not shown), the internally estimated temperature and composition can be far from the measured values. To overcome this, we increase the signal-to-noise ratio in each disturbance channel. See the use of `getindist` and `setindist` above. The default signal to noise is being increased by a factor of 10.
- We are using a zero weight on the measured temperature. See the above discussion of control objectives for the rationale.

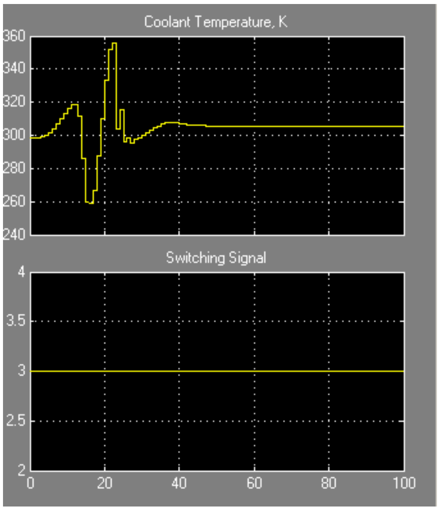


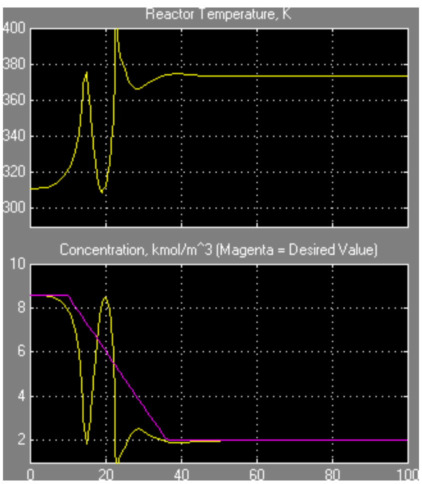


The above plots show the simulation results. The Multi MPC Controller block uses the three controllers sequentially as expected (see the switching signal). Tracking of the concentration setpoint is excellent and the reactor temperature is also controlled well.

To achieve this, the control system starts by increasing the coolant temperature, causing the reaction rate to increase. Once the reaction has achieved a high rate, it generates substantial heat and the coolant temperature must decrease to keep the reactor temperature under control. As the reactor concentration depletes, the reaction rate slows and the control system must again raise the coolant temperature, finally settling at 305 K, about 7 K above the initial condition.

For comparison the plots below show the results for the same scenario if we force `MPC3` to be active for the entire simulation. The CSTR eventually stabilizes at the desired steady-state but both the reactor temperature and composition exhibit large excursions away from the desired conditions.





References

[1] Seborg, D. E., T. F. Edgar, and D. A. Mellichamp *Process Dynamics and Control*, 2nd Edition (2004), Wiley, pp. 34–36.

Use Custom Constraints in Blending Process

This example shows how to design an MPC controller for a blending process using custom input and output constraints.

Blending Process

A continuous blending process combines three feeds in a well-mixed container to produce a blend having desired properties. The dimensionless governing equations are:

$$\frac{dv}{d\tau} = \sum_{i=1}^{3} \phi_i - \phi$$
$$V\frac{d\gamma_j}{d\tau} = \sum_{i=1}^{3} (\gamma_{ij} - \gamma_j)\,\phi_i$$

where

- V is the mixture inventory (in the container).
- ϕ_i is the plow rate for the ith feed.
- ϕ is the rate at which the blend is being removed from inventory, that is the demand.
- γ_{ij} is the concentration of constituent j in feed i.
- γ_j is the concentration of constituent j in the blend.
- τ is time.

In this example, there are two important constituents, j = 1 and 2.

The control objectives are targets for the two constituent concentrations in the blend, and the mixture inventory. The challenge is that the demand, ϕ, and feed compositions, γ_{ij}, vary. The inventory, blend compositions, and demand are measured, but the feed compositions are unmeasured.

At the nominal operating condition:

- Feed 1, ϕ_1, (mostly constituent 1) is 80% of the total inflow.
- Feed 2, ϕ_2, (mostly constituent 2) is 20%.
- Feed 3, ϕ_3, (pure constituent 1) is not used.

The process design allows manipulation of the total feed entering the mixing chamber, ϕ_T, and the individual rates of feeds 2 and 3. In other words, the rate of feed 1 is:

$$\phi_1 = \phi_T - \phi_2 - \phi_2$$

Each feed has limited availability:

$$0 \leq \phi_i \leq \phi_{i,\max}$$

The equations are normalized such that, at the nominal steady state, the mean residence time in the mixing container is $\tau = 1$.

The constraint $\phi_{1,\max} = 0.8$ is imposed by an upstream process, and the constraints $\phi_{2,\max} = \phi_{3,\max} = 0.6$ are imposed by physical limits.

Define Linear Plant Model

The blending process is mildly nonlinear, however you can derive a linear model at the nominal steady state. This approach is quite accurate unless the (unmeasured) feed compositions change. If the change is sufficiently large, the steady-state gains of the nonlinear process change sign and the closed-loop system can become unstable.

Specify the number of feeds, `ni`, and the number of constituents, `nc`.

```
ni = 3;
nc = 2;
```

Specify the nominal flow rates for the three input streams and the output stream, or demand. At the nominal operating condition, the output flow rate is equal to the sum of the input flow rates.

```
Fin_nom = [1.6,0.4,0];
F_nom  = sum(Fin_nom);
```

Define the nominal constituent compositions for the input feeds, where `cin_nom(i,j)` represents the composition of constituent `i` in feed `j`.

```
cin_nom = [0.7 0.2 0.8;0.3 0.8 0];
```

Define the nominal constituent compositions in the output feed.

```
cout_nom = cin_nom*Fin_nom'/F_nom;
```

Normalize the linear model such that the target demand is 1 and the product composition is 1.

```
fin_nom = Fin_nom/F_nom;
gij = [cin_nom(1,:)/cout_nom(1); cin_nom(2,:)/cout_nom(2)];
```

Create a state-space model with feed flows F1, F2, and F3 as MVs:

```
A = [zeros(1,nc+1); zeros(nc,1) -eye(nc)];
Bu = [ones(1,ni); gij-1];
```

Change the MV definition to [FT, F2, F3] where F1 = FT - F2 - F3

```
Bu = [Bu(:,1), Bu(:,2)-Bu(:,1), Bu(:,3)-Bu(:,1)];
```

Add the measured disturbance, blend demand, as the 4th model input.

```
Bv = [-1; zeros(nc,1)];
B = [Bu Bv];
```

Define all of the states as measurable. The states consist of the mixture inventory and the constituent concentrations.

```
C = eye(nc+1);
```

Specify that there is no direct feed-through from the inputs to the outputs.

```
D = zeros(nc+1,ni+1);
```

Construct the linear plant model.

```
Model = ss(A,B,C,D);
Model.InputName = {'F_T','F_2','F_3','F'};
Model.InputGroup.MV = 1:3;
Model.InputGroup.MD = 4;
Model.OutputName = {'V','c_1','c_2'};
```

Create MPC Controller

Specify the sample time, prediction horizon, and control horizon.

```
Ts = 0.1;
p = 10;
```

```
m = 3;
```

Create the controller.

```
mpcobj = mpc(Model,Ts,p,m);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.
```

The outputs are the inventory, `y(1)`, and the constituent concentrations, `y(2)` and `y(3)`. Specify nominal values of unity after normalization for all outputs.

```
mpcobj.Model.Nominal.Y = [1 1 1];
```

Specify the normalized nominal values the manipulated variables, `u(1)`, `u(2)` and `u(3)`, and the measured disturbance, `u(4)`.

```
mpcobj.Model.Nominal.U = [1 fin_nom(2) fin_nom(3) 1];
```

Specify output tuning weights. Larger weights are assigned to the first two outputs because we want to pay more attention to controlling the inventory, and the composition of the first constituent.

```
mpcobj.Weights.OV = [1 1 0.5];
```

Specify the hard bounds (physical limits) on the manipulated variables.

```
umin = [0 0 0];
umax = [2 0.6 0.6];
for i = 1:3
   mpcobj.MV(i).Min = umin(i);
   mpcobj.MV(i).Max = umax(i);
   mpcobj.MV(i).RateMin = -0.1;
   mpcobj.MV(i).RateMax =  0.1;
end
```

The total feed rate and the rates of feed 2 and feed 3 have upper bounds. Feed 1 also has an upper bound, determined by the upstream unit supplying it.

Specify Custom Constraints

Given the specified upper bounds on the feed 2 and 3 rates (0.6), it is possible that their sum could be as much as 1.2. Since the nominal total feed rate is 1.0, the controller can request a physically impossible condition, where the sum of feeds 2 and 3 exceeds the total feed rate, which implies a negative feed 1 rate.

The following constraint prevents the controller from requesting an unrealistic ϕ_1 value.

$$0 \leq \phi_1 = \phi_T - \phi_2 - \phi_3 \leq 0.8$$

Specify this constraint in the form $Eu + Fy \leq g$.

```
E = [-1 1 1; 1 -1 -1];
g = [0;0.8];
```

Since no outputs are specified in the mixed constraints, set their coefficients to zero.

```
F = zeros(2,3);
```

Specify that both constraints are hard (ECR = 0).

```
v = zeros(2,1);
```

Specify zero coefficients for the measured disturbance.

```
h = zeros(2,1);
```

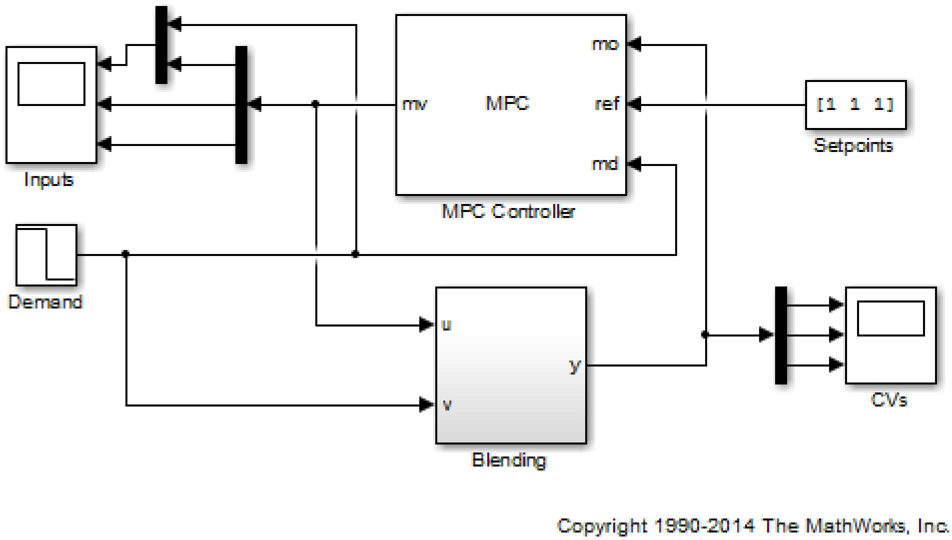
Set the custom constraints in the MPC controller.

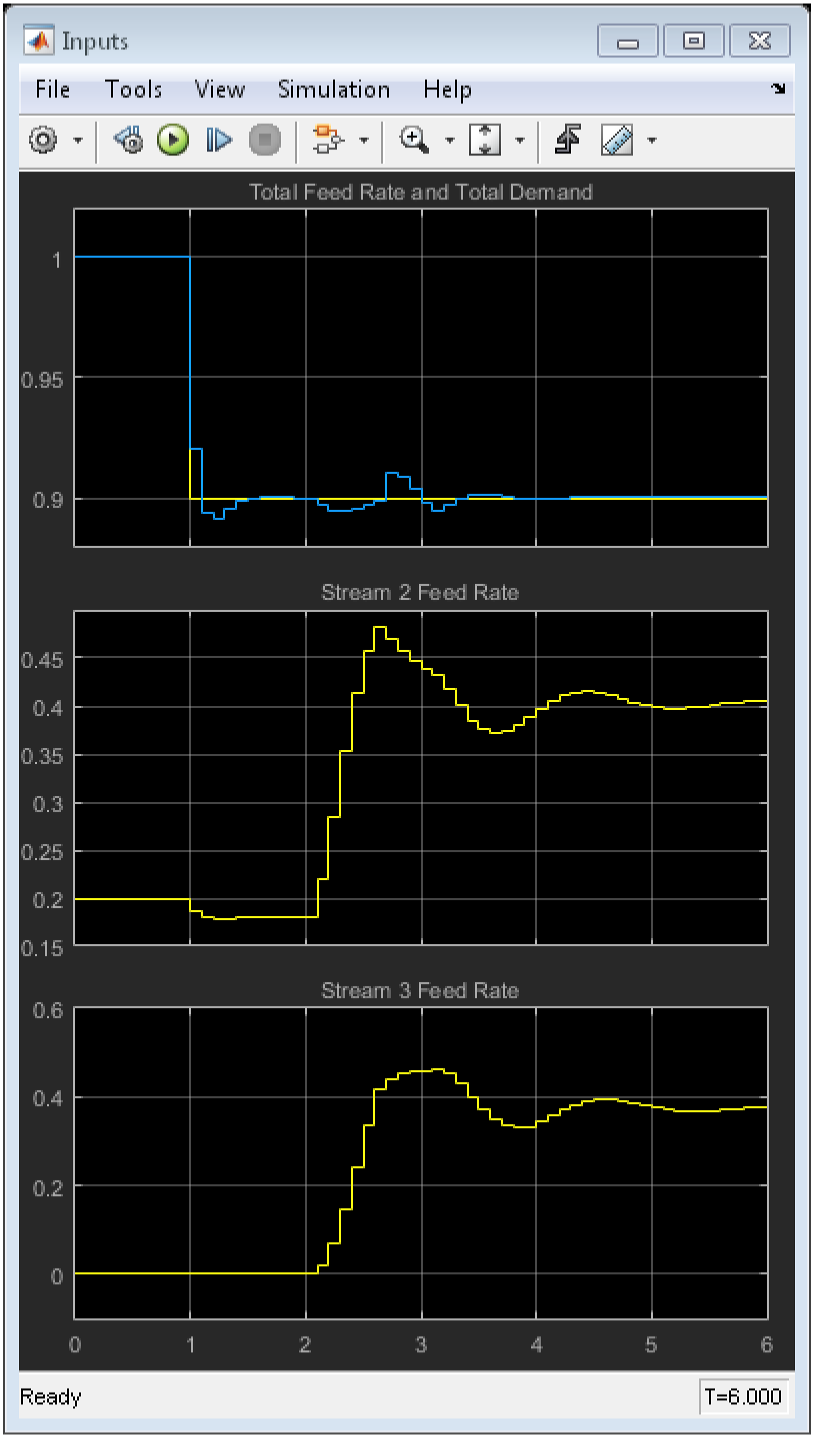
```
setconstraint(mpcobj,E,F,g,v,h)
```

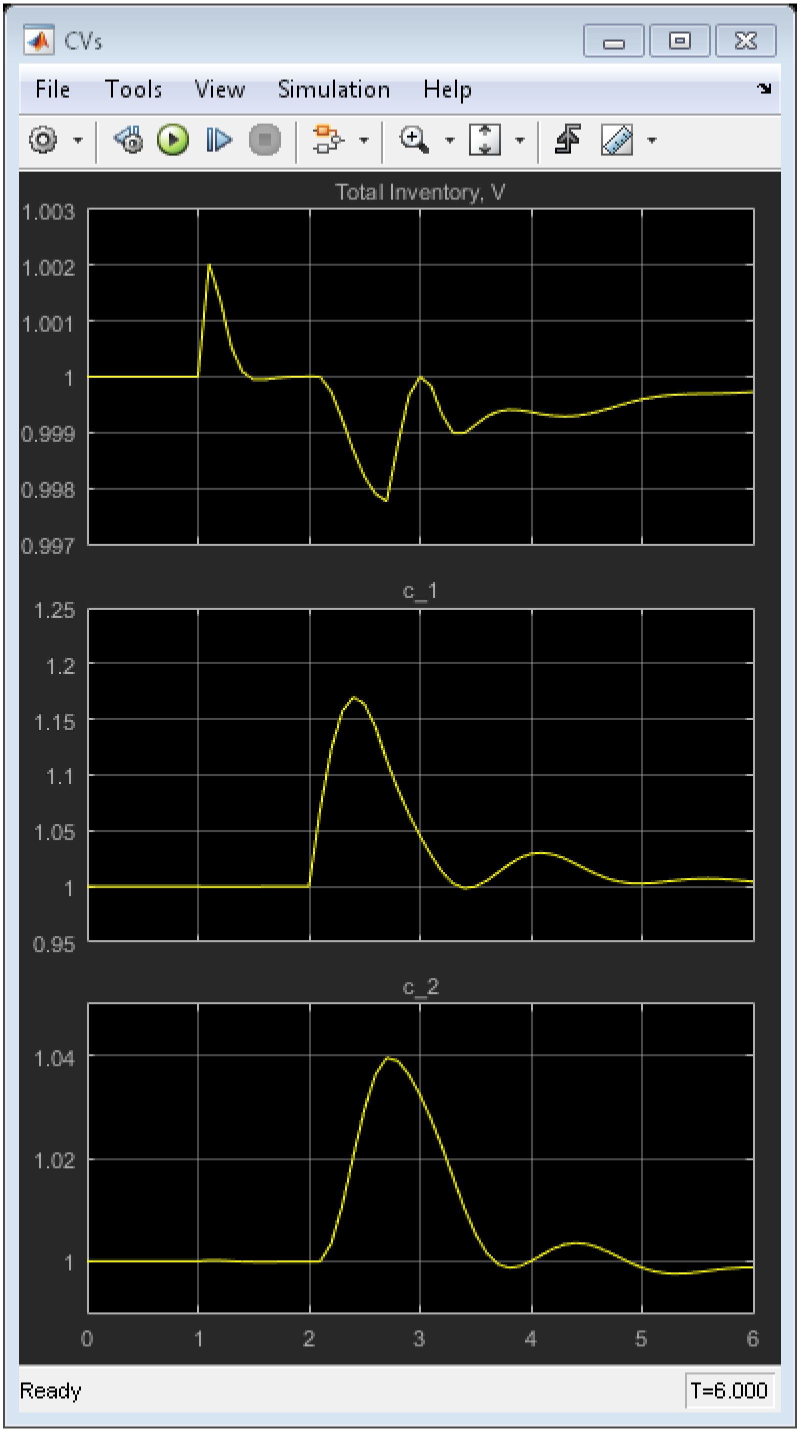
Open and Simulate Model in Simulink

```
sys = 'mpc_blendingprocess';
open_system(sys)
sim(sys)
```

```
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->Assuming output disturbance added to measured output channel #2 is integrated white
-->Assuming output disturbance added to measured output channel #3 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```


Inputs
mv
MPC
mo
ref
md
MPC Controller
[1 1 1]
Setpoints
Demand
u
y
v
Blending
CVs
Copyright 1990-2014 The MathWorks, Inc.


Inputs
File Tools View Simulation Help
Total Feed Rate and Total Demand
1
0.95
0.9
Stream 2 Feed Rate
0.45
0.4
0.35
0.3
0.25
0.2
0.15
Stream 3 Feed Rate
0.6
0.4
0.2
0
0 1 2 3 4 5 6
Ready
T=6.000


CVs
File Tools View Simulation Help
Total Inventory, V
1.003
1.002
1.001
1
0.999
0.998
0.997
c_1
1.25
1.2
1.15
1.1
1.05
1
0.95
c_2
1.04
1.02
1
0 1 2 3 4 5 6
Ready
T=6.000

The MPC controller controls the blending process. The block labeled `Blending` incorporates the previously described model equations and includes an unmeasured step disturbance in the constituent 1 feed composition.

The `Demand`, ϕ, is modeled as a measured disturbance. The operator can vary the demand value, and the resulting signal goes to both the process and the controller.

The model simulates the following scenario:

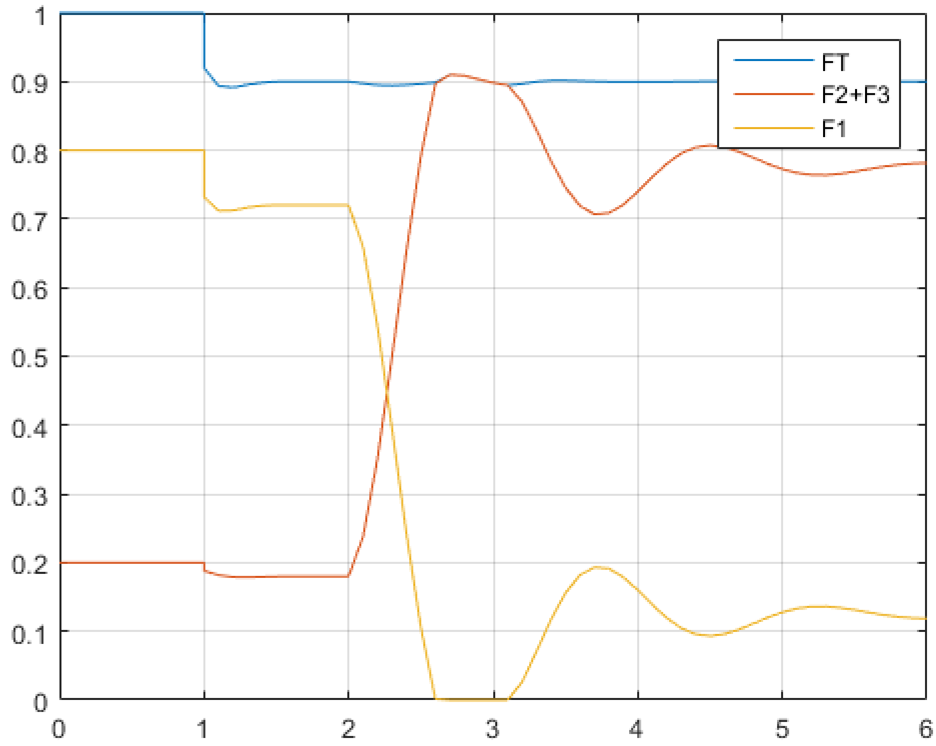
- At $\tau = 0$, the process is operating at steady state.
- At $\tau = 1$, the `Total Demand` decreases from $\phi = 1.0$ to $\phi = 0.9$.
- At $\tau = 2$, there is a large step increase in the concentration of constituent 1 in feed 1, from 1.17 to 2.17.

The controller maintains the inventory very close to its setpoint, but the severe disturbance in the feed composition causes a prediction error and a large disturbance in the blend composition, especially for constituent 1, `c_1`. However, the controller recovers and drives the blend composition back to its setpoint.

Verify Effect of Custom Constraints

Plot the feed rate signals.

```
figure
plot(MVs.time,[MVs.signals(1).values(:,2), ...
    (MVs.signals(2).values + MVs.signals(3).values), ...
    (MVs.signals(1).values(:,2)-MVs.signals(2).values-MVs.signals(3).values)])
grid
legend('FT','F2+F3','F1')
```



The total feed rate, FT, and the sum of feed rates F2 and F3 coincide for $1.7 \leq \tau \leq 2.2$. If the custom input constraints had not been included, the controller would have requested an impossible negative feed 1 rate, F1, during this period.

```
bdclose(sys)
```

See Also

`setconstraint`

Related Examples

- MPC Control with Constraints on a Combination of Input and Output Signals

- MPC Control of a Nonlinear Blending Process

More About

- “Constraints on Linear Combinations of Inputs and Outputs” on page 2-33

Providing LQR Performance Using Terminal Penalty

This example, from Scokaert and Rawlings [1], shows how to make a finite-horizon Model Predictive Controller equivalent to an infinite-horizon linear quadratic regulator (LQR).

The"Standard Cost Function" on page 2-2 is similar to that used in an LQR controller with output weighting, as shown in the following equation:

$$J(u) = \sum_{i=1}^{\infty} y(k+i)^T Q y(k+i) + u(k+i-1)^T R u(k+i-1)$$

The LQR and MPC cost functions differ in the following ways:

- The LQR cost function forces y and u towards zero whereas the MPC cost function forces y and u toward nonzero setpoints.

  You can shift the MPC prediction model's origin to eliminate this difference and achieve zero setpoints at nominal condition.
- The LQR cost function uses an infinite prediction horizon in which the manipulated variable changes at each sampling instant. In the standard MPC cost function, the horizon length is p, and the manipulated variable changes m times, where m is the control horizon.

The two cost functions are equivalent if the MPC cost function is:

$$J(u) = \sum_{i=1}^{p-1} y(k+i)^T Q y(k+i) + u(k+i-1)^T R u(k+i-1) + x(k+p)^T Q_p x(k+p)$$

where Q_p is a penalty applied at the last (i.e., terminal) prediction horizon step, and the prediction and control horizons are equal, i.e., $p = m$. The required Q_p is the Ricatti matrix that you can calculate using the Control System Toolbox `lqr` and `lqry` commands. The value is a positive definite symmetric matrix.

The following procedure shows how to design an unconstrained MPC controller that provides performance equivalent to a LQR controller:

1 Define a plant with one input and two outputs.

The plant is a double-integrator, represented as a state-space model in discrete-time with sampling interval 0.1 seconds.

```
A = [1 0;0.1 1];
B = [0.1;0.005];
C = eye(2);
D = zeros(2,1);
Ts = 0.1;
Plant = ss(A,B,C,D,Ts);
Plant.InputName = {'u'};
Plant.OutputName = {'x_1','x_2'};
```

2 Design an LQR controller with output feedback for the plant.

```
Q = eye(2);
R = 1;
[K,Qp] = lqry(Plant,Q,R);
```

`Q` and `R` are output and input weight matrices, respectively. $\mathtt{Q_p}$ is the Ricatti matrix.

3 Design an MPC controller equivalent to the LQR controller.

To implement Equation 4-2, compute L, the Cholesky decomposition of Q_p, such that $L^TL = Q_p$. Then, define auxiliary unmeasured output variables $y_a(k) = Lx(k)$ such that $y_a^T y_a = x^T Q_p x$. For the first `p-1` prediction horizon steps, the standard Q and R weights apply to the original u and y, and y_a has a zero penalty. On step p, the original u and y have zero penalties, and y_a has a unity penalty.

a Augment the plant model, and specify the augmented outputs as unmeasured.

```
NewPlant = Plant;
cholP = chol(Qp);
set(NewPlant,'C',[C;cholP],'D',[D;zeros(2,1)],...
    'OutputName',{'x_1','x_2','Cx_1','Cx_2'})
NewPlant.InputGroup.MV = 1;
NewPlant.OutputGroup.MO = [1 2];
NewPlant.OutputGroup.UO = [3 4];
```

b Create an MPC controller with equal prediction and control horizons.

```
P = 3;
M = 3;
MPCobj = mpc(NewPlant,Ts,P,M);

-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assumi
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. As
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming de
```

```
for output(s) y1 and zero weight for output(s) y2 y3 y4
```

When there are no constraints, you can use a rather short horizon (in this case, $p \geq 1$ gives identical results).

c Specify weights for manipulated variables (MV) and output variables (OV).

```
ywt = sqrt(diag(Q))';
uwt = sqrt(diag(R))';
MPCobj.Weights.OV = [ywt 0 0];
MPCobj.Weights.MV = uwt;
MPCobj.Weights.MVrate = 1e-6;
```

The two augmented outputs have zero weights during the prediction horizon.

d Specify terminal weights.

To obtain the desired effect, define unity weights for these at the final point in the horizon.

```
U = struct('Weight', uwt);
Y = struct('Weight', [0 0 1 1]);
setterminal(MPCobj, Y, U)
```

The first two states receive zero weight at the terminal point, and the input weight is unchanged.

e Remove default state estimator.

The model states are measured directly, so the default MPC state estimator is unnecessary.

```
setoutdist(MPCobj,'model',tf(zeros(4,1)))
setEstimator(MPCobj,[],C)
```

The `setoutdist` command removes the output disturbances from the output channels, and the `setEstimator` command sets the controller state estimates equal to the measured output values.

4 Compare the control performance of LQR, MPC with terminal weights, and a standard MPC.

a Compute closed-loop response with LQR controller.

```
clsys = feedback(Plant,K);
```

```
Tstop = 6;
x0 = [0.2;0.2];
[yLQR,tLQR] = initial(clsys,x0,Tstop);
```

b Compute closed-loop response with MPC with terminal weights.

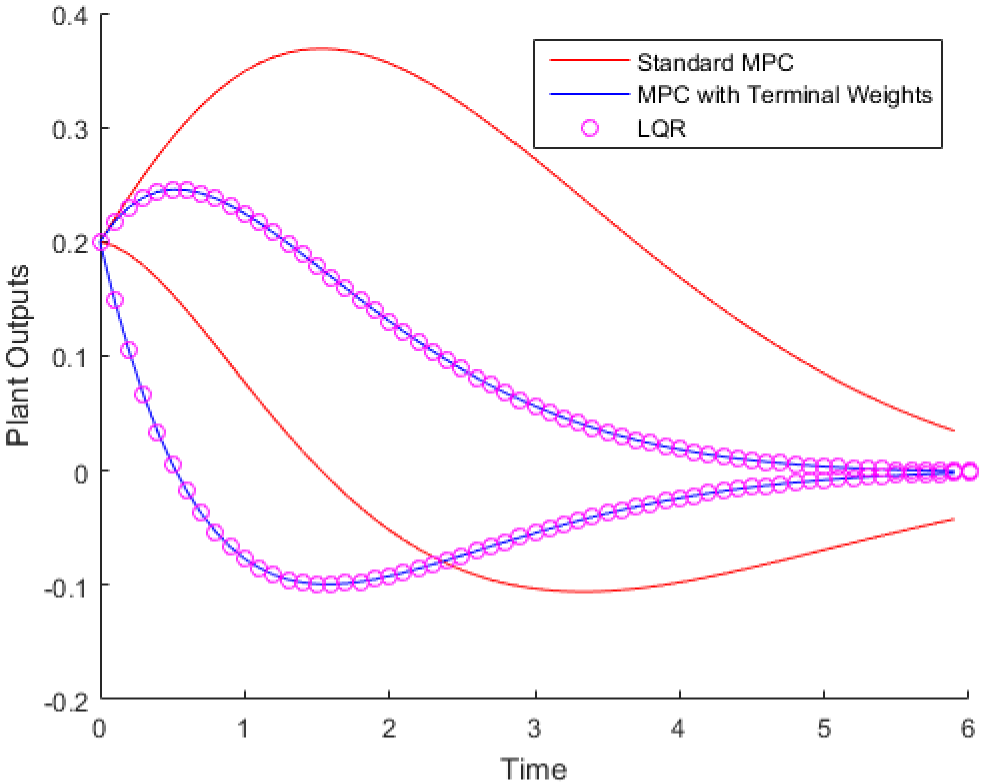
```
SimOptions = mpcsimopt(MPCobj);
SimOptions.PlantInitialState = x0;
r = zeros(1,4);
[y,t,u] = sim(MPCobj,ceil(Tstop/Ts),r,SimOptions);
Cost = sum(sum(y(:,1:2)*diag(ywt).*y(:,1:2))) + sum(u*diag(uwt).*u);

-->The "Model.Noise" property of the "mpc" object is empty. Assuming white nois
```

c Compute closed-loop response with standard MPC controller.

```
MPCobjSTD = mpc(Plant,Ts); % Default P = 10, M = 2
MPCobjSTD.Weights.MV = uwt;
MPCobjSTD.Weights.MVrate = 1e-6;
MPCobjSTD.Weights.OV = ywt;
SimOptions = mpcsimopt(MPCobjSTD);
SimOptions.PlantInitialState = x0;
r = zeros(1,2);
[ySTD,tSTD,uSTD] = sim(MPCobjSTD,ceil(Tstop/Ts),r,SimOptions);
CostSTD = sum(sum(ySTD*diag(ywt).*ySTD)) + sum(uSTD*uwt.*uSTD);

-->The "PredictionHorizon" property of "mpc" object is empty. Trying Prediction
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assumi
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. As
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming de
   for output(s) y1 and zero weight for output(s) y2
-->Assuming output disturbance added to measured output channel #1 is integrate
   Assuming no disturbance added to measured output channel #2.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white nois
```

d Compare the responses.

```
figure
h1 = line(tSTD,ySTD,'color','r');
h2 = line(t,y(:,1:2),'color','b');
h3 = line(tLQR,yLQR,'color','m','marker','o','linestyle','none');
xlabel('Time')
ylabel('Plant Outputs')
legend([h1(1) h2(1) h3(1)],'Standard MPC','MPC with Terminal Weights','LQR','Lo
```



The plot shows that the MPC controller with the terminal weights provides faster settling to the origin than the standard MPC. The LQR controller and MPC with terminal weights provide identical control performance.

As reported by Scokaert and Rawlings [1], the computed `Cost` value is 2.23, identical to that provided by the LQR controller. The computed `CostSTD` value for the standard MPC is 4.82, more than double compared to `Cost`.

You can improve the standard MPC by retuning. For example, use the same state estimation strategy. If the prediction and control horizons are then increased, it provides essentially the same performance.

This example shows that using a terminal penalty can eliminate the need to tune the MPC prediction and control horizons for the unconstrained case. If your application includes constraints, using a terminal weight is insufficient to guarantee nominal stability. You must also choose appropriate horizons and possibly add terminal constraints. For an in-depth discussion, see Rawlings and Mayne [2].

Although you can design and implement such a controller in Model Predictive Control Toolbox software, you might find designing the standard MPC controller more convenient.

References

[1] Scokaert, P. O. M. and J. B. Rawlings "Constrained linear quadratic regulation" *IEEE Transactions on Automatic Control* (1998), Vol. 43, No. 8, pp. 1163-1169.

[2] Rawlings, J. B., and David Q. Mayne "Model Predictive Control: Theory and Design" Nob Hill Publishing, 2010.

Related Examples

- "Designing Model Predictive Controller Equivalent to Infinite-Horizon LQR"

More About

- "Terminal Weights and Constraints" on page 2-30

Real-Time Control with OPC Toolbox

This example shows how to implement an online model predictive controller application using the OPC client supplied with the OPC Toolbox™.

The example uses the Matrikon™ Simulation OPC server to simulate the behavior of an industrial process on Windows® operating system.

Download the Matrikon™ OPC Simulation Server from "www.matrikon.com"

Download and install the server and set it running either as a service or as an application.

This example needs OPC Toolbox™.

```
if ~mpcchecktoolboxinstalled('opc')
    disp('The example needs OPC Toolbox(TM).')
end
```

```
The example needs OPC Toolbox(TM).
```

Establish a Connection to the OPC Server

Use OPC Toolbox commands to connect to the Matrikon OPC Simulation Server.

```
if mpcchecktoolboxinstalled('opc')
    % Clear any existing opc connections.
    opcreset
    % Flush the callback persistent variables.
    clear mpcopcPlantStep;
    clear mpcopcMPCStep;
    try
        h = opcda('localhost','Matrikon.OPC.Simulation.1');
        connect(h);
    catch ME
        disp('The Matrikon(TM) OPC Simulation Server must be running on the local machi
        return
    end
end
```

Set up the Plant OPC I/O

In practice the plant would be a physical process, and the OPC tags which define its I/O would already have been created on the OPC server. However, since in this case a simulation OPC server is being used, the plant behavior must be simulated. This is achieved by defining tags for the plant manipulated and measured variables and creating a callback (mpcopcPlantStep) to simulate plant response to changes in the manipulated variables. Two OPC groups are required, one to represent the two manipulated variables to be read by the plant simulator and another to write back the two measured plant outputs storing the results of the plant simulation.

```
if mpcchecktoolboxinstalled('opc')
    % Build an opc group for 2 plant inputs and initialize them to zero.
    plant_read = addgroup(h,'plant_read');
    imv1 = additem(plant_read,'Bucket Brigade.Real8', 'double');
    writeasync(imv1,0);
    imv2 = additem(plant_read,'Bucket Brigade.Real4', 'double');
    writeasync(imv2,0);
    % Build an opc group for plant outputs.
    plant_write = addgroup(h,'plant_write');
    opv1 = additem(plant_write,'Bucket Brigade.Time', 'double');
    opv2 = additem(plant_write,'Bucket Brigade.Money', 'double');
    plant_write.WriteAsyncFcn = []; % Suppress command line display.
end
```

Specify the MPC Controller Which Will Control the Simulated Plant

Create plant model.

```
plant_model = ss([-.2 -.1; 0 -.05],eye(2,2),eye(2,2),zeros(2,2));
disc_plant_model = c2d(plant_model,1);
% We assume no model mismatch, a control horizon 6 samples and
% prediction horizon 20 samples.
mpcobj = mpc(disc_plant_model,1,20,6);
mpcobj.weights.ManipulatedVariablesRate = [1 1];
% Build an internal MPC object structure so that the MPC object
% is not rebuilt each callback execution.
state = mpcstate(mpcobj);
y1 = mpcmove(mpcobj,state,[1;1]',[1 1]');
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->Assuming output disturbance added to measured output channel #2 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Build the OPC I/O for the MPC Controller

Build two OPC groups, one to read the two measured plant outputs and the other to write back the two manipulated variables.

```
if mpcchecktoolboxinstalled('opc')
    % Build an opc group for MPC inputs.
    mpc_read = addgroup(h,'mpc_read');
    impcpv1 = additem(mpc_read,'Bucket Brigade.Time', 'double');
    writeasync(impcpv1,0);
    impcpv2 = additem(mpc_read,'Bucket Brigade.Money', 'double');
    writeasync(impcpv2,0);
    impcref1 = additem(mpc_read,'Bucket Brigade.Int2', 'double');
    writeasync(impcref1,1);
    impcref2 = additem(mpc_read,'Bucket Brigade.Int4', 'double');
    writeasync(impcref2,1);
    % Build an opc group for mpc outputs.
    mpc_write = addgroup(h,'mpc_write');
    additem(mpc_write,'Bucket Brigade.Real8', 'double');
    additem(mpc_write,'Bucket Brigade.Real4', 'double');
    % Suppress command line display.
    mpc_write.WriteAsyncFcn = [];
end
```

Build OPC Groups to Trigger Execution of the Plant Simulator & Controller

Build two opc groups based on the same external opc timer to trigger execution of both plant simulation and MPC execution when the contents of the OPC time tag changes.

```
if mpcchecktoolboxinstalled('opc')
    gtime = addgroup(h,'time');
    time_tag = additem(gtime,'Triangle Waves.Real8');
    gtime.UpdateRate = 1;
    gtime.DataChangeFcn = {@mpcopcPlantStep plant_read plant_write disc_plant_model};
    gmpctime = addgroup(h,'mpctime');
    additem(gmpctime,'Triangle Waves.Real8');
    gmpctime.UpdateRate = 1;
    gmpctime.DataChangeFcn = {@mpcopcMPCStep mpc_read mpc_write mpcobj};
end
```

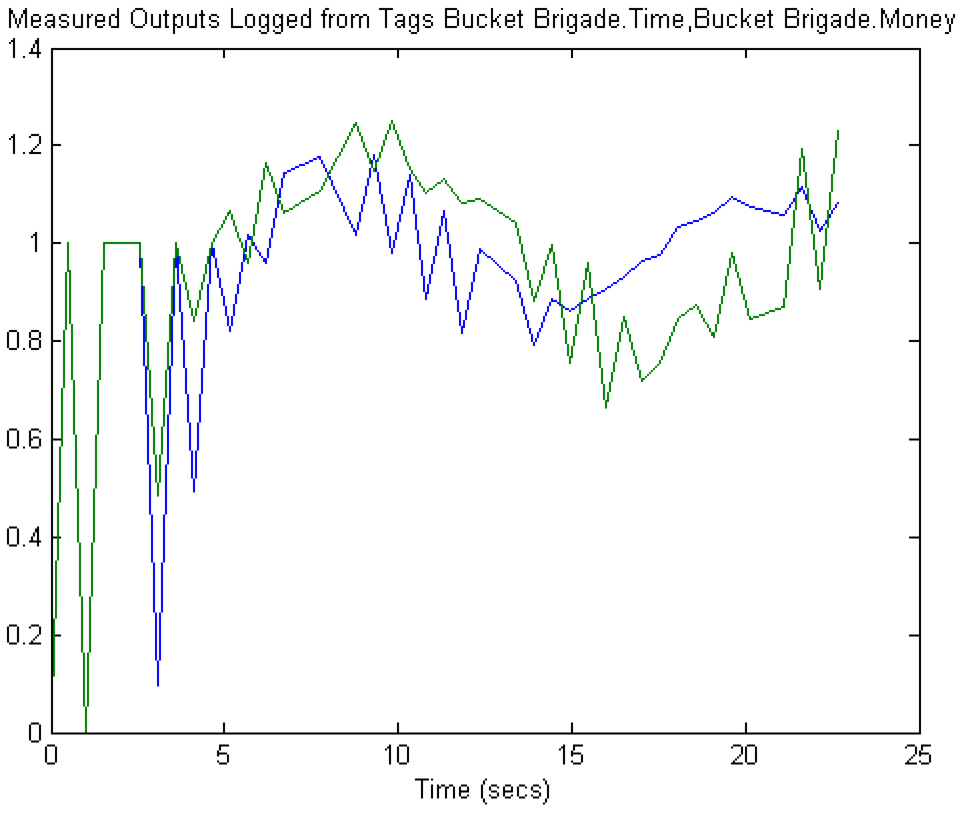
Log Data from the Plant Measured Outputs

Log the plant measured outputs from tags 'Bucket Brigade.Money' and 'Bucket Brigade.Money'.

```
if mpcchecktoolboxinstalled('opc')
    mpc_read.RecordsToAcquire = 40;
    start(mpc_read);
    while mpc_read.RecordsAcquired < mpc_read.RecordsToAcquire
       pause(3)
       fprintf('Logging data: Record %d / %d',mpc_read.RecordsAcquired,mpc_read.Records
    end
    stop(mpc_read);
end
```

Extract and Plot the Logged Data

```
if mpcchecktoolboxinstalled('opc')
    [itemID, value, quality, timeStamp, eventTime] = getdata(mpc_read,'double');
    plot((timeStamp(:,1)-timeStamp(1,1))*24*60*60,value)
    title('Measured Outputs Logged from Tags Bucket Brigade.Time,Bucket Brigade.Money')
    xlabel('Time (secs)');
end
```


Measured Outputs Logged from Tags Bucket Brigade.Time,Bucket Brigade.Money
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Time (secs)

Simulation and Code Generation Using Simulink Coder

This example shows how to simulate and generate real-time code for an MPC Controller block with Simulink Coder. Code can be generated in both single and double precisions.

Required Products

To run this example, Simulink® and Simulink® Coder™ are required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('simulinkcoder')
    disp('Simulink(R) Coder(TM) is required to run this example.');
    return
end
```

```
Simulink(R) Coder(TM) is required to run this example.
```

Setup Environment

You must have write-permission to generate the relevant files and the executable. So, before starting simulation and code generation, change the current directory to a temporary directory.

```
cwd = pwd;
tmpdir = tempname;
mkdir(tmpdir);
cd(tmpdir);
```

Define Plant Model and MPC Controller

Define a SISO plant.

```
plant = ss(tf([3 1],[1 0.6 1]));
```

Define the MPC controller for the plant.

```
Ts = 0.1;   %Sampling time
p = 10;     %Prediction horizon
m = 2;      %Control horizon
Weights = struct('MV',0,'MVRate',0.01,'OV',1); % Weights
```

```
MV = struct('Min',-Inf,'Max',Inf,'RateMin',-100,'RateMax',100); % Input constraints
OV = struct('Min',-2,'Max',2); % Output constraints
mpcobj = mpc(plant,Ts,p,m,Weights,MV,OV);
```

Simulate and Generate Code in Double-Precision

By default, MPC Controller blocks use double-precision in simulation and code generation.

Simulate the model in Simulink.

```
mdl1 = 'mpc_rtwdemo';
open_system(mdl1);
sim(mdl1);
```

The controller effort and the plant output are saved into base workspace as variables **u** and **y**, respectively.

Build the model with the `rtwbuild` command.

```
disp('Generating C code... Please wait until it finishes.');
set_param(mdl1,'RTWVerbose','off');
rtwbuild(mdl1);
```

On a Windows system, an executable file named "mpc_rtwdemo.exe" appears in the temporary directory after the build process finishes.

Run the executable.

```
if ispc
    disp('Running executable...');
    status = system(mdl1);
else
    disp('The example only runs the executable on Windows system.');
end
```

After the executable completes successfully (status=0), a data file named "mpc_rtwdemo.mat" appears in the temporary directory.

Compare the responses from the generated code (**rt_u** and **rt_y**) with the responses from the previous simulation in Simulink (**u** and **y**).

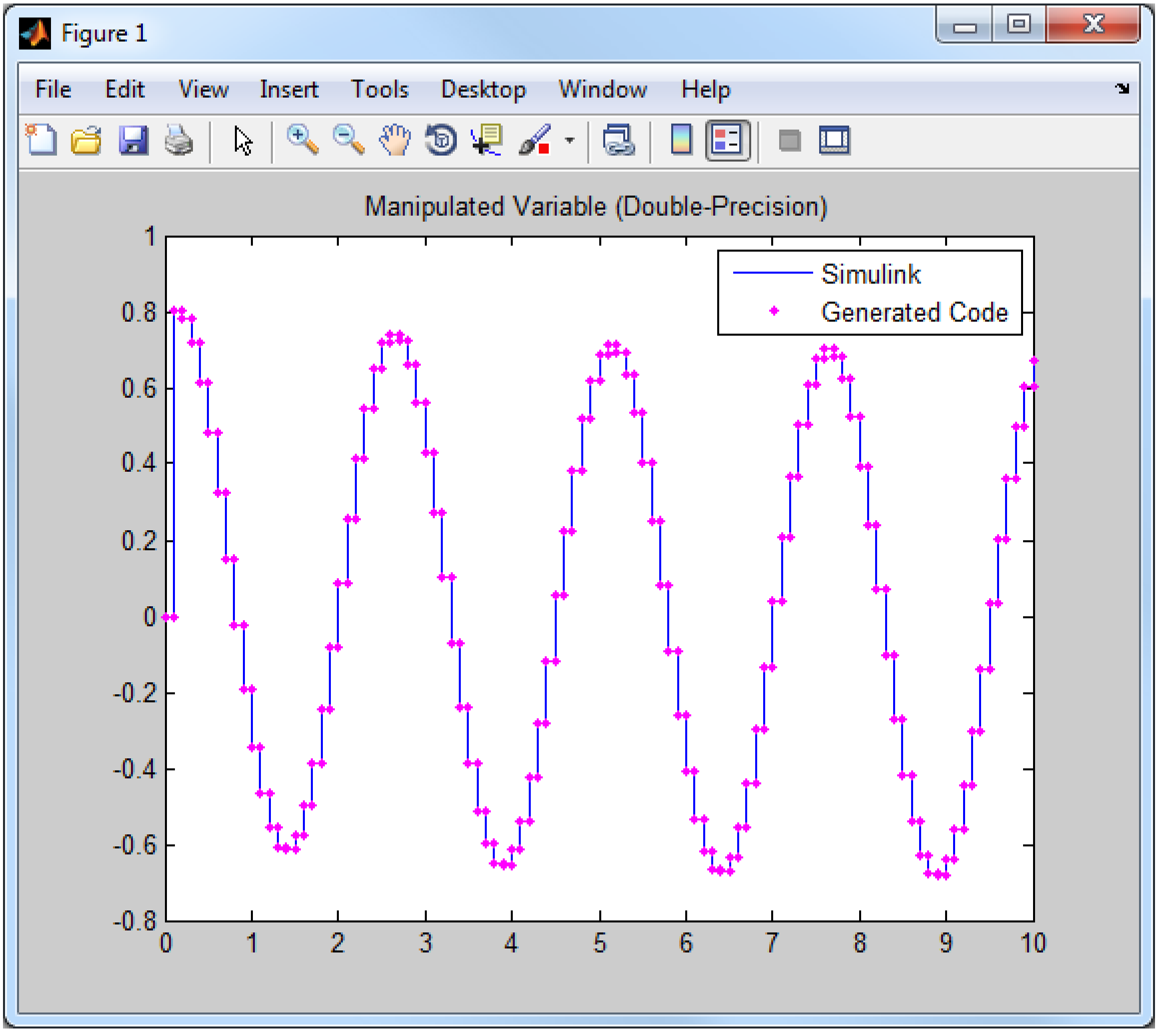
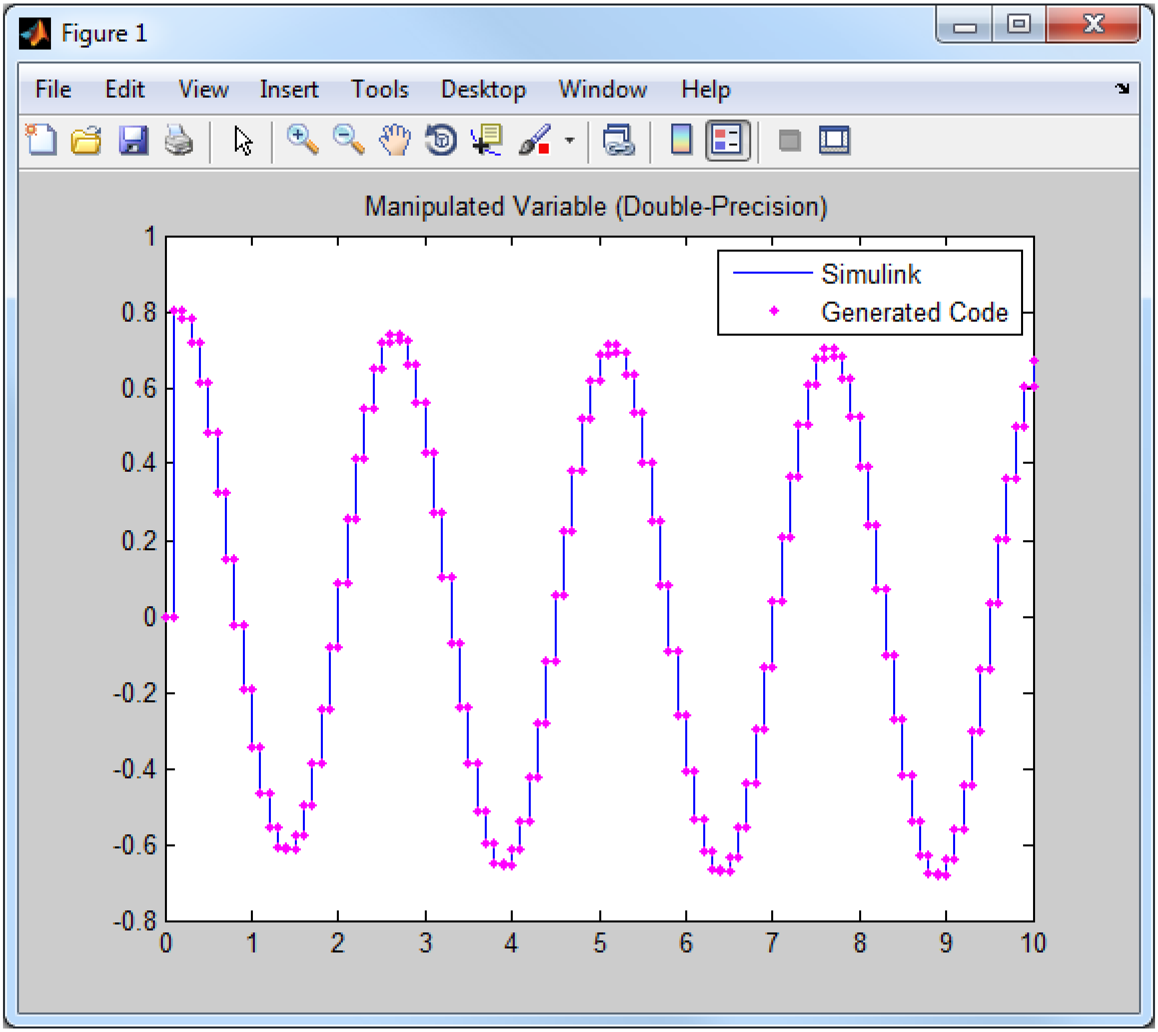


Figure 1
File Edit View Insert Tools Desktop Window Help
Manipulated Variable (Double-Precision)
Simulink
Generated Code
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 1 2 3 4 5 6 7 8 9 10



They are numerically equal.

Simulate and Generate Code in Single-Precision

You can also configure the MPC block to use single-precision in simulation and code generation.

```
mdl2 = 'mpc_rtwdemo_single';
```

```
open_system(mdl2);
```

To do that, open the MPC block dialog and select "single" as the "output data type" at the bottom of the dialog.

```
open_system([mdl2 '/MPC Controller']);
```

Simulate the model in Simulink.

```
close_system([mdl2 '/MPC Controller']);
sim(mdl2);
```

The controller effort and the plant output are saved into base workspace as variables **u1** and **y1**, respectively.

Build the model with the `rtwbuild` command.

```
disp('Generating C code... Please wait until it finishes.');
set_param(mdl2,'RTWVerbose','off');
rtwbuild(mdl2);
```

On a Windows system, an executable file named "mpc_rtwdemo_single.exe" appears in the temporary directory after the build process finishes.

Run the executable.

```
if ispc
    disp('Running executable...');
    status = system(mdl2);
else
    disp('The example only runs the executable on Windows system.');
end
```

After the executable completes successfully (status=0), a data file named "mpc_rtwdemo_single.mat" appears in the temporary directory.

Compare the responses from the generated code (**rt_u1** and **rt_y1**) with the responses from the previous simulation in Simulink (**u1** and **y1**).

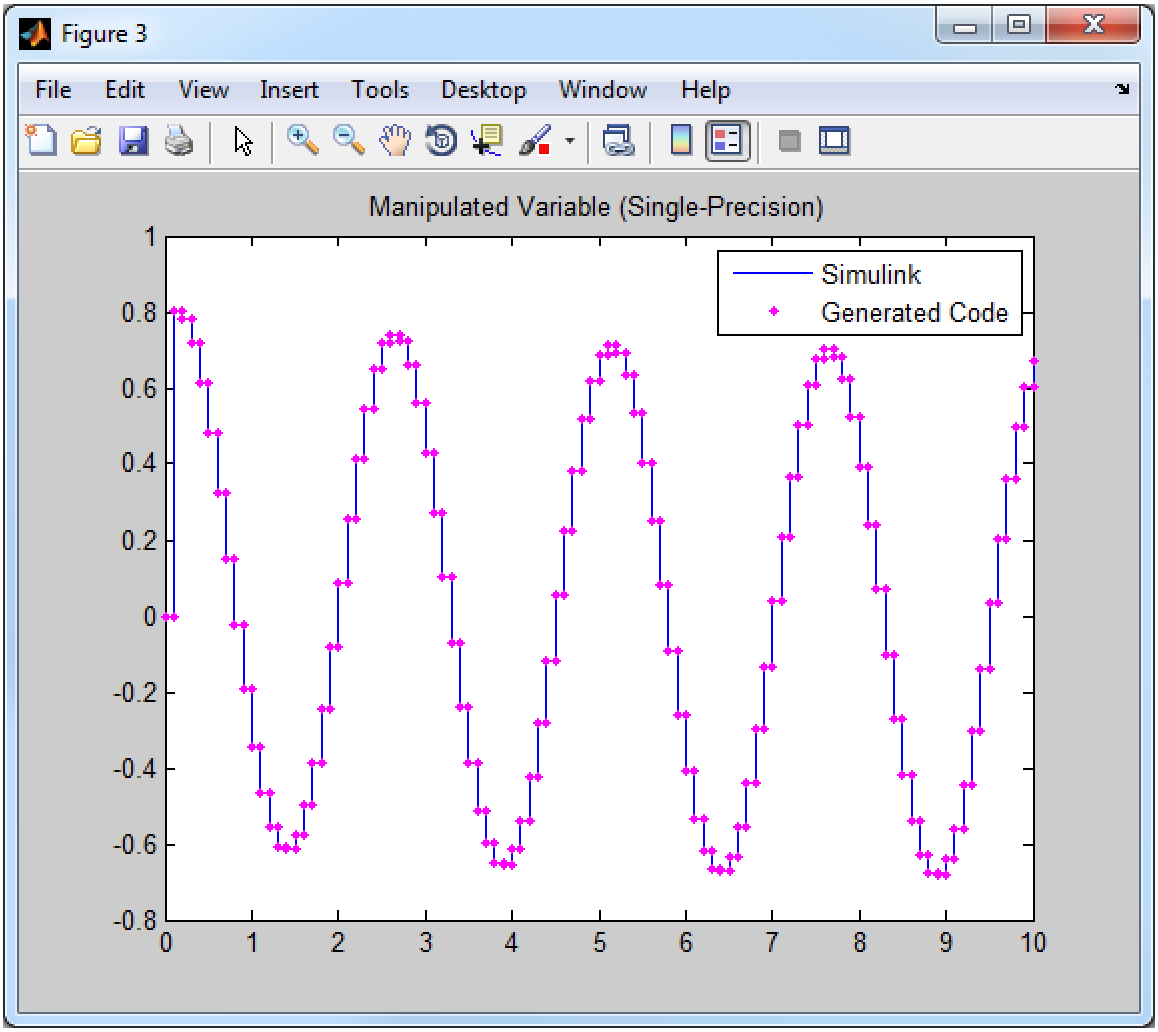
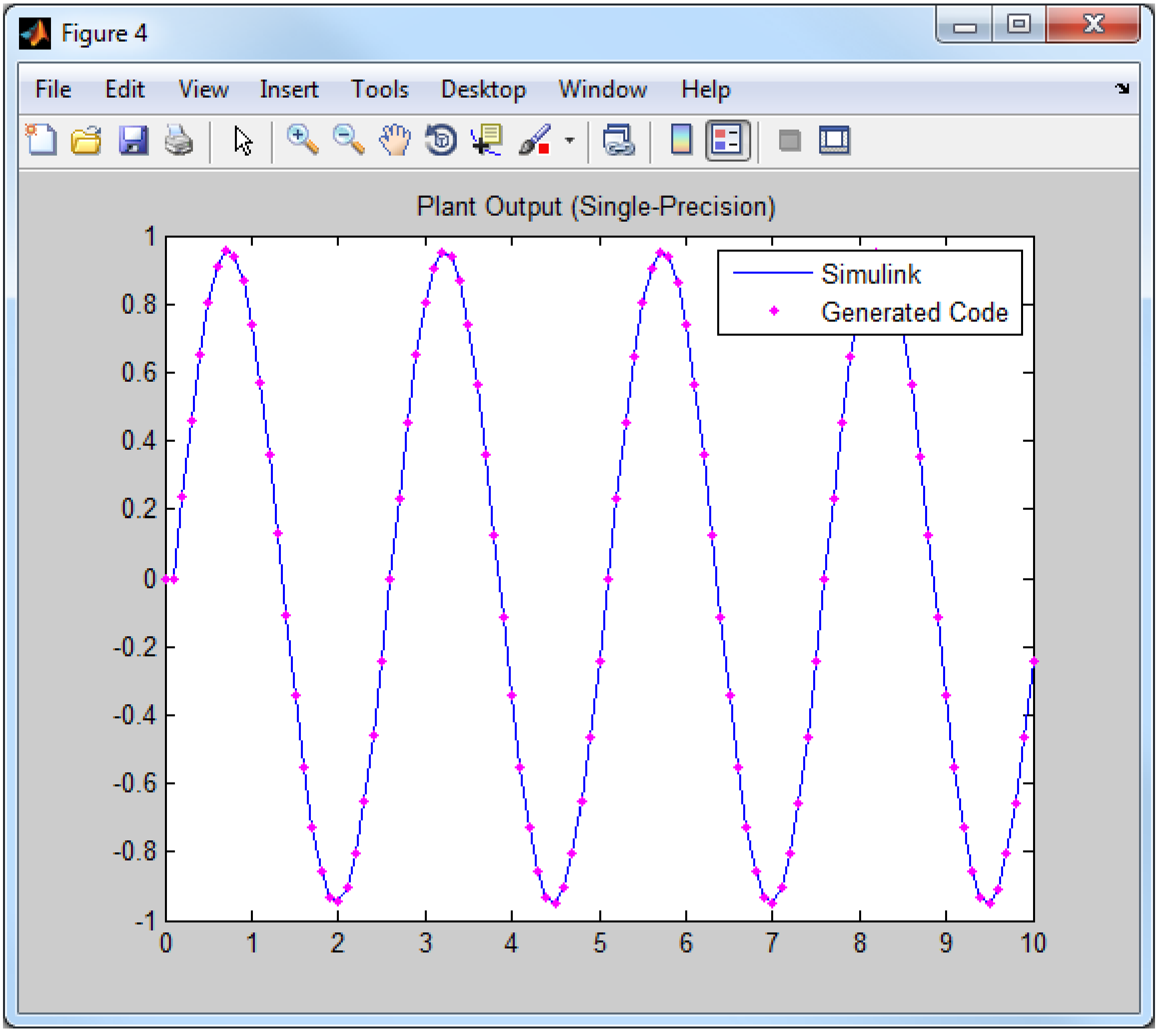


Figure 3
File Edit View Insert Tools Desktop Window Help
Manipulated Variable (Single-Precision)
Simulink
Generated Code
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 1 2 3 4 5 6 7 8 9 10



They are numerically equal.

Close the Simulink model.

```
bdclose(mdl1);
bdclose(mdl2);

cd(cwd)
```

Simulation and Structured Text Generation Using PLC Coder

This example shows how to simulate and generate Structured Text for an MPC Controller block using PLC Coder software. The generated code uses single-precision.

Required Products

To run this example, Simulink® and Simulink® PLC Coder™ are required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('plccoder')
    disp('Simulink(R) PLC Coder(TM) is required to run this example.');
    return
end
```

```
Simulink(R) PLC Coder(TM) is required to run this example.
```

Setup Environment

You must have write-permission to generate the relevant files and the executable. So, before starting simulation and code generation, change the current directory to a temporary directory.

```
cwd = pwd;
tmpdir = tempname;
mkdir(tmpdir);
cd(tmpdir);
```

Define Plant Model and MPC Controller

Define a SISO plant.

```
plant = ss(tf([3 1],[1 0.6 1]));
```

Define the MPC controller for the plant.

```
Ts = 0.1;   %Sampling time
p = 10;     %Prediction horizon
m = 2;      %Control horizon
Weights = struct('MV',0,'MVRate',0.01,'OV',1); % Weights
```

```
MV = struct('Min',-Inf,'Max',Inf,'RateMin',-100,'RateMax',100); % Input constraints
OV = struct('Min',-2,'Max',2); % Output constraints
mpcobj = mpc(plant,Ts,p,m,Weights,MV,OV);
```

Simulate and Generate Structured Text

Open the Simulink model.

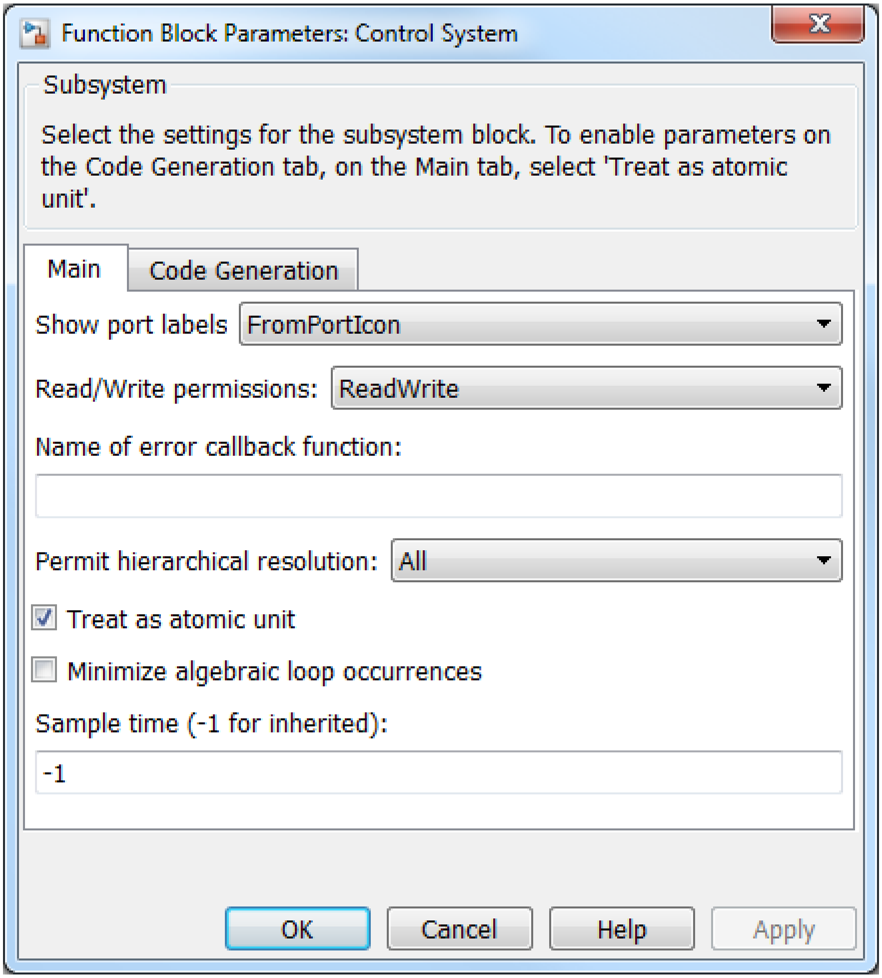
```
mdl = 'mpc_plcdemo';
open_system(mdl);
```

To generate structured text for the MPC Controller block, complete the following two steps:

- Configure the MPC block to use single precision. Select "single" in the "Output data type" combo box in the MPC block dialog.

```
open_system([mdl '/Control System/MPC Controller']);
```

- Put MPC block inside a subsystem block and treat the subsystem block as an atomic unit. Select the "Treat as atomic unit" checkbox in the subsystem block dialog.



Simulate the model in Simulink.

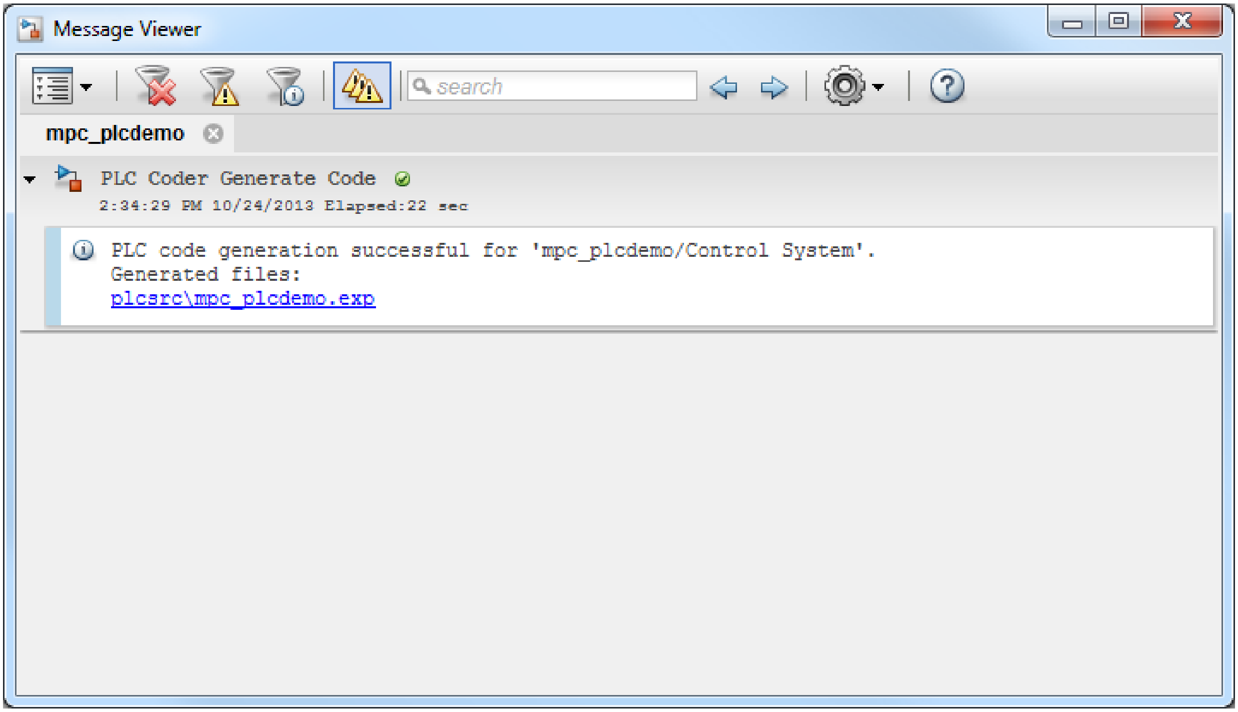
```
close_system([mdl '/Control System/MPC Controller']);
open_system([mdl '/Outputs//References']);
open_system([mdl '/Inputs']);
sim(mdl);
```

To generate code with the PLC Coder, use the `plcgeneratecode` command.

```
disp('Generating PLC structure text... Please wait until it finishes.');
```

```
plcgeneratecode([mdl '/Control System']);
```

The Message Viewer dialog box shows that PLC code generation was successful.



Close the Simulink model.

```
bdclose(mdl);
```

```
cd(cwd)
```

Setting Targets for Manipulated Variables

This example shows how to design a model predictive controller for a plant with two inputs and one output with target set-point for a manipulated variable.

Define Plant Model

The linear plant model has two inputs and two outputs.

```
N1 = [3 1];
D1 = [1 2*.3 1];
N2 = [2 1];
D2 = [1 2*.5 1];
plant = ss(tf({N1,N2},{D1,D2}));
A = plant.a;
B = plant.b;
C = plant.c;
D = plant.d;
x0 = [0 0 0 0]';
```

Design MPC Controller

Create MPC controller.

```
Ts = 0.4;                     % Sampling time
mpcobj = mpc(plant,Ts,20,5);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

Specify weights.

```
mpcobj.weights.manipulated = [0.3 0]; % weight difference MV#1 - Target#1
mpcobj.weights.manipulatedrate = [0 0];
mpcobj.weights.output = 1;
```

Define input specifications.

```
mpcobj.MV = struct('RateMin',{-0.5;-0.5},'RateMax',{0.5;0.5});
```

Specify target set-point u=2 for the first manipulated variable.

```
mpcobj.MV(1).Target=2;
```

Simulation Using Simulink®

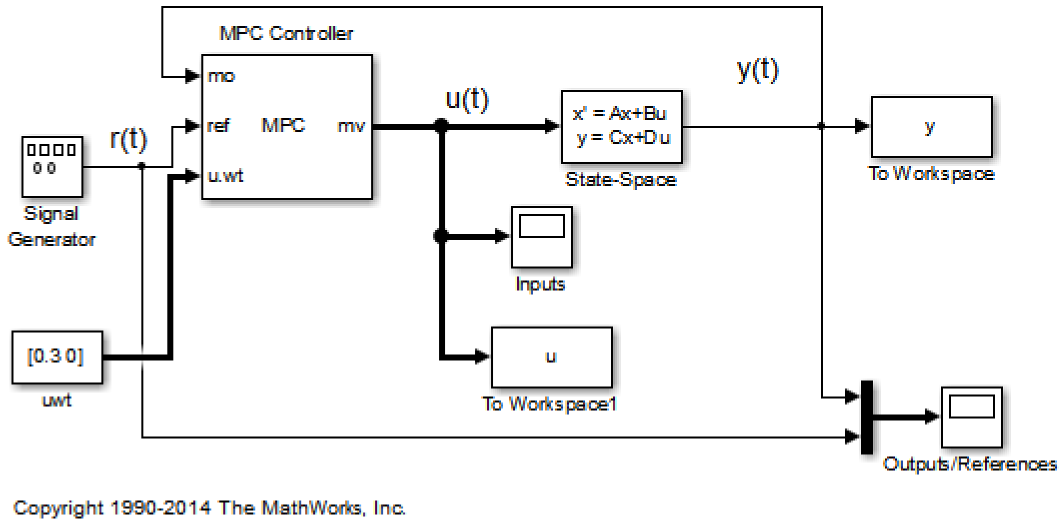
To run this example, Simulink® is required.

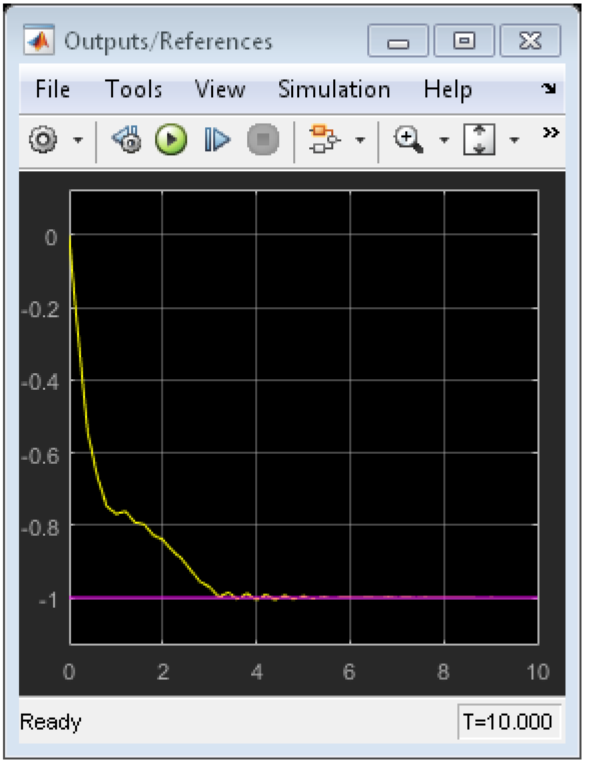
```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
```

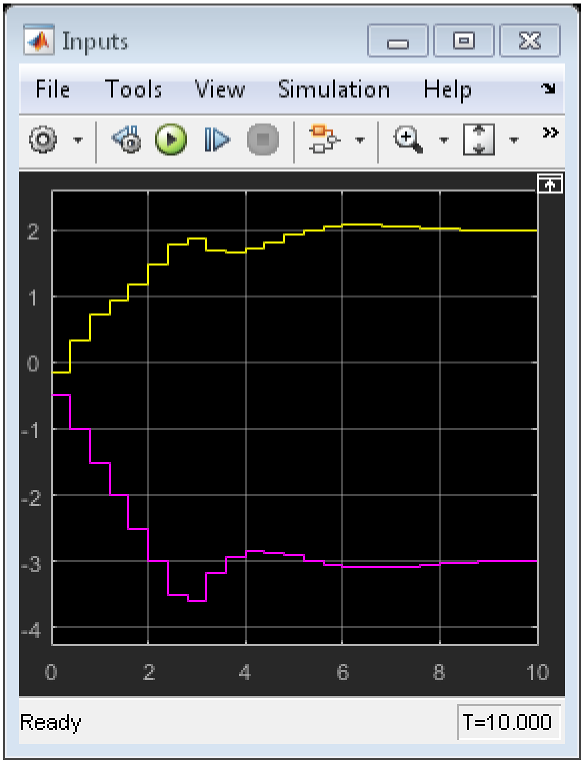
Simulate.

```
mdl = 'mpc_utarget';
open_system(mdl)       % Open Simulink(R) Model
sim(mdl);              % Start Simulation
```

```
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```




Outputs/References
File Tools View Simulation Help
0
-0.2
-0.4
-0.6
-0.8
-1
0 2 4 6 8 10
Ready
T=10.000


Inputs
File Tools View Simulation Help
2
1
0
-1
-2
-3
-4
0 2 4 6 8 10
Ready
T=10.000

```
bdclose(mdl)
```

Specifying Alternative Cost Function with Off-Diagonal Weight Matrices

This example shows how to use non-diagonal weight matrices in a model predictive controller.

Define Plant Model and MPC Controller

The linear plant model has two inputs and two outputs.

```
plant = ss(tf({1,1;1,2},{[1 .5 1],[.7 .5 1];[1 .4 2],[1 2]}));
[A,B,C,D] = ssdata(plant);
Ts = 0.1;               % sampling time
plant = c2d(plant,Ts);  % convert to discrete time
```

Create MPC controller.

```
p=20;        % prediction horizon
m=2;         % control horizon
mpcobj = mpc(plant,Ts,p,m);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

Define constraints on the manipulated variable.

```
mpcobj.MV = struct('Min',{-3;-2},'Max',{3;2},'RateMin',{-100;-100},'RateMax',{100;100})
```

Define non-diagonal output weight. Note that it is specified inside a cell array.

```
OW = [1 -1]'*[1 -1];
% Non-diagonal output weight, corresponding to ((y1-r1)-(y2-r2))^2
mpcobj.Weights.OutputVariables = {OW};
% Non-diagonal input weight, corresponding to (u1-u2)^2
mpcobj.Weights.ManipulatedVariables = {0.5*OW};
```

Simulate Using SIM Command

Specify simulation options.

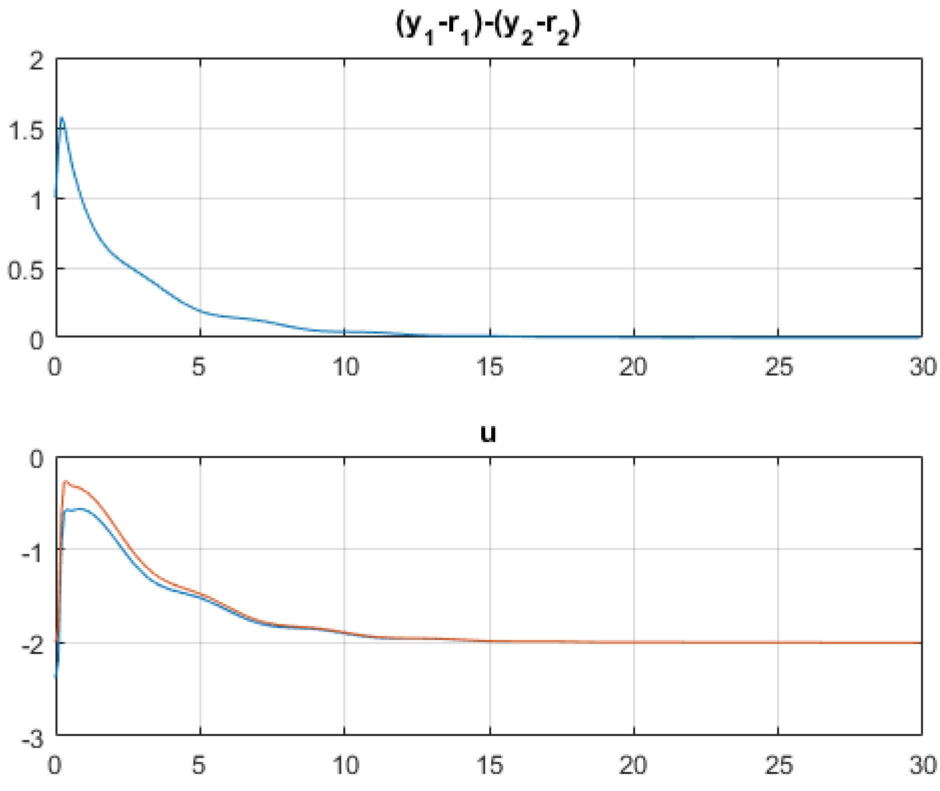
```
Tstop = 30;               % simulation time
Tf = round(Tstop/Ts);     % number of simulation steps
```

```
r = ones(Tf,1)*[1 2];     % reference trajectory
```

Run the closed-loop simulation and plot results.

```
[y,t,u] = sim(mpcobj,Tf,r);
subplot(211)
plot(t,y(:,1)-r(1,1)-y(:,2)+r(1,2));grid
title('(y_1-r_1)-(y_2-r_2)');
subplot(212)
plot(t,u);grid
title('u');
```

```
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->Assuming output disturbance added to measured output channel #2 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```



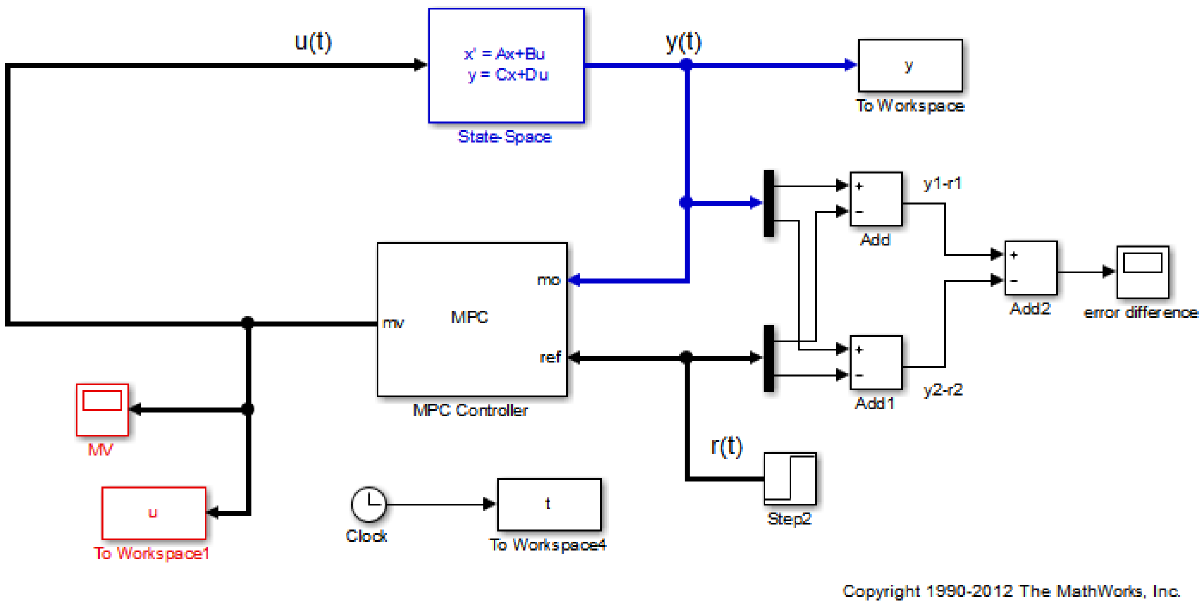
Simulate Using Simulink®

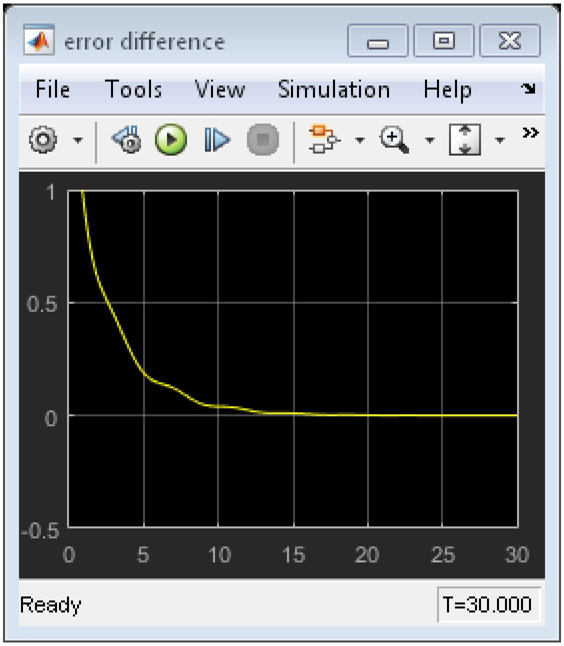
To run this example, Simulink® is required.

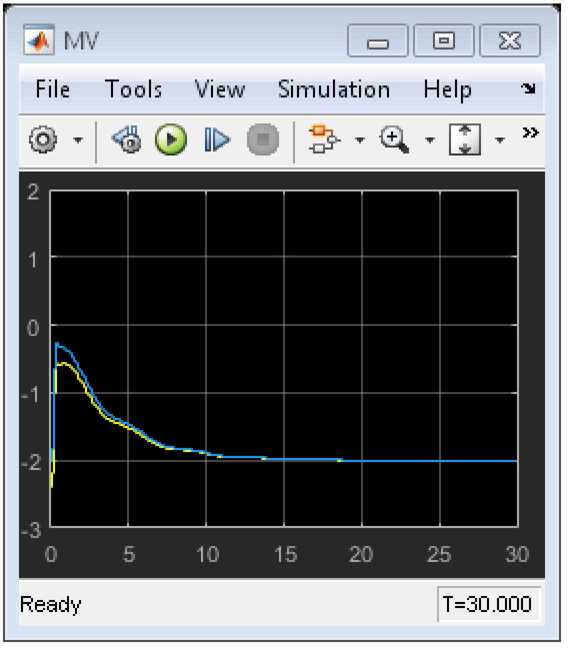
```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this part of the example.')
    return
end
```

Now simulate closed-loop MPC in Simulink®.

```
mdl = 'mpc_weightsdemo';
open_system(mdl);
sim(mdl)
```




error difference
File Tools View Simulation Help
1
0.5
0
-0.5
0 5 10 15 20 25 30
Ready
T=30.000


MV
File Tools View Simulation Help
2
1
0
-1
-2
-3
0 5 10 15 20 25 30
Ready
T=30.000

```
bdclose(mdl);
```

Review Model Predictive Controller for Stability and Robustness Issues

This example shows how to use the `review` command to detect potential issues with a model predictive controller design.

The Fuel Gas Blending Process

The example application is a fuel gas blending process. The objective is to blend six gases to obtain a fuel gas, which is then burned to provide process heating. The fuel gas must satisfy three quality standards in order for it to burn reliably and with the expected heat output. The fuel gas header pressure must also be controlled. Thus, there are four controlled output variables. The manipulated variables are the six feed gas flow rates.

Inputs:

```
1. Natural Gas (NG)
2. Reformed Gas (RG)
3. Hydrogen (H2)
4. Nitrogen (N2)
5. Tail Gas 1 (T1)
6. Tail Gas 2 (T2)
```

Outputs:

```
1. High Heating Value (HHV)
2. Wobbe Index (WI)
3. Flame Speed Index (FSI)
4. Header Pressure (P)
```

The fuel gas blending process was studied by Muller et al.: "Modeling, validation, and control of an industrial fuel gas blending system", C.J. Muller, I.K. Craig, N.L. Ricker, J. of Process Control, in press, 2011.

Linear Plant Model

Use the following linear plant model as the prediction model for the controller. This state-space model, applicable at a typical steady-state operating point, uses the time unit of hours.

```
a = diag([-28.6120, -28.6822, -28.5134,  -0.0281, -23.2191, -23.4266, ...
```

```
          -22.9377, - 0.0101, -26.4877, -26.7950, -27.2210,  -0.0083, ...
          -23.0890, -23.0062, -22.9349,  -0.0115, -25.8581, -25.6939, ...
          -27.0793,  -0.0117, -22.8975, -22.8233, -21.1142,  -0.0065]);
b = zeros(24,6);
b( 1: 4,1) = [4, 4, 8, 32]';
b( 5: 8,2) = [2, 2, 4, 32]';
b( 9:12,3) = [2, 2, 4, 32]';
b(13:16,4) = [4, 4, 8, 32]';
b(17:20,5) = [2, 2, 4, 32]';
b(21:24,6) = [1, 2, 1, 32]';
c = [diag([ 6.1510,  7.6785, -5.9312, 34.2689]), ...
     diag([-2.2158, -3.1204,  2.6220, 35.3561]), ...
     diag([-2.5223,  1.1480,  7.8136, 35.0376]), ...
     diag([-3.3187, -7.6067, -6.2755, 34.8720]), ...
     diag([-1.6583, -2.0249,  2.5584, 34.7881]), ...
     diag([-1.6807, -1.2217,  1.0492, 35.0297])];
d = zeros(4,6);
Plant = ss(a, b, c, d);
```

By default, all the plant inputs are manipulated variables.

```
Plant.InputName = {'NG', 'RG', 'H2', 'N2', 'T1', 'T2'};
```

By default, all the plant outputs are measured outputs.

```
Plant.OutputName = {'HHV', 'WI', 'FSI', 'P'};
```

Transport delay is added to plant outputs to reflect the delay in the sensors.

```
Plant.OutputDelay = [0.00556  0.0167  0.00556  0];
```

Initial Controller Design

Construct an initial model predictive controller based on design requirements.

Specify sampling time, horizons and steady-state values.

The sampling time is that of the sensors (20 seconds). The prediction horizon is approximately equal to the plant settling time (39 intervals). The control horizon uses four blocked moves that have lengths of 2, 6, 12 and 19 intervals respectively. The nominal operating conditions are non-zero. The output measurement noise is white noise with magnitude of 0.001.

```
MPC_verbosity = mpcverbosity('off'); % Disable MPC message displaying at command line
```

```
Ts = 20/3600;   % Time units are hours.
Obj = mpc(Plant, Ts, 39, [2, 6, 12, 19]);
Obj.Model.Noise = ss(0.001*eye(4));
Obj.Model.Nominal.Y = [16.5, 25, 43.8, 2100];
Obj.Model.Nominal.U = [1.4170, 0, 2, 0, 0, 26.5829];
```

Specify lower and upper bounds on manipulated variables.

Since all the manipulated variables are flow rates of gas streams, their lower bounds are zero. All the MV constraints are hard (MinECR and MaxECR = 0) by default.

```
MVmin = zeros(1,6);
MVmax = [15, 20, 5, 5, 30, 30];
for i = 1:6
    Obj.MV(i).Min = MVmin(i);
    Obj.MV(i).Max = MVmax(i);
end
```

Specify lower and upper bounds on manipulated variable increments.

The bounds are set large enough to allow full range of movement in one interval. All the MV rate constraints are hard (RateMinECR and RateMaxECR = 0) by default.

```
for i = 1:6
    Obj.MV(i).RateMin = -MVmax(i);
    Obj.MV(i).RateMax =  MVmax(i);
end
```

Specify lower and upper bounds on plant outputs.

All the OV constraints are soft (MinECR and MaxECR = 0) by default.

```
OVmin = [16.5, 25, 39, 2000];
OVmax = [18.0, 27, 46, 2200];
for i = 1:4
    Obj.OV(i).Min = OVmin(i);
    Obj.OV(i).Max = OVmax(i);
end
```

Specify weights on manipulated variables.

MV weights are specified based on the known costs of each feed stream. This tells MPC controller how to move the six manipulated variables in order to minimize the cost of the blended fuel gas. The weights are normalized so the maximum weight is approximately 1.0.

```
Obj.Weights.MV = [54.9, 20.5, 0, 5.73, 0, 0]/55;
```

Specify weights on manipulated variable increments.

They are small relative to the maximum MV weight so the MVs are free to vary.

```
Obj.Weights.MVrate = 0.1*ones(1,6);
```

Specify weights on plant outputs.

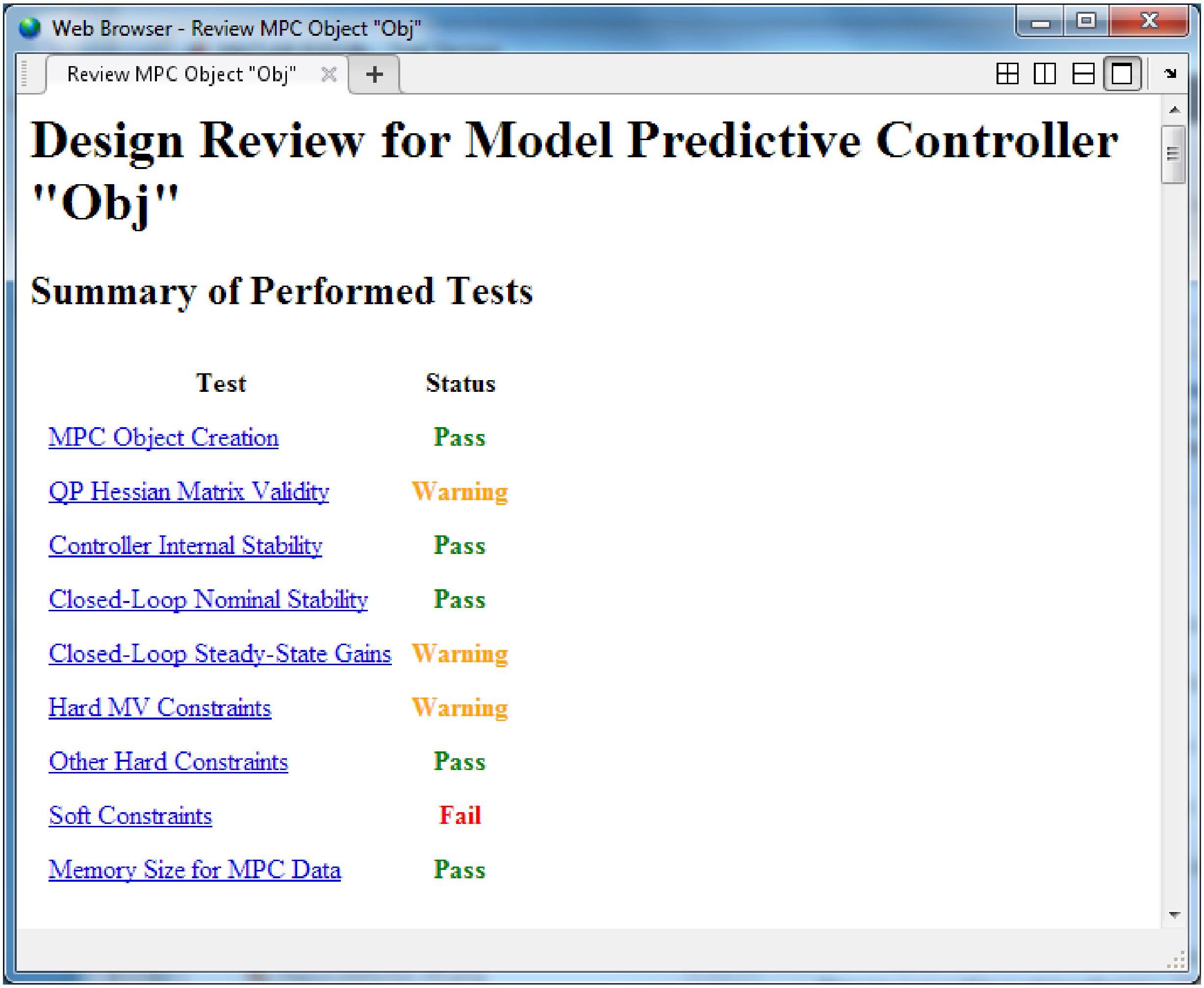
The OV weights penalize deviations from specified setpoints and would normally be "large" relative to the other weights. Let us first consider the default values, which equal the maximum MV weight specified above.

```
Obj.Weights.OV = [1, 1, 1, 1];
```

Using the `review` Command to Improve the Initial Design

Review the initial controller design.

```
review(Obj)
```



The summary table shown above lists three warnings and one error. Let's consider these in turn. Click **QP Hessian Matrix Validity** and scroll down to display the warning. It indicates that the plant signal magnitudes differ significantly. Specifically, the pressure response is much larger than the others.

Scale Factors

Scaling converts the relationship between output variables and manipulated variables to dimensionless form. Scale factor specifications can improve QP numerical accuracy. They also make it easier to specify tuning weight magnitudes.

In order for the outputs to be controllable, each must respond to at least one manipulated variable within the prediction horizon. If the plant is well scaled, the maximum absolute value of such responses should be of order unity.

Outputs whose maximum absolute scaled responses are outside the range [0.1,10] appear below. The table shows the maximum absolute response of each such OV with respect to each MV.

	NG	RG	H2	N2	T1	T2
P	236.876	244.868	242.709	241.478	240.892	242.702

Warning: at least one output variable response indicates poor scaling. Consider adjusting MV and OV ScaleFactors.

Examination of the specified OV bounds shows that the spans are quite different, and the pressure span is two orders of magnitude larger than the others. It is good practice to specify MPC scale factors to account for the expected differences in signal magnitudes. We are already weighting MVs based on relative cost, so we focus on the OVs only.

Calculate OV spans

```
OVspan = OVmax - OVmin;
%
% Use these as the specified scale factors
for i = 1:4
    Obj.OV(i).ScaleFactor = OVspan(i);
end
% Use review to verify that the scale factor warning has disappeared.
review(Obj);
%
% <<reviewDemo03.png>>
```

The next warning indicates that the controller does not drive the OVs to their targets at steady state. Click **Closed-Loop Steady-State Gains** to see a list of the non-zero gains.

Closed-Loop Steady-State Gains

`cloffset` is used to determine whether the controller forces all controlled output variables to their targets at steady state, in the absence of constraints.

The command calculates the impact of a sustained disturbance on each measured output variable (OV) in terms of an input/output gain. If a gain is zero, the controller eliminates steady-state tracking error for that disturbance-to-output mapping.

The gains with magnitudes exceeding 1e-05 are as follows:

Disturbed OV	Affected OV	Gain
HHV	HHV	0.0860281
WI	HHV	-0.0344992
FSI	HHV	0.0665757
HHV	WI	-0.036145
WI	WI	0.014495
FSI	WI	-0.027972
HHV	FSI	0.279361
WI	FSI	-0.11203
FSI	FSI	0.216193
HHV	P	0.0468767
WI	P	-0.0187986
FSI	P	0.036277

Warning: your design allows non-zero steady-state tracking errors in at least one controlled output. If this was not your intent, possible causes are as follows:

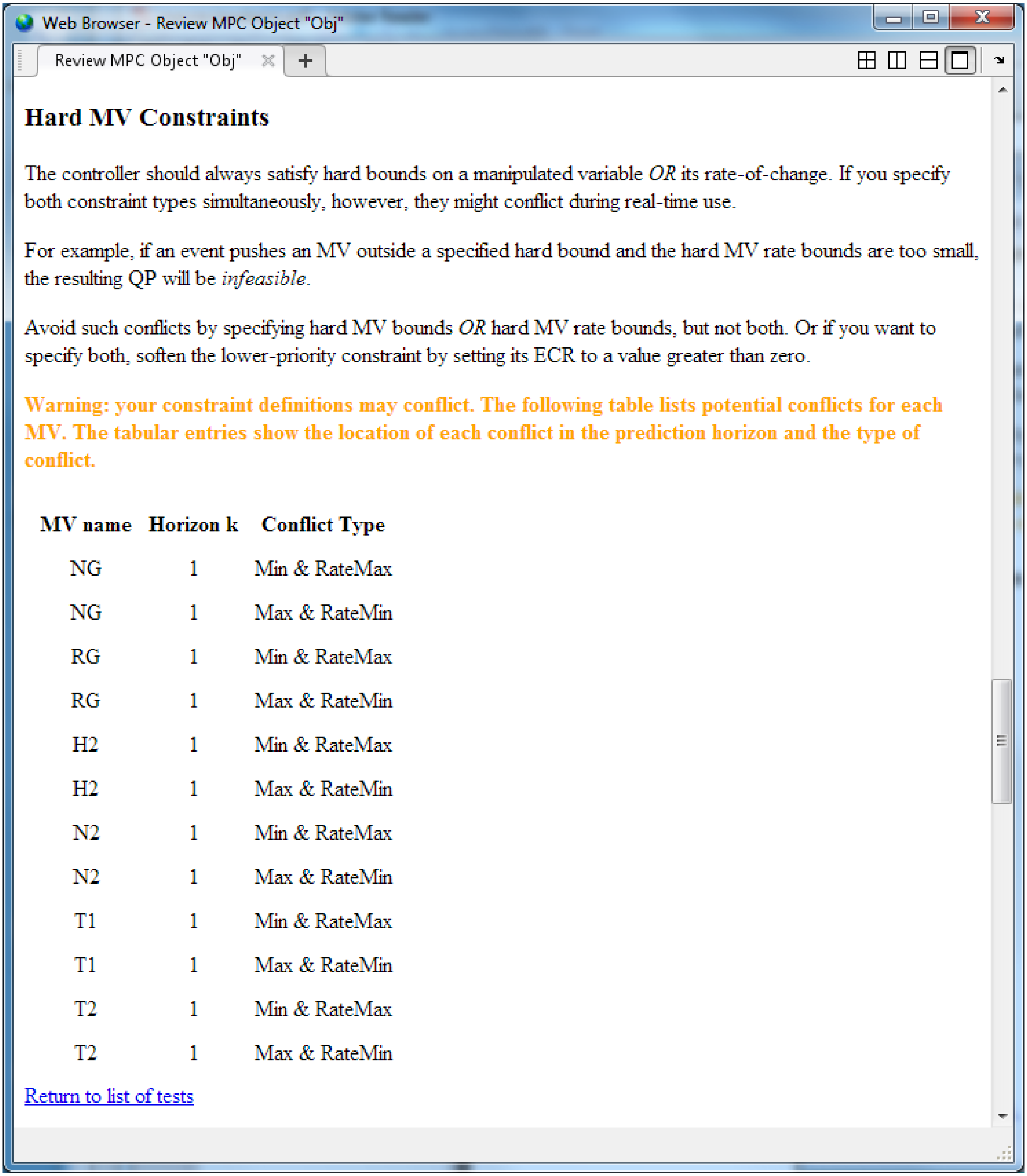
- Zero penalty weight on a plant output. Check the Weights.OV property.
- Non-zero penalty weight on a manipulated variable. Check the Weights.MV property.
- State estimator that does not include integration of output tracking error. The default estimator includes integration. If you have modified or replaced it, review your estimator design.

The first entry in the list shows that adding a sustained disturbance of unit magnitude to the HHV output would cause the HHV to deviate 0.0860 units from its steady-state target, assuming no constraints are active. The second entry shows that a unit disturbance in WI would cause a steady-state deviation ("offset") of -0.0345 in HHV, etc.

Since there are six MVs and only four OVs, excess degrees of freedom are available and you might be surprised to see non-zero steady-state offsets. The non-zero MV weights we have specified in order to drive the plant toward the most economical operating condition are causing this.

Non-zero steady-state offsets are often undesirable but are acceptable in this application because: # The primary objective is to minimize the blend cost. The gas quality (HHV, etc.) can vary freely within the specified OV limits. # The small offset gain magnitudes indicate that the impact of disturbances would be small. # The OV limits are soft constraints. Small, short-term violations are acceptable.

View the second warning by clicking **Hard MV Constraints**, which indicates a potential hard-constraint conflict.



Hard MV Constraints

The controller should always satisfy hard bounds on a manipulated variable *OR* its rate-of-change. If you specify both constraint types simultaneously, however, they might conflict during real-time use.

For example, if an event pushes an MV outside a specified hard bound and the hard MV rate bounds are too small, the resulting QP will be *infeasible*.

Avoid such conflicts by specifying hard MV bounds *OR* hard MV rate bounds, but not both. Or if you want to specify both, soften the lower-priority constraint by setting its ECR to a value greater than zero.

Warning: your constraint definitions may conflict. The following table lists potential conflicts for each MV. The tabular entries show the location of each conflict in the prediction horizon and the type of conflict.

MV name	Horizon k	Conflict Type
NG	1	Min & RateMax
NG	1	Max & RateMin
RG	1	Min & RateMax
RG	1	Max & RateMin
H2	1	Min & RateMax
H2	1	Max & RateMin
N2	1	Min & RateMax
N2	1	Max & RateMin
T1	1	Min & RateMax
T1	1	Max & RateMin
T2	1	Min & RateMax
T2	1	Max & RateMin

Return to list of tests

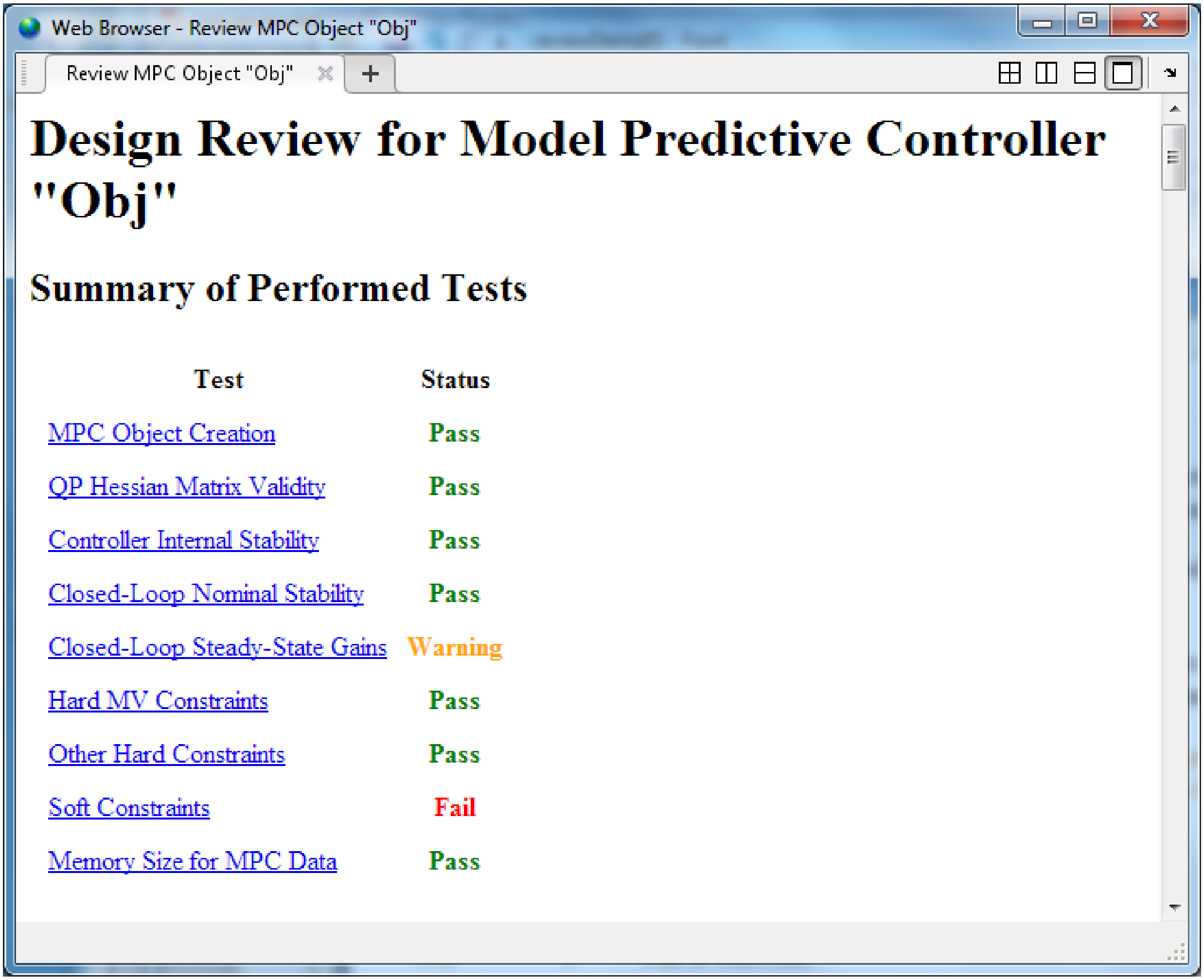
If an external event causes the NG to go far below its specified minimum, the constraint on its rate of increase might make it impossible to return the NG within bounds in one interval. In other words, when you specify both MV.Min and MV.RateMax, the controller would not be able to find an optimal solution if the most recent MV value is less than (MV.Min - MV.RateMax). Similarly, there is a potential conflict when you specify both MV.Max and MV.RateMin.

An MV constraint conflict would be extremely unlikely in the gas blending application, but it's good practice to eliminate the possibility by softening one of the two constraints. Since the MV minimum and maximum values are physical limits and the increment bounds are not, we soften the increment bounds as follows:

```
for i = 1:6
    Obj.MV(i).RateMinECR = 0.1;
    Obj.MV(i).RateMaxECR = 0.1;
end
```

Review the new controller design.

```
review(Obj)
```



The MV constraint conflict warning has disappeared. Now click **Soft Constraints** to view the error message.

Impact of delays

Delays can make it impossible to satisfy output constraints. The presence of unattainable constraints usually degrades performance. Let j be the location (within the prediction horizon) of the first finite constraint value (Min or Max) for OV(i). If all delays for OV(i) exceed j, the constraint is unattainable.

The following table lists each output constraint that is impossible to satisfy. The first column is the location (within the prediction horizon) of the first finite constraint value. The second column is the minimum delay for that output variable.

Constraint	Begins	Delay
WI.Min	1	3
WI.Max	1	3

Error: at least one output variable constraint is impossible to satisfy.

We see that the delay in the WI output makes it impossible to satisfy bounds on that variable until at least three control intervals have elapsed. The WI bounds are soft but it is poor practice to include unattainable constraints in a design. We therefore modify the WI bound specifications such that it is unconstained until the 4th prediction horizon step.

```
Obj.OV(2).Min = [-Inf(1,3), OVmin(2)];
Obj.OV(2).Max = [ Inf(1,3), OVmax(2)];

% Ee-issuing the review command to verifies that this eliminates the
% error message (see the next step).
```

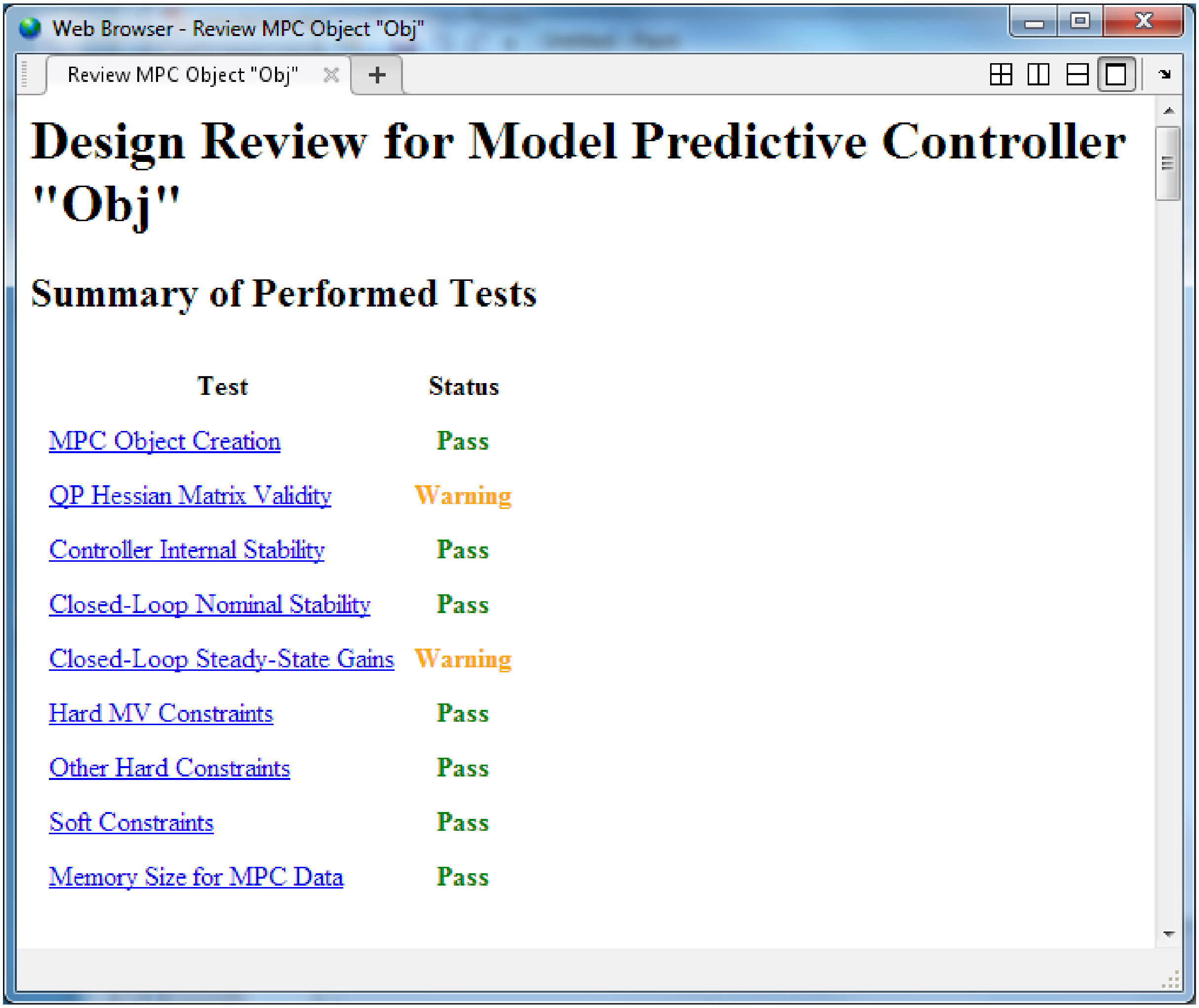
Diagnosing the Impact of Zero Output Weights

Given that the design requirements allow the OVs to vary freely within their limits, consider zeroing their penalty weights:

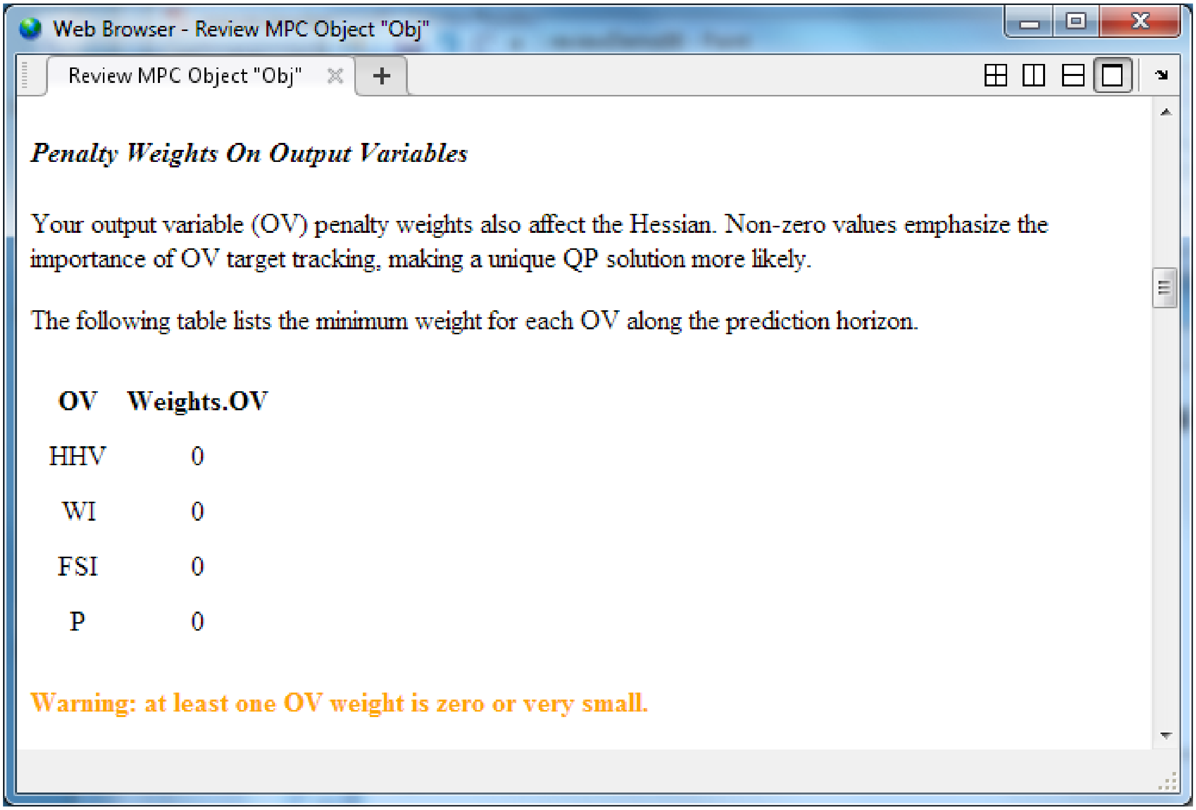
```
Obj.Weights.OV = zeros(1,4);
```

Review the impact of this design change.

```
review(Obj)
```



A new warning regarding QP Hessian Matrix Validity has appeared. Click **QP Hessian Matrix Validity** warning to see the details.



The review has flagged the zero weights on all four output variables. Since the zero weights are consistent with the design requirement and the other Hessian tests indicate that the quadratic programming problem has a unique solution, this warning can be ignored.

Click **Closed-Loop Steady-State Gains** to see the second warning. It shows another consequence of setting the four OV weights to zero. When an OV is not penalized by a weight, any output disturbance added to it will be ignored, passing through with no attenuation.

Closed-Loop Steady-State Gains

`cloffset` is used to determine whether the controller forces all controlled output variables to their targets at steady state, in the absence of constraints.

The command calculates the impact of a sustained disturbance on each measured output variable (OV) in terms of an input/output gain. If a gain is zero, the controller eliminates steady-state tracking error for that disturbance-to-output mapping.

The gains with magnitudes exceeding 1e-05 are as follows:

Disturbed OV	Affected OV	Gain
HHV	HHV	1
WI	WI	1
FSI	FSI	1
P	P	1

Since it is a design requirement, non-zero steady-state offsets are acceptable provided that MPC is able to hold all the OVs within their specified bounds. It is therefore a good idea to examine how easily the soft OV constraints can be violated when disturbances are present.

Reviewing Soft Constraints

Click **Soft Constraints** to see a list of soft constraints -- in this case an upper and lower bound on each OV.

Soft Constraints

ECR Parameters

This test evaluates the constraint ECR parameters to help you achieve the proper balance of using hard and soft constraints. If a constraint is too soft, an unacceptable violation may occur. If it is too hard, the controller might pay it too much attention. Moreover, making a constraint harder cannot prevent a violation if the constraint is fundamentally infeasible.

You have defined 8 soft constraints. The table below lists these and shows potential violations based on specified variable bounds and other factors.

Impact Factor: the increase in the MPC cost function caused by this constraint violation relative to the average such increase. Rows are sorted in order of decreasing impact.

Sensitivity Ratio: the increase in the MPC cost function caused by this constraint violation relative to the typical cost function magnitude when there are no violations.

We consider a possible constraint violation equal to 10% of the nominal OV range. It then estimates the impact of such a violation on the MPC objective function relative to the impact of other violations. A large impact factor indicates a high-priority controller objective, and vice versa.

Constraint	Assumed Violation	Impact Factor	Sensitivity Ratio
Lower limit: P	20	1509	1000
Upper limit: P	20	1509	1000
Lower limit: FSI	0.7	1.849	1.225
Upper limit: FSI	0.7	1.849	1.225
Lower limit: WI	0.2	0.1509	0.1
Upper limit: WI	0.2	0.1509	0.1
Lower limit: HHV	0.15	0.08491	0.05625
Upper limit: HHV	0.15	0.08491	0.05625

A sensitivity ratio greater than 1e+08 may degrade QP solution accuracy.

The Impact Factor column shows that using the default MinECR and MaxECR values give the pressure (P) a much higher priority than the other OVs. If we want the priorities to be more comparable, we should increase the pressure constraint ECR values and adjust the others too. For example, we consider

```
Obj.OV(1).MinECR = 0.5;
Obj.OV(1).MaxECR = 0.5;
Obj.OV(3).MinECR = 3;
Obj.OV(3).MaxECR = 3;
Obj.OV(4).MinECR = 80;
Obj.OV(4).MaxECR = 80;
```

Review the impact of this design change.

```
review(Obj)
```

Constraint	Assumed Violation	Impact Factor	Sensitivity Ratio
Lower limit: HHV	0.15	1.539	0.225
Upper limit: HHV	0.15	1.539	0.225
Lower limit: P	20	1.069	0.1563
Upper limit: P	20	1.069	0.1563
Lower limit: FSI	0.7	0.9311	0.1361
Upper limit: FSI	0.7	0.9311	0.1361
Lower limit: WI	0.2	0.6841	0.1
Upper limit: WI	0.2	0.6841	0.1

Notice from the Sensitivity Ratio column that all the sensitivity ratios are now less than unity. This means that the soft constraints will receive less attention than other terms in the MPC objective function, such as deviations of the MVs from their target values. Thus, it is likely that an output constraint violation would occur.

In order to give the output constraints higher priority than other MPC objectives, increase the Weights.ECR parameter from default 1e5 to a higher value to harden all the soft OV constraints.

```
Obj.Weights.ECR = 1e8;
```

Review the impact of this design change.

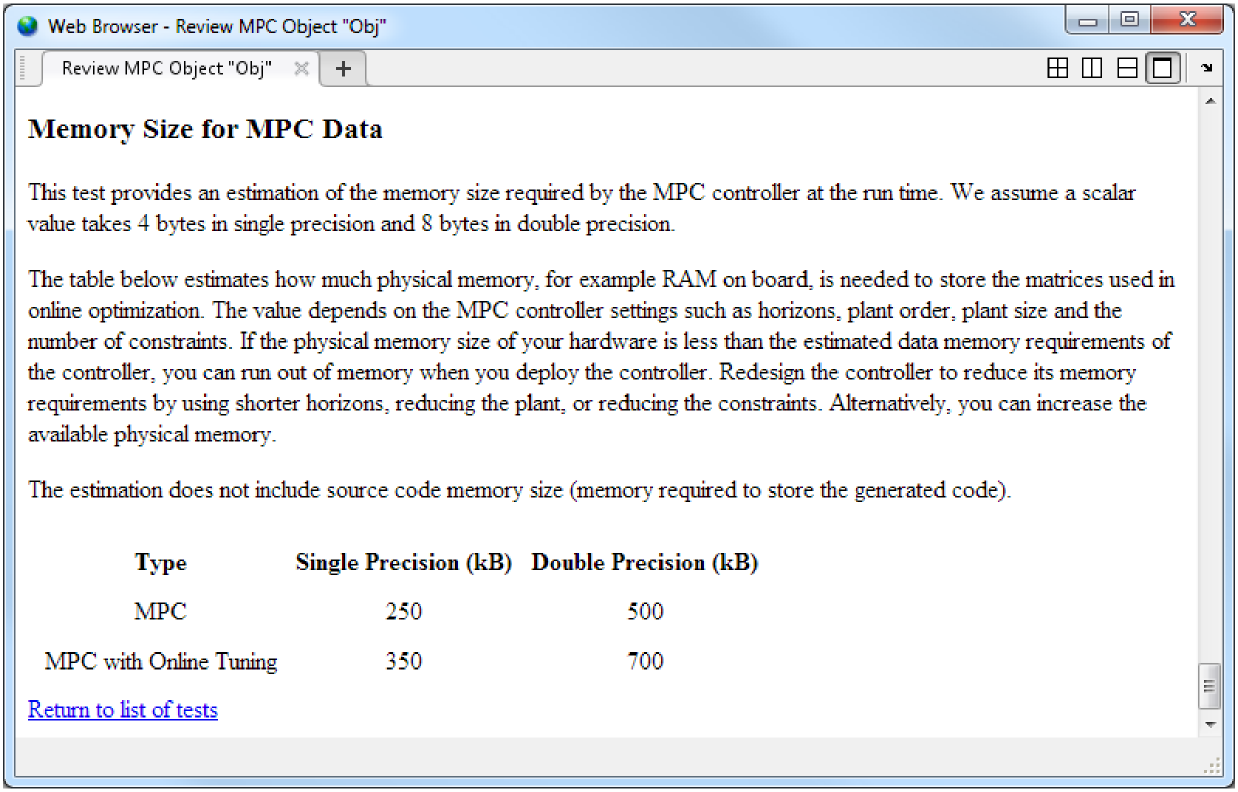
```
review(Obj)
```

Constraint	Assumed Violation	Impact Factor	Sensitivity Ratio
Lower limit: HHV	0.15	1.539	225
Upper limit: HHV	0.15	1.539	225
Lower limit: P	20	1.069	156.3
Upper limit: P	20	1.069	156.3
Lower limit: FSI	0.7	0.9311	136.1
Upper limit: FSI	0.7	0.9311	136.1
Lower limit: WI	0.2	0.6841	100
Upper limit: WI	0.2	0.6841	100

The controller is now a factor of 100 more sensitive to output constraint violations than to errors in target tracking.

Reviewing Data Memory Size

Click **Memory Size for MPC Data** to see the estimated memory size needed to store the MPC data matrices used on the hardware.



Memory Size for MPC Data

This test provides an estimation of the memory size required by the MPC controller at the run time. We assume a scalar value takes 4 bytes in single precision and 8 bytes in double precision.

The table below estimates how much physical memory, for example RAM on board, is needed to store the matrices used in online optimization. The value depends on the MPC controller settings such as horizons, plant order, plant size and the number of constraints. If the physical memory size of your hardware is less than the estimated data memory requirements of the controller, you can run out of memory when you deploy the controller. Redesign the controller to reduce its memory requirements by using shorter horizons, reducing the plant, or reducing the constraints. Alternatively, you can increase the available physical memory.

The estimation does not include source code memory size (memory required to store the generated code).

Type	Single Precision (kB)	Double Precision (kB)
MPC	250	500
MPC with Online Tuning	350	700

Return to list of tests

In this example, if the controller is running using single precision, it requires 250 KB of memory to store its matrices. If the controller memory size exceeds the memory available on the target system, you must redesign the controller to reduce its memory requirements. Alternatively, increase the memory available on the target system.

```
mpcverbosity(MPC_verbosity);
[~, hWebBrowser] = web;
close(hWebBrowser);
```

Adaptive MPC Design

Adaptive MPC

In this section...

When to Use Adaptive MPC

MPC control predicts future behavior using a linear-time-invariant (LTI) dynamic model. In practice, such predictions are never exact, and a key tuning objective is to make MPC insensitive to prediction errors. In many applications, this approach is sufficient for robust controller performance.

If the plant is strongly nonlinear or its characteristics vary dramatically with time, LTI prediction accuracy might degrade so much that MPC performance becomes unacceptable. Adaptive MPC can address this degradation by adapting the prediction model for changing operating conditions. As implemented in the Model Predictive Control Toolbox software, adaptive MPC uses a fixed model structure, but allows the model's parameters to evolve with time. Ideally, whenever the controller requires a prediction (at the beginning of each control interval) it uses a model appropriate for the current conditions.

After you design an MPC controller for the average or most likely operating conditions of your control system, you can implement an adaptive MPC controller based on that design. For information about designing that initial controller, see "Controller Creation".

An alternative option for controlling a nonlinear or time-varying plant is to use gain-scheduled MPC control. See "Gain-Scheduled MPC" on page 7-2.)

Plant Model

The plant model used as the basis for Adaptive MPC must be an LTI discrete-time, state-space model. See "Basic Models" in the Control System Toolbox documentation or "Linearization Basics" in the Simulink Control Design documentation for information about creating and modifying such systems. The plant model structure is as follows:

$$x_p(k+1) = A_p x_p(k) + B_{pu} u(k) + B_{pv} v(k) + B_{pd} d(k)$$
$$y(k) = C_p x_p(k) + D_{pv} v(k) + D_{pd} d(k).$$

Here, the matrices A_p, B_{pu}, B_{pv}, B_{pd}, C_p, D_{pv} and D_{pd} are the parameters that can vary with time. The other variables in the expression are:

- k — Time index (current control interval).
- x_p — n_{xp} plant model states.
- u — n_u manipulated inputs (MVs). These are the one or more inputs that are adjusted by the MPC controller.
- v — n_v measured disturbance inputs.
- d — n_d unmeasured disturbance inputs.
- y — n_y plant outputs, including n_{ym} measured and n_{yu} unmeasured outputs. The total number of outputs, $n_y = n_{ym} + n_{yu}$. Also, $n_{ym} \geq 1$ (there is at least one measured output).

Additional requirements for the plant model in adaptive MPC control are:

- Sample time (`Ts`) is a constant and identical to the MPC control interval.
- Time delay (if any) is absorbed as discrete states (see, e.g., the Control System Toolbox `absorbDelay` command).
- n_{xp}, n_u, n_y, n_d, n_{ym}, and n_{yu} are all constants.
- Adaptive MPC prohibits direct feed-through from any manipulated variable to any plant output. Thus, $D_u = 0$ in the above model.

When you create the plant model, you specify it as an LTI state-space model with parameters A_p, B_p, C_p, D_p, and T_s. The sampling time is $T_s > 0$. The `InputGroup` and `OutputGroup` properties of the model designate input and output signal types (such as manipulated or measured). Each column in B_p and D_p represents a particular plant input variable. Grouping these columns according to input signal type yields the matrices B_{pu}, B_{pv}, B_{pd}, D_{pv} and D_{pd}. Similarly, each row in C_p, and D_p represent a particular output variable. Once you create these column and row assignments, you cannot change them as the system evolves in time.

For more details about creation of plant models for MPC control, see “Plant Specification”.

Nominal Operating Point

A traditional MPC controller includes a nominal operating point at which the plant model applies, such as the condition at which you linearize a nonlinear model to obtain the LTI approximation. The `Model.Nominal` property of the controller contains this information.

In adaptive MPC, as time evolves you should update the nominal operating point to be consistent with the updated plant model.

You can write the plant model in terms of deviations from the nominal conditions:

$$
\begin{aligned}
x_p(k+1) &= \bar{x}_p + A_p\left(x_p(k) - \bar{x}_p\right) + B_p\left(u_t(k) - \bar{u}_t\right) + \overline{\Delta x}_p \\
y(k) &= \bar{y} + C_p\left(x_p(k) - \bar{x}_p\right) + D_p\left(u_t(k) - \bar{u}_t\right).
\end{aligned}
$$

Here, the matrices A_p, B_p, C_p, and D_p are the parameter matrices to be updated. u_t is the combined plant input variable, comprising the u, v, and d variables defined above. The nominal conditions to be updated are:

- $\bar{x}_p$ — n_{xp} nominal states
- $\overline{\Delta x}_p$ — n_{xp} nominal state increments
- $\bar{u}_t$ — n_{ut} nominal inputs
- $\bar{y}$ — n_y nominal outputs

State Estimation

By default, MPC uses a static Kalman filter (KF) to update its controller states, which include the n_{xp} plant model states, n_d (≥ 0) disturbance model states, and n_n (≥ 0) measurement noise model states. This KF requires two gain matrices, L and M. By default, the MPC controller calculates them during initialization. They depend upon the plant, disturbance, and noise model parameters, and assumptions regarding the stochastic noise signals driving the disturbance and noise models. For more details about state estimation in traditional MPC, see “Controller State Estimation” on page 2-40.

Adaptive MPC uses a Kalman filter by default, and adjusts the gains, L and M, at each control interval to maintain consistency with the updated plant model. The result is a linear-time-varying Kalman filter (LTVKF):

$$
\begin{aligned}
L_k &= \left(A_k P_{k|k-1} C_{m,k}^T + N\right)\left(C_{m,k} P_{k|k-1} C_{m,k}^T + R\right)^{-1} \\
M_k &= P_{k|k-1} C_{m,k}^T \left(C_{m,k} P_{k|k-1} C_{m,k}^T + R\right)^{-1} \\
P_{k+1|k} &= A_k P_{k|k-1} A_k^T - \left(A_k P_{k|k-1} C_{m,k}^T + N\right) L_k^T + Q.
\end{aligned}
$$

Here, Q, R, and N are constant covariance matrices defined as in MPC state estimation. A_k and $C_{m,k}$ are state-space parameter matrices for the entire controller state, defined as for traditional MPC but with the portions affected by the plant model updated to time k. The value $P_{k|k-1}$ is the state estimate error covariance matrix at time k based on information available at time k–1. Finally, L_k and M_k are the updated KF gain matrices. For details on the KF formulation used in traditional MPC, see "Controller State Estimation" on page 2-40. By default, the initial condition, $P_{0|-1}$, is the static KF solution prior to any model updates.

The KF gain and the state error covariance matrix depend upon the model parameters and the assumptions leading to the constant Q, R, and N matrices. If the plant model is constant, the expressions for L_k and M_k converge to the equivalent static KF solution used in traditional MPC.

The equations for the controller state evolution at time k are identical to the KF formulation of traditional MPC described in "Controller State Estimation" on page 2-40, but with the estimator gains and state space matrices updated to time k.

You have the option to update the controller state using a procedure external to the MPC controller, and then supply the updated state to MPC at each control instant, k. In this case, the MPC controller skips all KF and LTVKF calculations.

Related Examples

- "Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization" on page 5-8
- "Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation" on page 5-21

More About

- "Model Updating Strategy" on page 5-6
- "Controller State Estimation" on page 2-40

Model Updating Strategy

In this section...

Overview

Typically, to implement adaptive MPC control, you employ one of the following model-updating strategies:

Successive linearization. Given a mechanistic plant model, e.g., a set of nonlinear ordinary differential and algebraic equations, derive its LTI approximation at the current operating condition. For example, Simulink Control Design software provides linearization tools for this purpose.

Using a Linear Parameter Varying (LPV) model. Control System Toolbox software provides a `LPV System` Simulink block that allows you to specify an array of LTI models with scheduling parameters. You can perform batch linearization offline to obtain an array of plant models at the desired operating points and then use them in the LPV System block to provide model updating to the `Adaptive MPC Controller` Simulink block.

Online parameter estimation. Given an empirical model structure and initial estimates of its parameters, use the available real-time plant measurements to estimate the current model parameters. For example, the System Identification Toolbox™ software provides real-time parameter estimation tools.

Other Considerations

There are several factors to keep in mind when designing and implementing an adaptive MPC controller.

- Before attempting adaptive MPC, define and tune an MPC controller for the most typical (nominal) operating condition. Make sure the system can tolerate some prediction error. Test this tolerance via simulations in which the MPC prediction model differs from the plant. See "MPC Design".

- An adaptive MPC controller requires more real-time computations than traditional MPC. In addition to the state estimation calculation, you must also implement and test a model-updating strategy, which might be computationally intensive.
- You must determine MPC tuning constants that provide robust performance over the expected range of model parameters. See “Tuning Weights” on page 1-16.
- Model updating via online parameter estimation is most effective when parameter variations occur gradually.
- When implementing adaptive MPC control, adapt only parameters defining the `Model.Plant` property of the controller. The disturbance and noise models, if any, remain constant.

See Also

`Adaptive MPC Controller`

Related Examples

- “Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization” on page 5-8
- “Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation” on page 5-21

More About

- “Adaptive MPC” on page 5-2

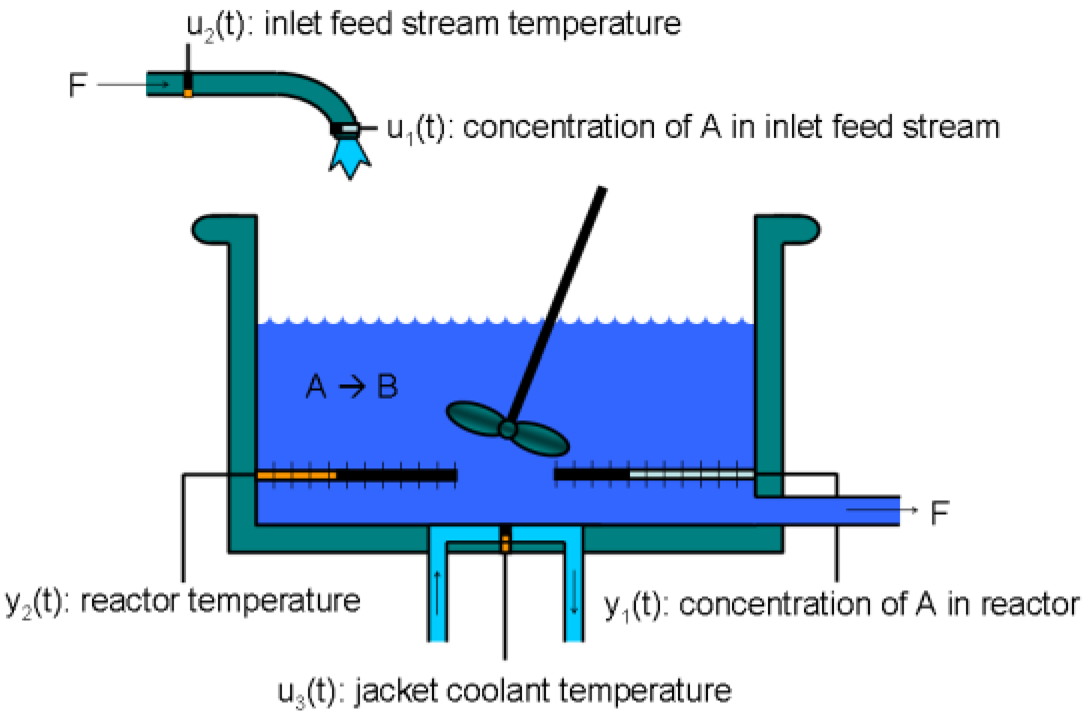
Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization

This example shows how to use an Adaptive MPC controller to control a nonlinear continuous stirred tank reactor (CSTR) as it transitions from low conversion rate to high conversion rate.

A first principle nonlinear plant model is available and being linearized at each control interval. The adaptive MPC controller then updates its internal predictive model with the linearized plant model and achieves nonlinear control successfully.

About the Continuous Stirred Tank Reactor

A Continuously Stirred Tank Reactor (CSTR) is a common chemical system in the process industry. A schematic of the CSTR system is:



This is a jacketed non-adiabatic tank reactor described extensively in Seborg's book, "Process Dynamics and Control", published by Wiley, 2004. The vessel is assumed to be perfectly mixed, and a single first-order exothermic and irreversible reaction, A --> B, takes place. The inlet stream of reagent A is fed to the tank at a constant volumetric rate. The product stream exits continuously at the same volumetric rate and liquid density is constant. Thus the volume of reacting liquid is constant.

The inputs of the CSTR model are:

$$\begin{array}{ll} u_1 = CA_i & \text{Concentration of A in inlet feed stream}[kgmol/m^3] \\ u_2 = T_i & \text{Inlet feed stream temperature}[K] \\ u_3 = T_c & \text{Jacket coolant temperature}[K] \end{array}$$

and the outputs (y(t)), which are also the states of the model (x(t)), are:

$$\begin{array}{ll} y_1 = x_1 = CA & \text{Concentration of A in reactor tank}[kgmol/m^3] \\ y_2 = x_2 = T & \text{Reactor temperature}[K] \end{array}$$

The control objective is to maintain the concentration of reagent A, CA at its desired setpoint, which changes over time when reactor transitions from low conversion rate to high conversion rate. The coolant temperature T_c is the manipulated variable used by the MPC controller to track the reference as well as reject the measured disturbance arising from the inlet feed stream temperature T_i. The inlet feed stream concentration, CA_i, is assumed to be constant. The Simulink model `mpc_cstr_plant` implements the nonlinear CSTR plant.

We also assume that direct measurements of concentrations are unavailable or infrequent, which is the usual case in practice. Instead, we use a "soft sensor" to estimate CA based on temperature measurements and the plant model.

About Adaptive Model Predictive Control

It is well known that the CSTR dynamics are strongly nonlinear with respect to reactor temperature variations and can be open-loop unstable during the transition from one operating condition to another. A single MPC controller designed at a particular operating condition cannot give satisfactory control performance over a wide operating range.

To control the nonlinear CSTR plant with linear MPC control technique, you have a few options:

- If a linear plant model cannot be obtained at run time, first you need to obtain several linear plant models offline at different operating conditions that cover the typical operating range. Next you can choose one of the two approaches to implement MPC control strategy:

(1) Design several MPC controllers offline, one for each plant model. At run time, use Multiple MPC Controller block that switches MPC controllers from one to another based on a desired scheduling strategy. See "Gain Scheduled MPC Control of Nonlinear Chemical Reactor" for more details. Use this approach when the plant models have different orders or time delays.

(2) Design one MPC controller offline at the initial operating point. At run time, use Adaptive MPC Controller block (updating predictive model at each control interval) together with Linear Parameter Varying (LPV) System block (supplying linear plant model with a scheduling strategy). See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Linear Parameter Varying System" for more details. Use this approach when all the plant models have the same order and time delay.

- If a linear plant model can be obtained at run time, you should use Adaptive MPC Controller block to achieve nonlinear control. There are two typical ways to obtain a linear plant model online:

(1) Use successive linearization as shown in this example. Use this approach when a nonlinear plant model is available and can be linearized at run time.

(2) Use online estimation to identify a linear model when loop is closed. See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation" for more details. Use this approach when linear plant model cannot be obtained from either an LPV system or successive linearization.

Obtain Linear Plant Model at Initial Operating Condition

To linearize the plant, Simulink® and Simulink Control Design® are required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('slcontrol')
    disp('Simulink Control Design(R) is required to run this example.')
    return
end
```

To implement an adaptive MPC controller, first you need to design a MPC controller at the initial operating point where CAi is 10 kgmol/m^3, Ti and Tc are 298.15 K.

Create operating point specification.

```
plant_mdl = 'mpc_cstr_plant';
op = operspec(plant_mdl);
```

Feed concentration is known at the initial condition.

```
op.Inputs(1).u = 10;
op.Inputs(1).Known = true;
```

Feed temperature is known at the initial condition.

```
op.Inputs(2).u = 298.15;
op.Inputs(2).Known = true;
```

Coolant temperature is known at the initial condition.

```
op.Inputs(3).u = 298.15;
op.Inputs(3).Known = true;
```

Compute initial condition.

```
[op_point, op_report] = findop(plant_mdl,op);
```

```
 Operating Point Search Report:
---------------------------------

 Operating Report for the Model mpc_cstr_plant.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) mpc_cstr_plant/CSTR/Integrator
      x:           311      dx:      8.12e-11 (0)
(2.) mpc_cstr_plant/CSTR/Integrator1
      x:          8.57      dx:     -6.87e-12 (0)

Inputs:
----------
(1.) mpc_cstr_plant/CAi
```

```
      u:           10
(2.) mpc_cstr_plant/Ti
      u:          298
(3.) mpc_cstr_plant/Tc
      u:          298

Outputs:
----------
(1.) mpc_cstr_plant/T
      y:          311    [-Inf Inf]
(2.) mpc_cstr_plant/CA
      y:         8.57    [-Inf Inf]
```

Obtain nominal values of x, y and u.

```
x0 = [op_report.States(1).x;op_report.States(2).x];
y0 = [op_report.Outputs(1).y;op_report.Outputs(2).y];
u0 = [op_report.Inputs(1).u;op_report.Inputs(2).u;op_report.Inputs(3).u];
```

Obtain linear plant model at the initial condition.

```
sys = linearize(plant_mdl, op_point);
```

Drop the first plant input CAi because it is not used by MPC.

```
sys = sys(:,2:3);
```

Discretize the plant model because Adaptive MPC controller only accepts a discrete-time plant model.

```
Ts = 0.5;
plant = c2d(sys,Ts);
```

Design MPC Controller

You design an MPC at the initial operating condition. When running in the adaptive mode, the plant model is updated at run time.

Specify signal types used in MPC.

```
plant.InputGroup.MeasuredDisturbances = 1;
plant.InputGroup.ManipulatedVariables = 2;
plant.OutputGroup.Measured = 1;
```

```
plant.OutputGroup.Unmeasured = 2;
plant.InputName = {'Ti','Tc'};
plant.OutputName = {'T','CA'};
```

Create MPC controller with default prediction and control horizons

```
mpcobj = mpc(plant);
```

```
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
   for output(s) y1 and zero weight for output(s) y2
```

Set nominal values in the controller

```
mpcobj.Model.Nominal = struct('X', x0, 'U', u0(2:3), 'Y', y0, 'DX', [0 0]);
```

Set scale factors because plant input and output signals have different orders of magnitude

```
Uscale = [30 50];
Yscale = [50 10];
mpcobj.DV(1).ScaleFactor = Uscale(1);
mpcobj.MV(1).ScaleFactor = Uscale(2);
mpcobj.OV(1).ScaleFactor = Yscale(1);
mpcobj.OV(2).ScaleFactor = Yscale(2);
```

Let reactor temperature T float (i.e. with no setpoint tracking error penalty), because the objective is to control reactor concentration CA and only one manipulated variable (coolant temperature Tc) is available.

```
mpcobj.Weights.OV = [0 1];
```

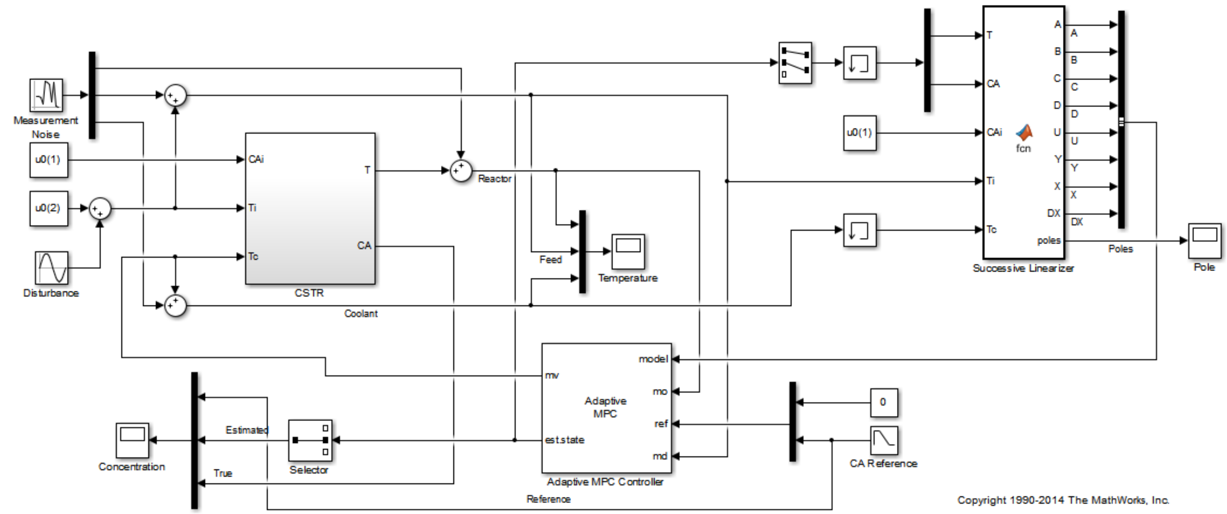
Due to the physical constraint of coolant jacket, Tc rate of change is bounded by degrees per minute.

```
mpcobj.MV.RateMin = -2;
mpcobj.MV.RateMax = 2;
```

Implement Adaptive MPC Control of CSTR Plant in Simulink (R)

Open the Simulink model.

```
mdl = 'ampc_cstr_linearization';
open_system(mdl);
```



The model includes three parts:

1. The "CSTR" block implements the nonlinear plant model.
2. The "Adaptive MPC Controller" block runs the designed MPC controller in the adaptive mode.
3. The "Successive Linearizer" block in a MATLAB Function block that linearizes a first principle nonlinear CSTR plant and provides the linear plant model to the "Adaptive MPC Controller" block at each control interval. Double click the block to see the MATLAB code. You can use the block as a template to develop appropriate linearizer for your own applications.

Note that the new linear plant model must be a discrete time state space system with the same order and sample time as the original plant model has. If the plant has time delay, it must also be same as the original time delay and absorbed into the state space model.

Validate Adaptive MPC Control Performance

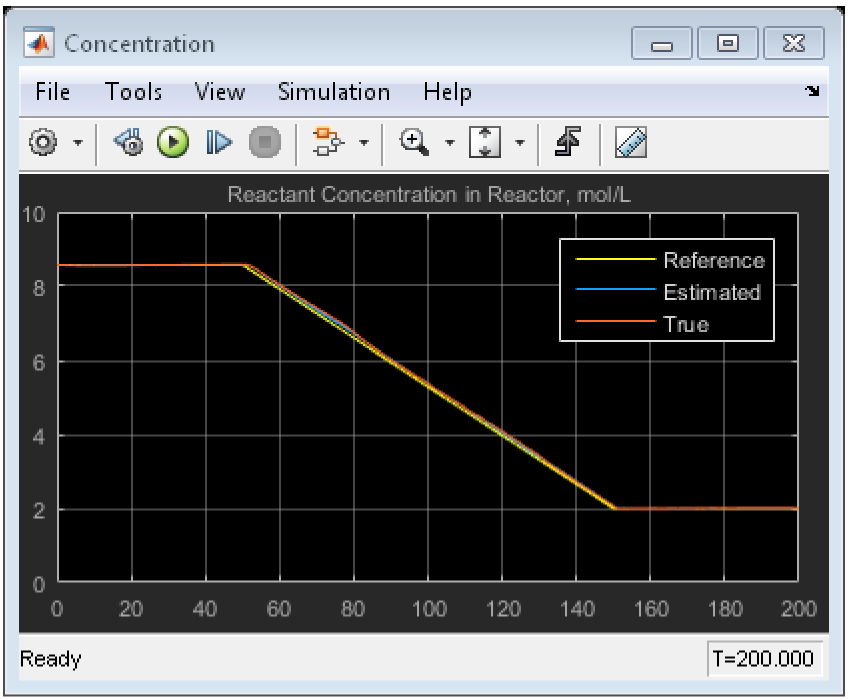
Controller performance is validated against both setpoint tracking and disturbance rejection.

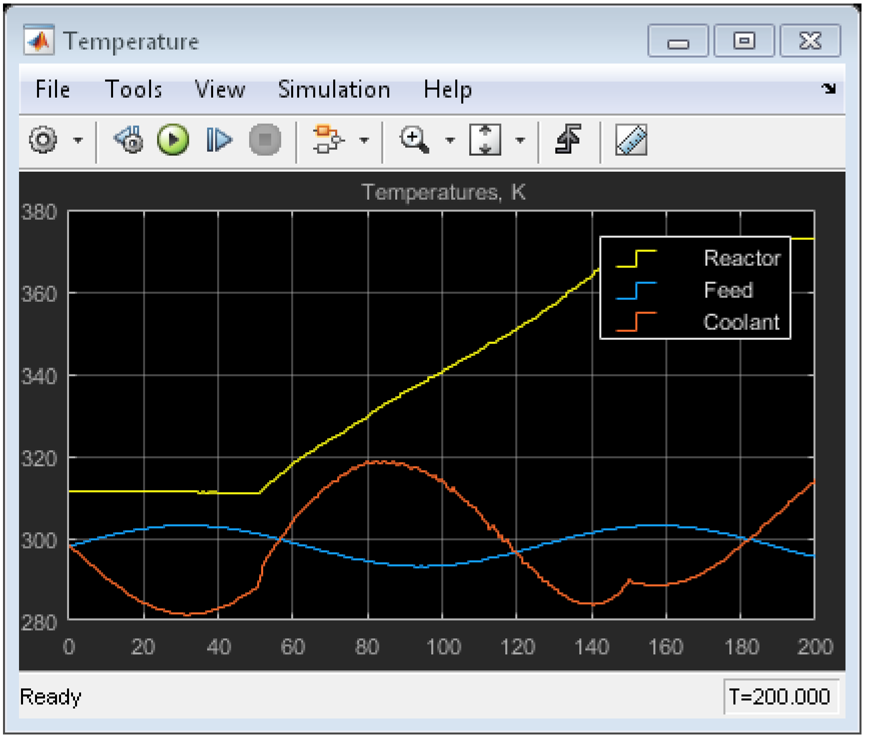
- Tracking: reactor concentration CA setpoint transitions from original 8.57 (low conversion rate) to 2 (high conversion rate) kgmol/m^3. During the transition, the plant first becomes unstable then stable again (see the poles plot).
- Regulating: feed temperature Ti has slow fluctuation represented by a sine wave with amplitude of 5 degrees, which is a measured disturbance fed to the MPC controller.

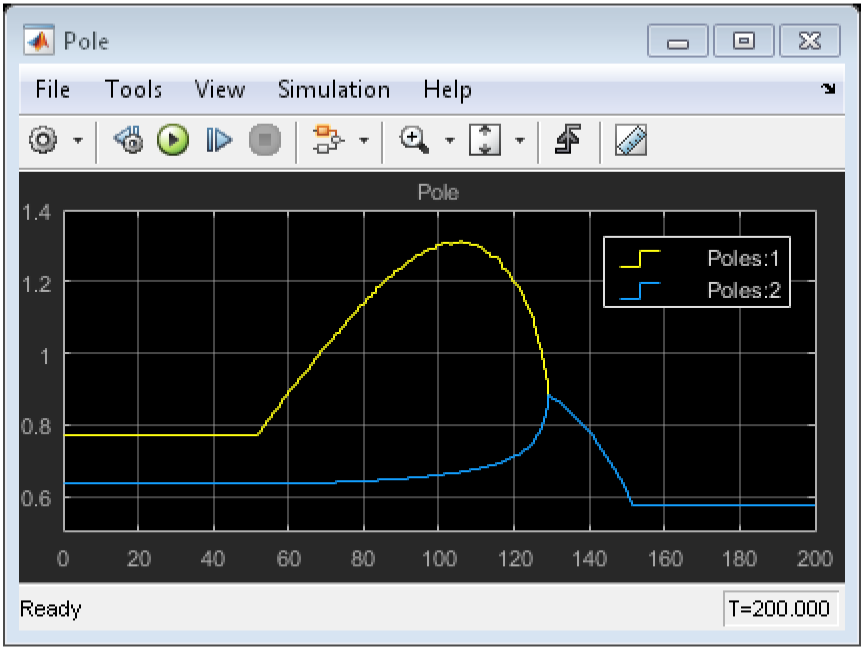
Simulate the closed-loop performance.

```
open_system([mdl '/Concentration'])
open_system([mdl '/Temperature'])
open_system([mdl '/Pole'])
sim(mdl);
```

```
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```




Temperature
File Tools View Simulation Help
Temperatures, K
380
360
340
320
300
280
0 20 40 60 80 100 120 140 160 180 200
Reactor
Feed
Coolant
Ready
T=200.000

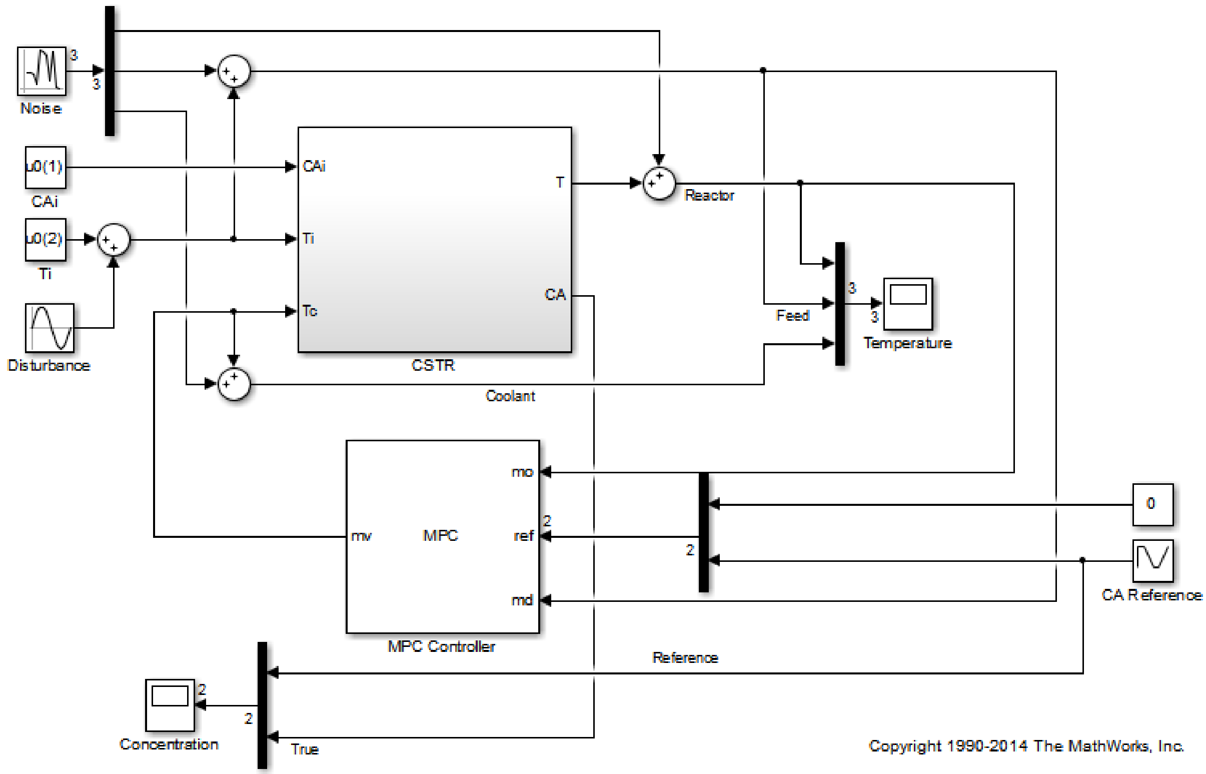


The tracking and regulating performance is very satisfactory. In an application to a real reactor, however, model inaccuracies and unmeasured disturbances could cause poorer tracking than shown here. Additional simulations could be used to study these effects.

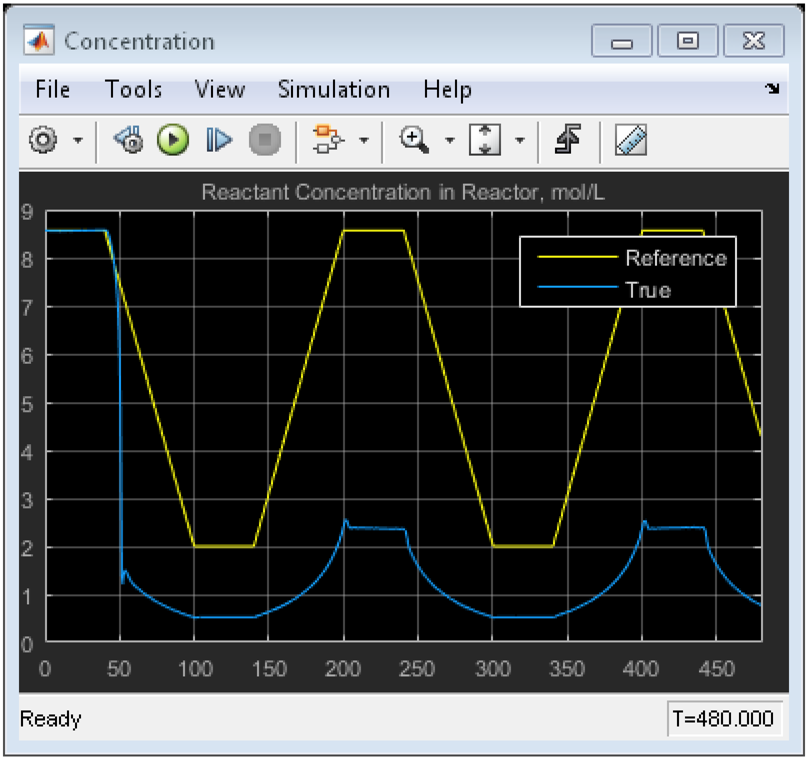
Compare with Non-Adaptive MPC Control

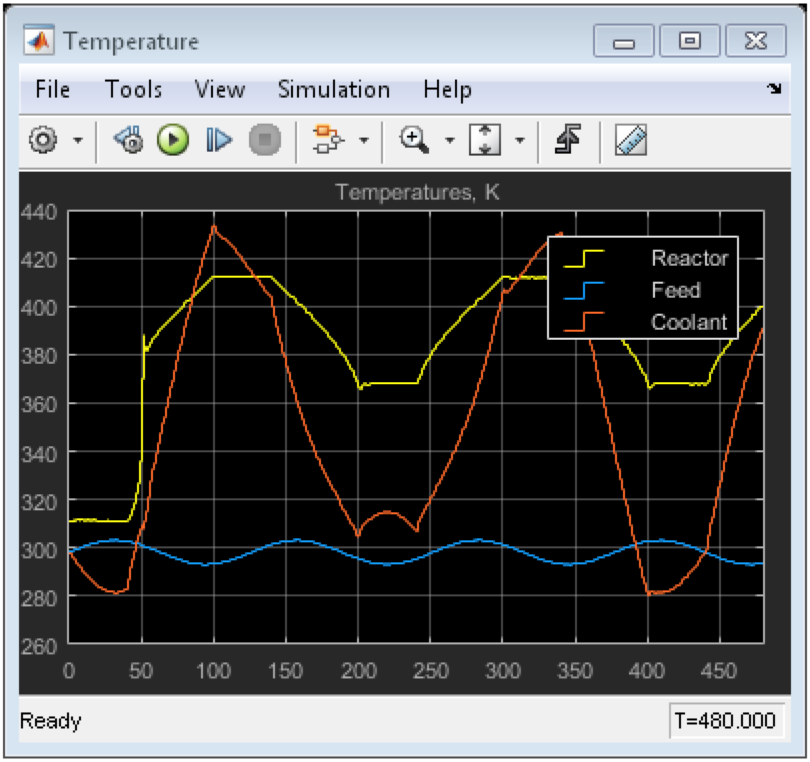
Adaptive MPC provides superior control performance than a non-adaptive MPC. To illustrate this point, the control performance of the same MPC controller running in the non-adaptive mode is shown below. The controller is implemented with a MPC Controller block.

```
mdl1 = 'ampc_cstr_no_linearization';
open_system(mdl1);
open_system([mdl1 '/Concentration'])
open_system([mdl1 '/Temperature'])
sim(mdl1);
```



Noise
3
3
u0(1)
CAi
u0(2)
Ti
Disturbance
CAi
Ti
Tc
T
CA
CSTR
Reactor
Feed
3
3
Temperature
Coolant
mo
ref
md
mv
MPC
2
MPC Controller
2
0
CA Reference
Reference
2
2
Concentration
True
Copyright 1990-2014 The MathWorks, Inc.


Concentration
File Tools View Simulation Help
Reactant Concentration in Reactor, mol/L
Reference
True
0
1
2
3
4
5
6
7
8
9
0 50 100 150 200 250 300 350 400 450
Ready
T=480.000



As expected, the tracking and regulating performance is unacceptable.

```
bdclose(mdl)
bdclose(mdl1)
```

See Also

`Adaptive MPC Controller`

Related Examples

- “Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation” on page 5-21

More About

- “Adaptive MPC” on page 5-2

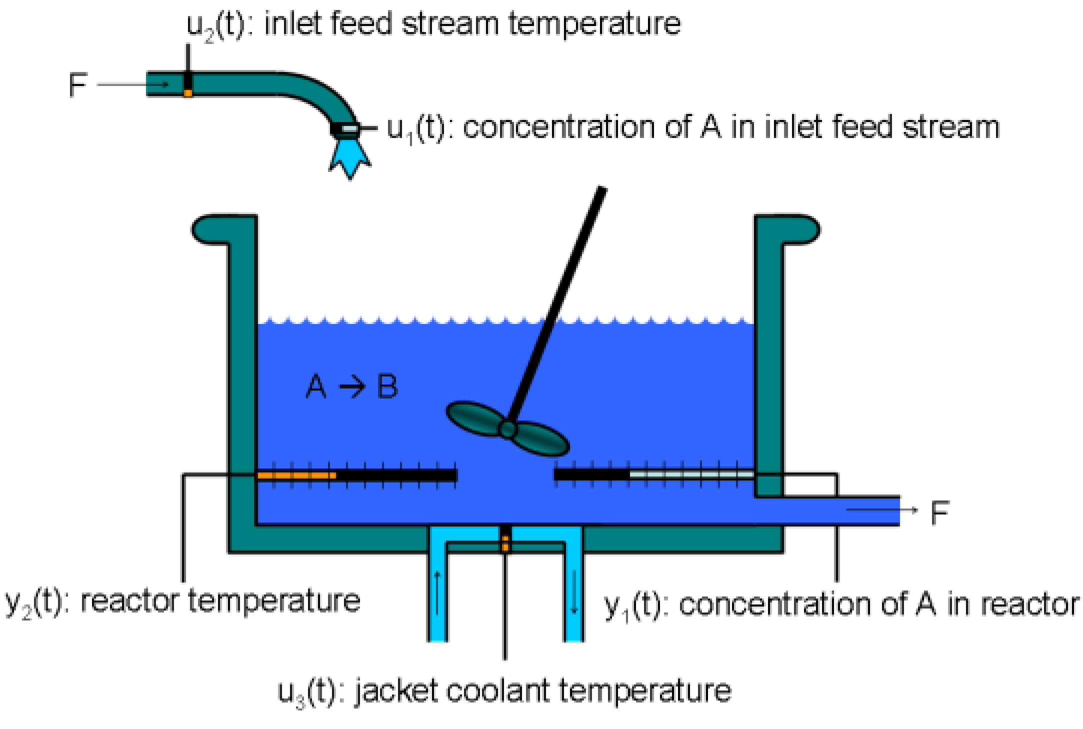
Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation

This example shows how to use an Adaptive MPC controller to control a nonlinear continuous stirred tank reactor (CSTR) as it transitions from low conversion rate to high conversion rate.

A discrete time ARX model is being identified online by the Recursive Polynomial Model Estimator block at each control interval. The adaptive MPC controller uses it to update internal plant model and achieves nonlinear control successfully.

About the Continuous Stirred Tank Reactor

A Continuously Stirred Tank Reactor (CSTR) is a common chemical system in the process industry. A schematic of the CSTR system is:



This is a jacketed non-adiabatic tank reactor described extensively in Seborg's book, "Process Dynamics and Control", published by Wiley, 2004. The vessel is assumed to be perfectly mixed, and a single first-order exothermic and irreversible reaction, A --> B, takes place. The inlet stream of reagent A is fed to the tank at a constant volumetric rate. The product stream exits continuously at the same volumetric rate and liquid density is constant. Thus the volume of reacting liquid is constant.

The inputs of the CSTR model are:

$$\begin{array}{ll} u_1 = CA_i & \text{Concentration of A in inlet feed stream}[kgmol/m^3] \\ u_2 = T_i & \text{Inlet feed stream temperature}[K] \\ u_3 = T_c & \text{Jacket coolant temperature}[K] \end{array}$$

and the outputs (y(t)), which are also the states of the model (x(t)), are:

$$\begin{array}{ll} y_1 = x_1 = CA & \text{Concentration of A in reactor tank}[kgmol/m^3] \\ y_2 = x_2 = T & \text{Reactor temperature}[K] \end{array}$$

The control objective is to maintain the reactor temperature T at its desired setpoint, which changes over time when reactor transitions from low conversion rate to high conversion rate. The coolant temperature T_c is the manipulated variable used by the MPC controller to track the reference as well as reject the measured disturbance arising from the inlet feed stream temperature T_i. The inlet feed stream concentration, CA_i, is assumed to be constant. The Simulink model `mpc_cstr_plant` implements the nonlinear CSTR plant.

About Adaptive Model Predictive Control

It is well known that the CSTR dynamics are strongly nonlinear with respect to reactor temperature variations and can be open-loop unstable during the transition from one operating condition to another. A single MPC controller designed at a particular operating condition cannot give satisfactory control performance over a wide operating range.

To control the nonlinear CSTR plant with linear MPC control technique, you have a few options:

- If a linear plant model cannot be obtained at run time, first you need to obtain several linear plant models offline at different operating conditions that cover the typical operating range. Next you can choose one of the two approaches to implement MPC control strategy:

(1) Design several MPC controllers offline, one for each plant model. At run time, use Multiple MPC Controller block that switches MPC controllers from one to another based on a desired scheduling strategy. See "Gain Scheduled MPC Control of Nonlinear Chemical Reactor" for more details. Use this approach when the plant models have different orders or time delays.

(2) Design one MPC controller offline at the initial operating point. At run time, use Adaptive MPC Controller block (updating predictive model at each control interval) together with Linear Parameter Varying (LPV) System block (supplying linear plant model with a scheduling strategy). See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Linear Parameter Varying System" for more details. Use this approach when all the plant models have the same order and time delay.

- If a linear plant model can be obtained at run time, you should use Adaptive MPC Controller block to achieve nonlinear control. There are two typical ways to obtain a linear plant model online:

(1) Use successive linearization. See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization" for more details. Use this approach when a nonlinear plant model is available and can be linearized at run time.

(2) Use online estimation to identify a linear model when loop is closed, as shown in this example. Use this approach when linear plant model cannot be obtained from either an LPV system or successive linearization.

Obtain Linear Plant Model at Initial Operating Condition

To linearize the plant, Simulink® and Simulink Control Design® are required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('slcontrol')
    disp('Simulink Control Design(R) is required to run this example.')
    return
end
```

To implement an adaptive MPC controller, first you need to design a MPC controller at the initial operating point where CAi is 10 kgmol/m^3, Ti and Tc are 298.15 K.

Create operating point specification.

```
plant_mdl = 'mpc_cstr_plant';
op = operspec(plant_mdl);
```

Feed concentration is known at the initial condition.

```
op.Inputs(1).u = 10;
op.Inputs(1).Known = true;
```

Feed temperature is known at the initial condition.

```
op.Inputs(2).u = 298.15;
op.Inputs(2).Known = true;
```

Coolant temperature is known at the initial condition.

```
op.Inputs(3).u = 298.15;
op.Inputs(3).Known = true;
```

Compute initial condition.

```
[op_point, op_report] = findop(plant_mdl,op);
```

```
 Operating Point Search Report:
---------------------------------

 Operating Report for the Model mpc_cstr_plant.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) mpc_cstr_plant/CSTR/Integrator
      x:           311      dx:      8.12e-11 (0)
(2.) mpc_cstr_plant/CSTR/Integrator1
      x:          8.57      dx:     -6.87e-12 (0)

Inputs:
----------
(1.) mpc_cstr_plant/CAi
      u:            10
(2.) mpc_cstr_plant/Ti
```

```
      u:           298
(3.) mpc_cstr_plant/Tc
      u:           298

Outputs:
----------
(1.) mpc_cstr_plant/T
      y:           311    [-Inf Inf]
(2.) mpc_cstr_plant/CA
      y:          8.57    [-Inf Inf]
```

Obtain nominal values of x, y and u.

```
x0 = [op_report.States(1).x;op_report.States(2).x];
y0 = [op_report.Outputs(1).y;op_report.Outputs(2).y];
u0 = [op_report.Inputs(1).u;op_report.Inputs(2).u;op_report.Inputs(3).u];
```

Obtain linear plant model at the initial condition.

```
sys = linearize(plant_mdl, op_point);
```

Drop the first plant input CAi and second output CA because they are not used by MPC.

```
sys = sys(1,2:3);
```

Discretize the plant model because Adaptive MPC controller only accepts a discrete-time plant model.

```
Ts = 0.5;
plant = c2d(sys,Ts);
```

Design MPC Controller

You design an MPC at the initial operating condition. When running in the adaptive mode, the plant model is updated at run time.

Specify signal types used in MPC.

```
plant.InputGroup.MeasuredDisturbances = 1;
plant.InputGroup.ManipulatedVariables = 2;
plant.OutputGroup.Measured = 1;
plant.InputName = {'Ti','Tc'};
plant.OutputName = {'T'};
```

Create MPC controller with default prediction and control horizons

```
mpcobj = mpc(plant);
```

```
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.
```

Set nominal values in the controller

```
mpcobj.Model.Nominal = struct('X', x0, 'U', u0(2:3), 'Y', y0(1), 'DX', [0 0]);
```

Set scale factors because plant input and output signals have different orders of magnitude

```
Uscale = [30 50];
Yscale = 50;
mpcobj.DV.ScaleFactor = Uscale(1);
mpcobj.MV.ScaleFactor = Uscale(2);
mpcobj.OV.ScaleFactor = Yscale;
```

Due to the physical constraint of coolant jacket, Tc rate of change is bounded by 2 degrees per minute.

```
mpcobj.MV.RateMin = -2;
mpcobj.MV.RateMax = 2;
```

Reactor concentration is not directly controlled in this example. If reactor temperature can be successfully controlled, the concentration will achieve desired performance requirement due to the strongly coupling between the two variables.

Implement Adaptive MPC Control of CSTR Plant in Simulink (R)

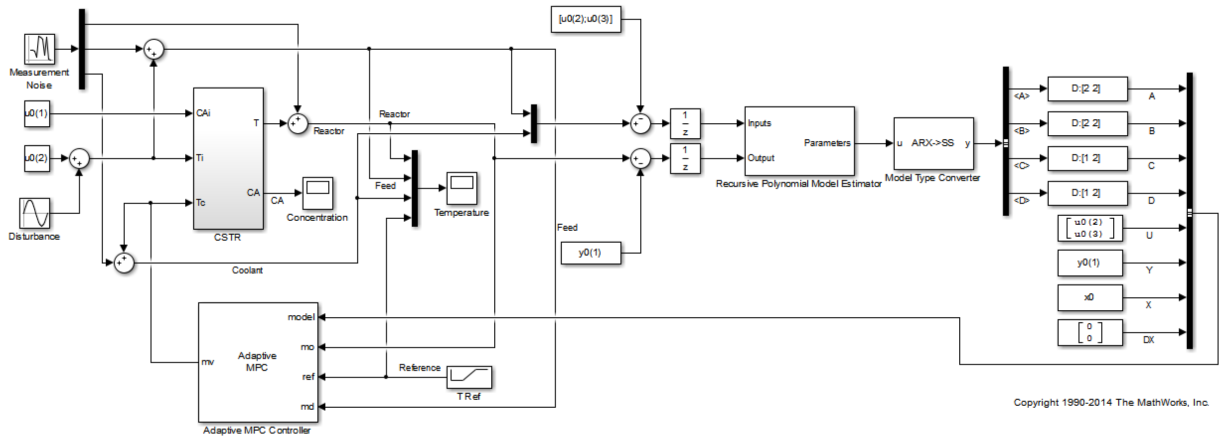
To run this example with online estimation, System Identification® is required.

```
if ~mpcchecktoolboxinstalled('ident')
    disp('System Identification(R) is required to run this example.')
    return
end
```

Open the Simulink model.

```
mdl = 'ampc_cstr_estimation';
```

```
open_system(mdl);
```



The model includes three parts:

1. The "CSTR" block implements the nonlinear plant model.
2. The "Adaptive MPC Controller" block runs the designed MPC controller in the adaptive mode.
3. The "Recursive Polynomial Model Estimator" block estimates a two-input (Ti and Tc) and one-output (T) discrete time ARX model based on the measured temperatures. The estimated model is then converted into state space form by the "Model Type Converter" block and fed to the "Adaptive MPC Controller" block at each control interval.

In this example, the initial plant model is used to initialize the online estimator with parameter covariance matrix set to 1. The online estimation method is "Kalman Filter" with noise covariance matrix set to 0.01. The online estimation result is sensitive to these parameters and you can further adjust them to achieve better estimation result.

Both "Recursive Polynomial Model Estimator" and "Model Type Converter" are provided by System Identification Toolbox. You can use the two blocks as a template to develop appropriate online model estimation for your own applications.

The initial value of A(q) and B(q) variables are populated with the numerator and denominator of the initial plant model.

```
[num, den] = tfdata(plant);
Aq = den{1};
Bq = num;
```

Note that the new linear plant model must be a discrete time state space system with the same order and sample time as the original plant model has. If the plant has time delay, it must also be same as the original time delay and absorbed into the state space model.

Validate Adaptive MPC Control Performance

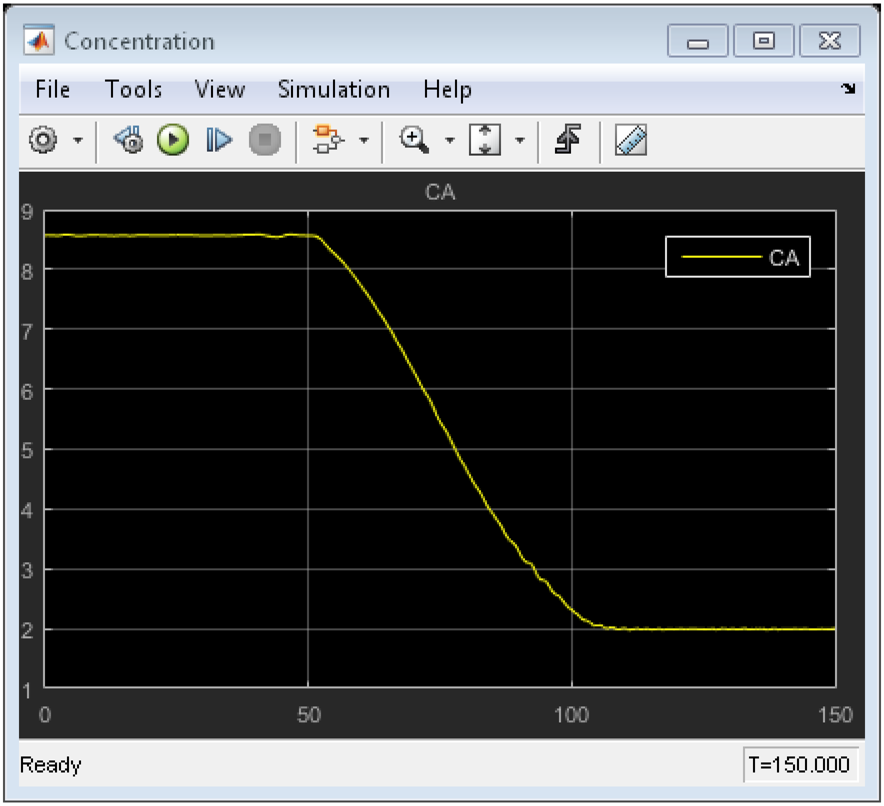
Controller performance is validated against both setpoint tracking and disturbance rejection.

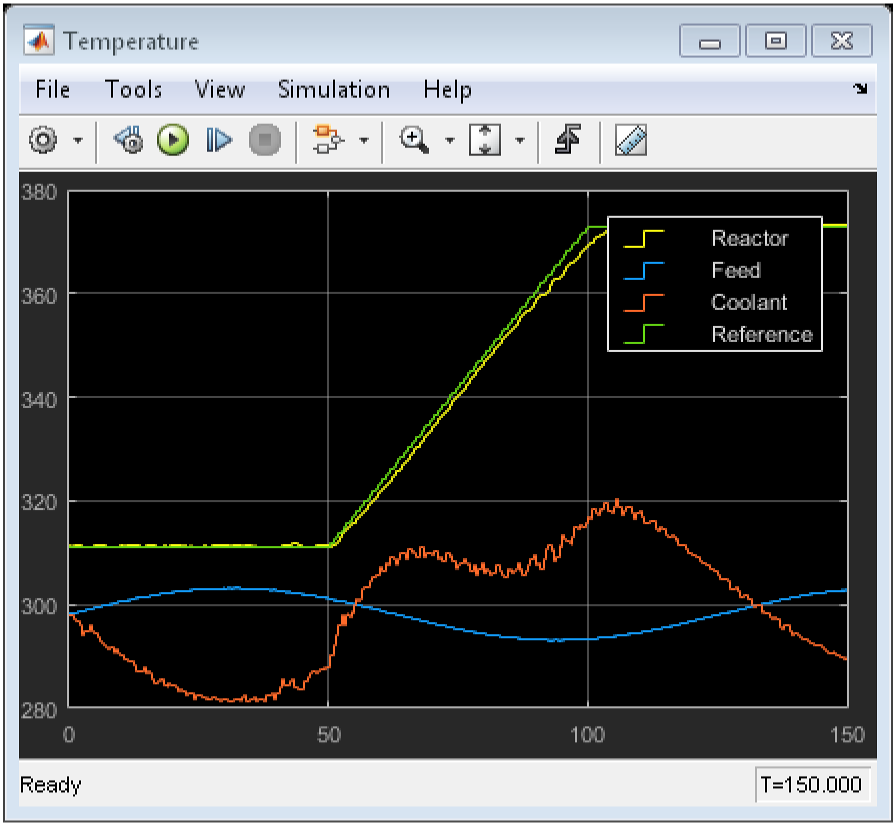
- Tracking: reactor temperature T setpoint transitions from original 311 K (low conversion rate) to 377 K (high conversion rate) kgmol/m^3. During the transition, the plant first becomes unstable then stable again (see the poles plot).
- Regulating: feed temperature Ti has slow fluctuation represented by a sine wave with amplitude of 5 degrees, which is a measured disturbance fed to MPC controller.

Simulate the closed-loop performance.

```
open_system([mdl '/Concentration'])
open_system([mdl '/Temperature'])
sim(mdl);
```

```
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```


Concentration
File Tools View Simulation Help
CA
9
8
7
6
5
4
3
2
1
0
50
100
150
CA
Ready
T=150.000

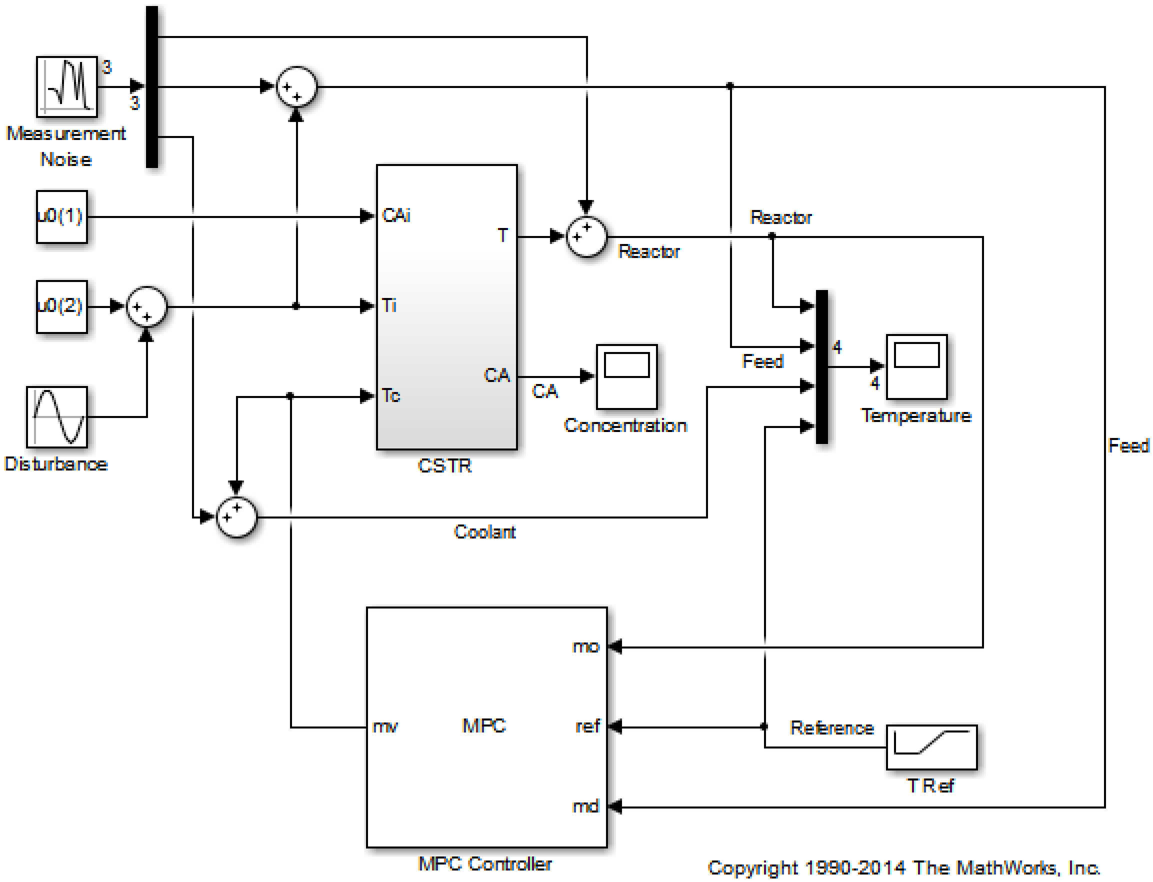


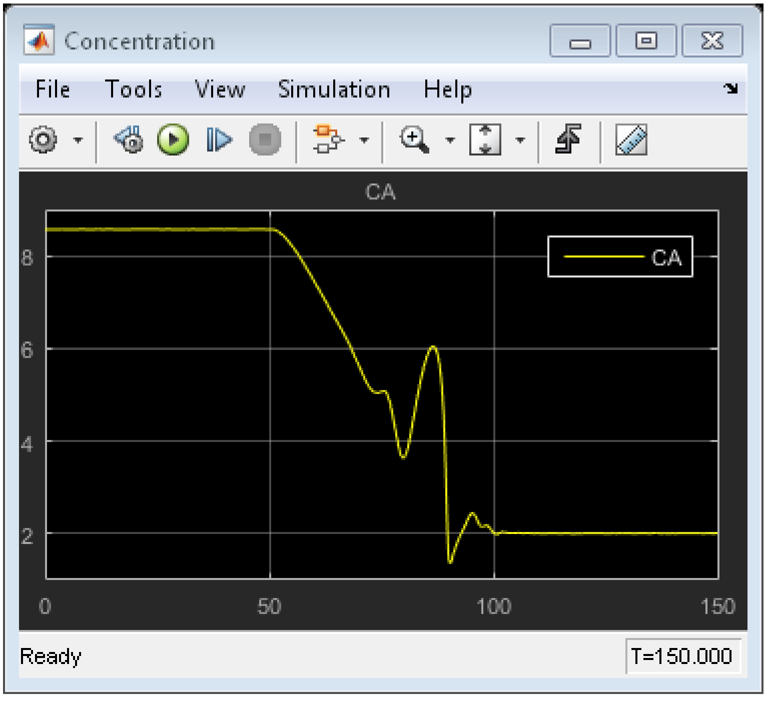
The tracking and regulating performance is very satisfactory.

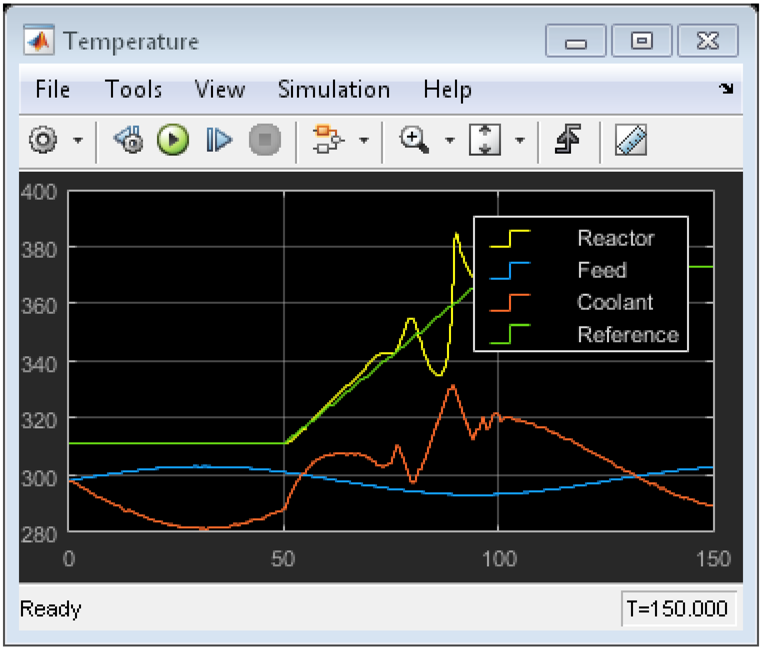
Compare with Non-Adaptive MPC Control

Adaptive MPC provides superior control performance than a non-adaptive MPC. To illustrate this point, the control performance of the same MPC controller running in the non-adaptive mode is shown below. The controller is implemented with a MPC Controller block.

```
mdl1 = 'ampc_cstr_no_estimation';
open_system(mdl1);
open_system([mdl1 '/Concentration'])
open_system([mdl1 '/Temperature'])
sim(mdl1);
```


Measurement Noise
u0(1)
u0(2)
Disturbance
CAi
Ti
Tc
T
CA
CSTR
Reactor
Reactor
Feed
CA
Concentration
Temperature
Coolant
Feed
mo
mv
MPC
ref
md
MPC Controller
Reference
TRef
Copyright 1990-2014 The MathWorks, Inc.


Concentration
File Tools View Simulation Help
CA
CA
8
6
4
2
0
50
100
150
Ready
T=150.000



As expected, the tracking and regulating performance is unacceptable.

```
bdclose(mdl)
bdclose(mdl1)
```

See Also

`Adaptive MPC Controller`

Related Examples

- "Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization" on page 5-8

More About

- "Adaptive MPC" on page 5-2

Explicit MPC Design

Explicit MPC

A traditional model predictive controller solves a quadratic program (QP) at each control interval to determine the optimal manipulated variable (MV) adjustments. These adjustments are the solution of the implicit nonlinear function $u=f(x)$.

The vector x contains the current controller state and other independent variables affecting the QP solution, such as the current output reference values. The Model Predictive Control Toolbox software imposes restrictions that force a unique QP solution.

Finding the optimal MV adjustments can be time consuming, and the required time can vary significantly from one control interval to the next. In applications that require a solution within a certain consistent time, which could be on the order of microseconds, the implicit MPC approach might be unsuitable.

As shown in “Optimization Problem” on page 2-2, if no QP inequality constraints are active for a given x vector, then the optimal MV adjustments become a linear function of x:

$$u = Fx + G.$$

where, F and G are constants. Similarly, if x remains in a region where a fixed subset of inequality constraints is active, the QP solution is also a linear function of x, but with different F and G constants.

Explicit MPC uses offline computations to determine all polyhedral regions where the optimal MV adjustments are a linear function of x, and the corresponding control-law constants. When the controller operates in real time, the explicit MPC controller performs the following steps at each control instant, k:

1. Estimate the controller state using available measurements, as in traditional MPC.
2. Form $x(k)$ using the estimated state and the current values of the other independent variables.
3. Identify the region in which $x(k)$ resides.
4. Looks up the predetermined F and G constants for this region.
5. Evaluate the linear function $u(k) = Fx(k) + G$.

You can establish a tight upper bound for the time required in each step. If the number of regions is not too large, the total computational time can be small. However, as the number of regions increases, the time required in step 3 dominates. Also, the memory required to store all the linear control laws and polyhedral regions becomes excessive. The number of regions characterizing $u = f(x)$ depends primarily on the QP inequality constraints that could be active at the solution. If an explicit MPC controller has many constraints, and thus requires significant computational effort or memory, a traditional (implicit) implementation may be preferable.

Related Examples

- “Explicit MPC Control of a Single-Input-Single-Output Plant” on page 6-9
- “Explicit MPC Control of an Aircraft with Unstable Poles” on page 6-21
- “Explicit MPC Control of DC Servomotor with Constraint on Unmeasured Output” on page 6-30

More About

- “Design Workflow for Explicit MPC” on page 6-4

Design Workflow for Explicit MPC

In this section...

To create an explicit MPC controller, you must first design a traditional (implicit) MPC controller. You then generate an explicit MPC controller based on the traditional controller design.

Traditional (Implicit) MPC Design

First design a traditional (implicit) MPC for your application and test it in simulations. Key considerations are as follows:

- The Model Predictive Control Toolbox software currently supports the following as independent variables for explicit MPC:
  - n_{xc} controller state variables (plant, disturbance, and measurement noise model states).
  - n_y (≥ 1) output reference values, where n_y is the number of plant output variables.
  - n_v (≥ 0) measured plant disturbance signals.

  Thus, you must fix most MPC design parameters prior to determining an explicit MPC. Fixed parameters include prediction models (plant, disturbance and measurement noise), scale factors, horizons, penalty weights, manipulated variable targets, and constraint bounds.

  For information about designing a traditional MPC controller, see "Controller Creation".

  For information about tuning traditional MPC controllers, see "Refinement".
- Reference and measured disturbance previewing are not supported. At each control interval, the current n_y reference and n_v measured disturbance signals apply for the entire prediction horizon.

- To limit the number of regions needed by explicit MPC, include only essential constraints.
  - When including a constraint on a manipulated variable (MV) use a short control horizon or MV blocking. See "Choosing Sample Time and Horizons" on page 1-6.
  - Avoid constraints on plant outputs. If such a constraint is essential, consider imposing it for selected prediction horizon steps rather than the entire prediction horizon.
- Establish upper and lower bounds for each of the $n_x = n_{xc} + n_y + n_v$ independent variables. You might know some of these bounds a priori. However, you must run simulations that record at least the n_{xc} controller states as the system operates over the range of expected conditions. It is very important that you not underestimate this range, because the explicit MPC control function is not defined for independent variables outside the range.

  For information about specifying bounds, see `generateExplicitRange`.

  For information about simulating a traditional MPC controller, see "Simulation".

Explicit MPC Generation

Given the constant MPC design parameters and the n_x upper and lower bounds on the control law's independent variables, i.e.,

$$x_l \le x(k) \le x_u,$$

the `generateExplicitMPC` command determines n_r regions. Each of these regions is defined by an inequality constraint and the corresponding control law constants:

$$\begin{aligned} H_i x(k) &\le K_i, \quad i = 1, n_r \\ u(k) &= F_i x(k) + G_i, \quad i = 1, n_r. \end{aligned}$$

The Explicit MPC Controller object contains the constants H_i, K_i, F_i, and G_i for each region. The Explicit MPC Controller object also holds the original (implicit) design and independent variable bounds. Provided that $x(k)$ stays within the specified bounds and you retain all n_r regions, the explicit MPC object should provide the same optimal MV adjustments, $u(k)$, as the equivalent implicit MPC object.

For details about explicit MPC, see [1]. For details about how the explicit MPC controller is generated, see [2].

Explicit MPC Simplification

Even a relatively simple explicit MPC controller might need $n_r >> 100$ to characterize the QP solution completely. If the number of regions is large, consider the following:

- Visualize the solution using the `plotSection` command.
- Use the `simplify` command to reduce the number of regions. In some cases, this can be done with no (or negligible) impact on control law optimality. For example, pairs of adjacent regions might employ essentially the same F_i and K_i constants. If so, and if the union of the two regions forms a convex set, they can be merged into a single region.

  Alternatively, you can eliminate relatively small regions or retain selected regions only. If during operation the current $x(k)$ is not contained in any of the retained regions, the explicit MPC will return a suboptimal $u(k)$, as follows:

  $$u(k) = F_j x(k) + G_j.$$

  Here, j is the index of the region whose bounding constraint, $H_j x(k) \leq K_j$, is least violated.

Implementation

During operation, for a given $x(k)$, the explicit MPC controller performs the following steps:

1. Verifies that $x(k)$ satisfies the specified bounds, $x_l \leq \mathrm{x}(k) \leq x_u$. If not, the controller returns an error status and sets $u(k) = u(k–1)$.
2. Beginning with region $i = 1$, tests the regions one by one to determine whether $x(k)$ belongs. If $H_i x(k) \leq K_i$, then $x(k)$ belongs to region i. If $x(k)$ belongs to region i, then the controller:

   - Obtains F_i and G_i from memory, and computes $u(k) = F_i x(k) + G_i$.
   - Signals successful completion, by returning a status code and the index i.
   - Returns without testing the remaining regions.

If $x(k)$ does not belong to region i, the controller:

- Computes the violation term v_i, which is the largest (positive) component of the vector $(H_i x(k) - K_i)$.
- If v_i is the minimum violation for this $x(k)$, the controller sets $j = i$, and sets $v_{min} = v_i$.
- The controller then increments i and tests the next region.

3 If all regions have been tested and $x(k)$ does not belong to any region (for example, due to a numerical precision issue), the controller:

- Obtains F_j and G_j from memory, and computes $u(k) = F_j x(k) + G_j$.
- Sets status to indicate a suboptimal solution and returns.

Thus, the maximum computational time per control interval is that needed to test each region, computing the violation term in each case, and then calculating the suboptimal control adjustment.

Simulation

You can perform command-line simulations using the `sim` or `mpcmoveExplicit` commands.

You can use the `Explicit MPC Controller` block to connect an explicit MPC to a plant modeled in Simulink.

References

[1] A. Bemporad, M. Morari, V. Dua, and E.N. Pistikopoulos, “The explicit linear quadratic regulator for constrained systems,” *Automatica*, vol. 38, no. 1, pp. 3–20, 2002.

[2] A. Bemporad, “A multi-parametric quadratic programming algorithm with polyhedral computations based on nonnegative least squares,” 2014, Submitted for publication.

See Also

`Explicit MPC Controller` | `generateExplicitMPC` | `mpcmoveExplicit`

Related Examples

- “Explicit MPC Control of a Single-Input-Single-Output Plant” on page 6-9
- “Explicit MPC Control of an Aircraft with Unstable Poles” on page 6-21
- “Explicit MPC Control of DC Servomotor with Constraint on Unmeasured Output” on page 6-30

More About

- “Explicit MPC” on page 6-2

Explicit MPC Control of a Single-Input-Single-Output Plant

This example shows how to control a double integrator plant under input saturation in Simulink® using explicit MPC.

See also MPCDOUBLEINT.

Define Plant Model

The linear open-loop dynamic model is a double integrator:

```
plant = tf(1,[1 0 0]);
```

Design MPC Controller

Create the controller object with sampling period, prediction and control horizons:

```
Ts = 0.1;
p = 10;
m = 3;
mpcobj = mpc(plant, Ts, p, m);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

Specify actuator saturation limits as MV constraints.

```
mpcobj.MV = struct('Min',-1,'Max',1);
```

Generate Explicit MPC Controller

Explicit MPC executes the equivalent explicit piecewise affine version of the MPC control law defined by the traditional MPC. To generate an Explicit MPC from a traditional MPC, you must specify range for each controller state, reference signal, manipulated variable and measured disturbance so that the multi-parametric quadratic programming problem is solved in the parameter space defined by these ranges.

Obtain a range structure for initialization

Use `generateExplicitRange` command to obtain a range structure where you can specify range for each parameter afterwards.

```
range = generateExplicitRange(mpcobj);
```

```
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Specify ranges for controller states

MPC controller states include states from plant model, disturbance model and noise model in that order. Setting the range of a state variable is sometimes difficult when the state does not correspond to a physical parameter. In that case, multiple runs of open-loop plant simulation with typical reference and disturbance signals are recommended in order to collect data that reflect the ranges of states.

```
range.State.Min(:) = [-10;-10];
range.State.Max(:) = [10;10];
```

Specify ranges for reference signals

Usually you know the practical range of the reference signals being used at the nominal operating point in the plant. The ranges used to generate Explicit MPC must be at least as large as the practical range.

```
range.Reference.Min = -2;
range.Reference.Max = 2;
```

Specify ranges for manipulated variables

If manipulated variables are constrained, the ranges used to generate Explicit MPC must be at least as large as these limits.

```
range.ManipulatedVariable.Min = -1.1;
range.ManipulatedVariable.Max = 1.1;
```

Construct the Explicit MPC controller

Use `generateExplicitMPC` command to obtain the Explicit MPC controller with the parameter ranges previously specified.

```
mpcobjExplicit = generateExplicitMPC(mpcobj, range);
display(mpcobjExplicit);
```

```
Regions found / unexplored:       19/       0


Explicit MPC Controller
---------------------------------------------
Controller sample time:    0.1 (seconds)
Polyhedral regions:        19
Number of parameters:      4
Is solution simplified:    No
State Estimation:          Default Kalman gain
---------------------------------------------
Type 'mpcobjExplicit.MPC' for the original implicit MPC design.
Type 'mpcobjExplicit.Range' for the valid range of parameters.
Type 'mpcobjExplicit.OptimizationOptions' for the options used in multi-parametric QP c
Type 'mpcobjExplicit.PiecewiseAffineSolution' for regions and gain in each solution.
```

Use `simplify` command with the "exact" method to join pairs of regions whose corresponding gains are the same and whose union is a convex set. This practice can reduce memory footprint of the Explicit MPC controller without sacrifice any performance.

```
mpcobjExplicitSimplified = simplify(mpcobjExplicit, 'exact');
display(mpcobjExplicitSimplified);
```

```
Regions to analyze:       15/      15


Explicit MPC Controller
---------------------------------------------
Controller sample time:    0.1 (seconds)
Polyhedral regions:        15
Number of parameters:      4
Is solution simplified:    Yes
State Estimation:          Default Kalman gain
---------------------------------------------
Type 'mpcobjExplicitSimplified.MPC' for the original implicit MPC design.
Type 'mpcobjExplicitSimplified.Range' for the valid range of parameters.
Type 'mpcobjExplicitSimplified.OptimizationOptions' for the options used in multi-paran
Type 'mpcobjExplicitSimplified.PiecewiseAffineSolution' for regions and gain in each sc
```

The number of piecewise affine region has been reduced.

Plot Piecewise Affine Partition

You can review any 2-D section of the piecewise affine partition defined by the Explicit MPC control law.

Obtain a plot parameter structure for initialization

Use `generatePlotParameters` command to obtain a parameter structure where you can specify which 2-D section to plot afterwards.

```
params = generatePlotParameters(mpcobjExplicitSimplified);
```

Specify parameters for a 2-D plot

In this example, you plot the 1th state variable vs. the 2nd state variable. All the other parameters must be fixed at a value within its range.

```
params.State.Index = [];
params.State.Value = [];
```

Fix other reference signals

```
params.Reference.Index = 1;
params.Reference.Value = 0;
```

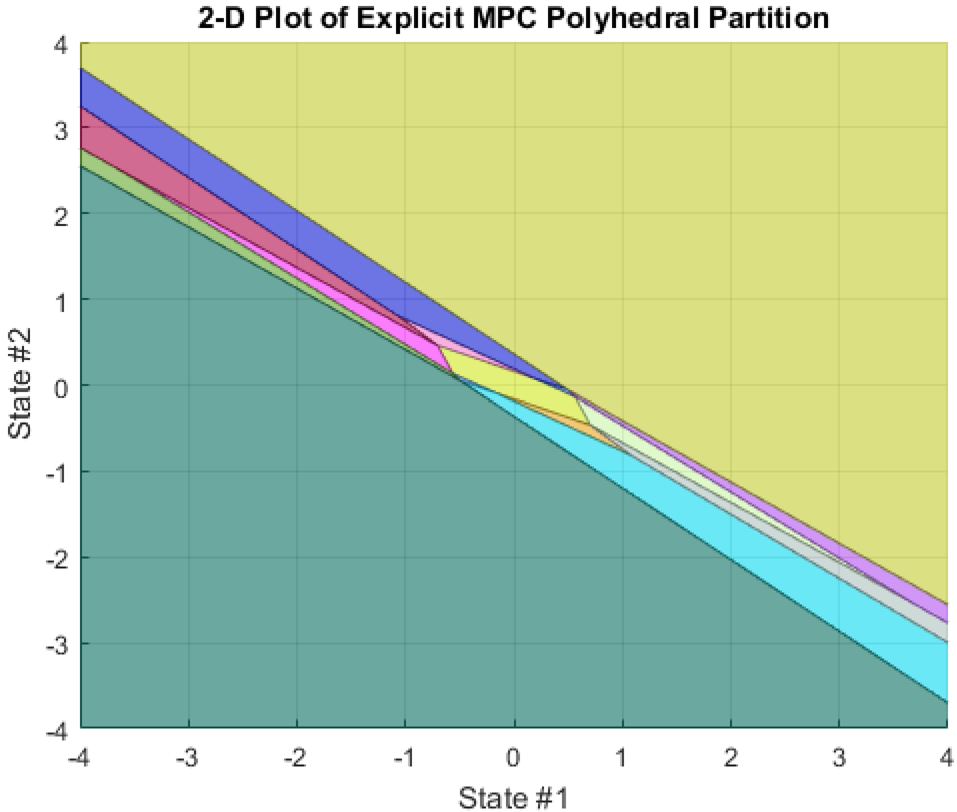
Fix manipulated variables

```
params.ManipulatedVariable.Index = 1;
params.ManipulatedVariable.Value = 0;
```

Plot the 2-D section

Use `plotSection` command to plot the 2-D section defined previously.

```
plotSection(mpcobjExplicitSimplified, params);
axis([-4 4 -4 4]);
grid
xlabel('State #1');
ylabel('State #2');
```



Simulate Using MPCMOVE Command

Compare closed-loop simulation between tradition MPC (as referred as Implicit MPC) and Explicit MPC using `mpcmove` and `mpcmoveExplicit` commands respectively.

Prepare to store the closed-loop MPC responses.

```
Tf = round(5/Ts);
YY = zeros(Tf,1);
YYExplicit = zeros(Tf,1);
UU = zeros(Tf,1);
UUExplicit = zeros(Tf,1);
```

Prepare the real plant used in simulation

```
sys = c2d(ss(plant),Ts);
xsys = [0;0];
xsysExplicit = xsys;
```

Use MPCSTATE object to specify the initial states for both controllers

```
xmpc = mpcstate(mpcobj);
xmpcExplicit = mpcstate(mpcobjExplicitSimplified);
```

Simulate closed-loop response in each iteration.

```
for t = 0:Tf
    % update plant measurement
    ysys = sys.C*xsys;
    ysysExplicit = sys.C*xsysExplicit;
    % compute traditional MPC action
    u = mpcmove(mpcobj,xmpc,ysys,1);
    % compute Explicit MPC action
    uExplicit = mpcmoveExplicit(mpcobjExplicit,xmpcExplicit,ysysExplicit,1);
    % store signals
    YY(t+1)=ysys;
    YYExplicit(t+1)=ysysExplicit;
    UU(t+1)=u;
    UUExplicit(t+1)=uExplicit;
    % update plant state
    xsys = sys.A*xsys + sys.B*u;
    xsysExplicit = sys.A*xsysExplicit + sys.B*uExplicit;
end
fprintf('\nDifference between traditional and Explicit MPC responses using MPCMOVE comm
```

```
Difference between traditional and Explicit MPC responses using MPCMOVE command is 1.69
```

Simulate Using SIM Command

Compare closed-loop simulation between tradition MPC and Explicit MPC using `sim` commands respectively.

```
Tf = 5/Ts;                      % simulation iterations
[y1,t1,u1] = sim(mpcobj,Tf,1);  % simulation with tradition MPC
[y2,t2,u2] = sim(mpcobjExplicitSimplified,Tf,1);   % simulation with Explicit MPC
```

```
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

The simulation results are identical.

```
fprintf('\nDifference between traditional and Explicit MPC responses using SIM command
```

```
Difference between traditional and Explicit MPC responses using SIM command is 1.68991e
```

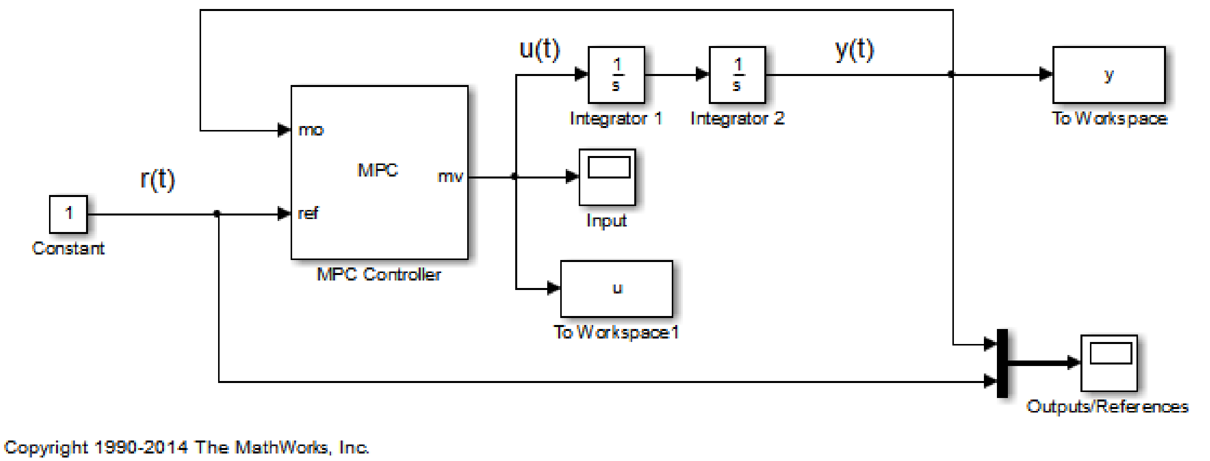
Simulate Using Simulink®

To run this example, Simulink® is required.

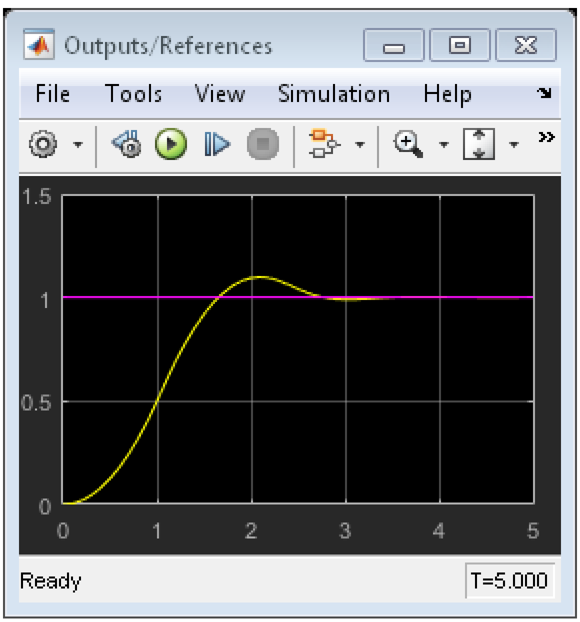
```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
```

Simulate with traditional MPC controller in Simulink. Controller "mpcobj" is specified in the block dialog.

```
mdl = 'mpc_doubleint';
open_system(mdl);
sim(mdl);
```



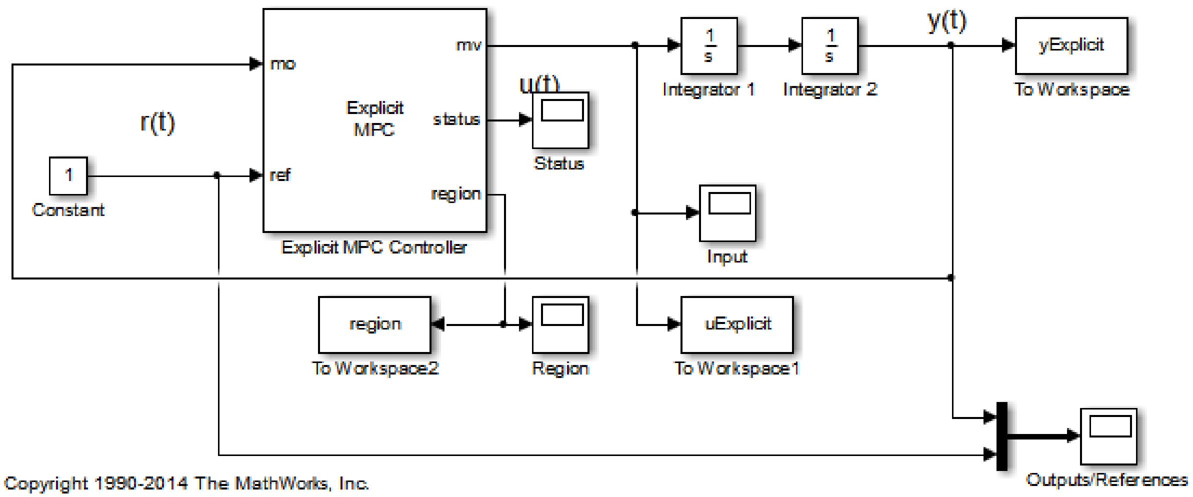
u(t)
y(t)
r(t)
1
s
Integrator 1
Integrator 2
y
To Workspace
mo
MPC
mv
ref
MPC Controller
1
Constant
Input
u
To Workspace1
Outputs/References
Copyright 1990-2014 The MathWorks, Inc.



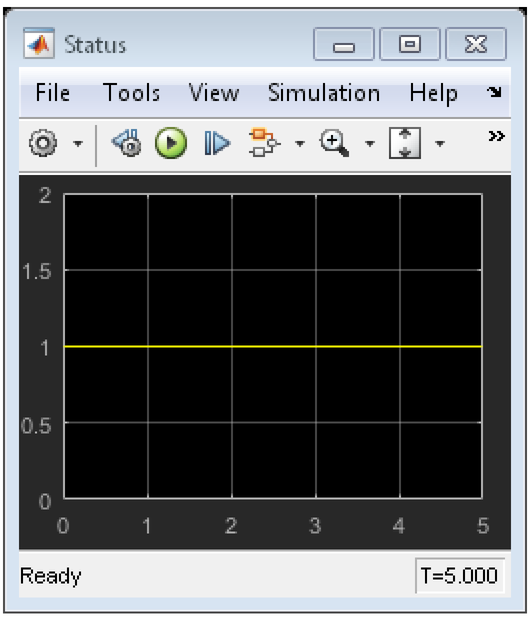
Outputs/References
File Tools View Simulation Help
1.5
1
0.5
0
0 1 2 3 4 5
Ready
T=5.000

Simulate with Explicit MPC controller in Simulink. Controller "mpcobjExplicitSimplified" is specified in the block dialog.

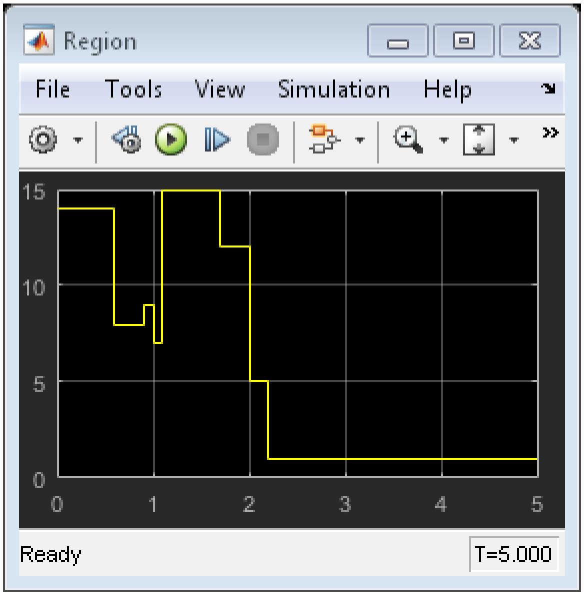
```
mdlExplicit = 'empc_doubleint';
open_system(mdlExplicit);
sim(mdlExplicit);
```

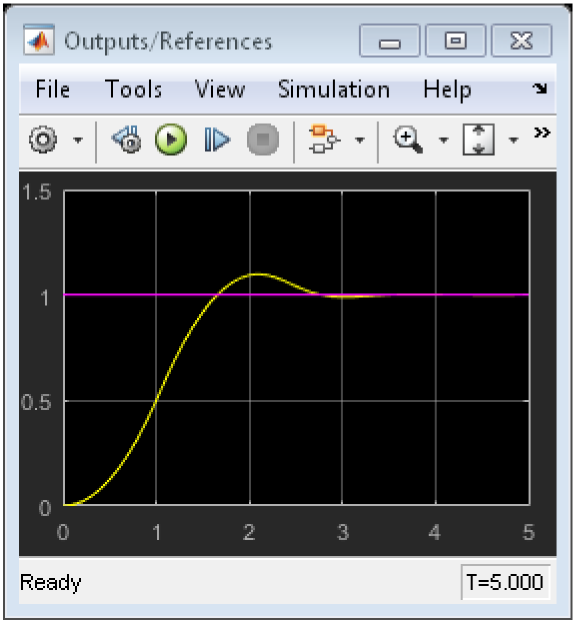


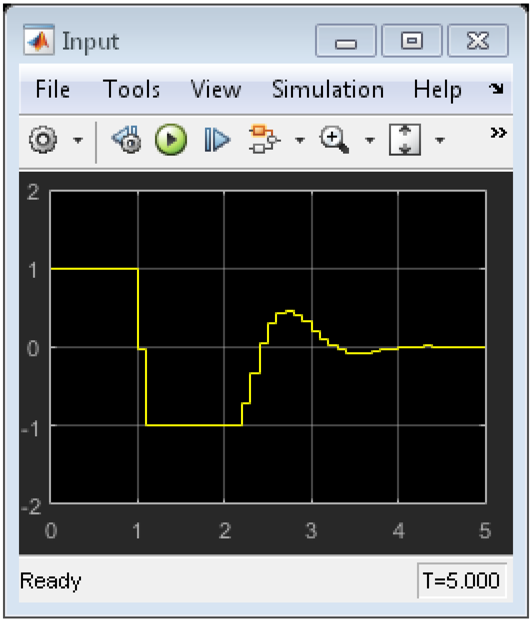
r(t)
1
Constant
mo
ref
Explicit
MPC
mv
status
region
Explicit MPC Controller
u(t)
Status
1
s
Integrator 1
1
s
Integrator 2
y(t)
yExplicit
To Workspace
Input
region
To Workspace2
Region
uExplicit
To Workspace1
Outputs/References
Copyright 1990-2014 The MathWorks, Inc.



Status
File Tools View Simulation Help
2
1.5
1
0.5
0
0 1 2 3 4 5
Ready
T=5.000


Region
File
Tools
View
Simulation
Help
15
10
5
0
0
1
2
3
4
5
Ready
T=5.000


Outputs/References
File
Tools
View
Simulation
Help
1.5
1
0.5
0
0
1
2
3
4
5
Ready
T=5.000



The closed-loop responses are identical.

```
fprintf('\nDifference between traditional and Explicit MPC responses in Simulink is %g'
```

```
Difference between traditional and Explicit MPC responses in Simulink is 1.5821e-13
```

```
bdclose(mdl)
bdclose(mdlExplicit)
```

Related Examples

- “Explicit MPC Control of an Aircraft with Unstable Poles” on page 6-21
- “Explicit MPC Control of DC Servomotor with Constraint on Unmeasured Output” on page 6-30

More About

- “Explicit MPC” on page 6-2

Explicit MPC Control of an Aircraft with Unstable Poles

This example shows how to control an unstable aircraft with saturating actuators using Explicit MPC.

Reference:

[1] P. Kapasouris, M. Athans and G. Stein, "Design of feedback control systems for unstable plants with saturating actuators", Proc. IFAC Symp. on Nonlinear Control System Design, Pergamon Press, pp.302--307, 1990

[2] A. Bemporad, A. Casavola, and E. Mosca, "Nonlinear control of constrained linear systems via predictive reference management", IEEE® Trans. Automatic Control, vol. AC-42, no. 3, pp. 340-349, 1997.

See also MPCAIRCRAFT.

Define Aircraft Model

The linear open-loop dynamic model is as follows:

```
A = [-0.0151 -60.5651 0 -32.174;
     -0.0001 -1.3411 0.9929 0;
      0.00018 43.2541 -0.86939 0;
      0      0       1      0];
B = [-2.516 -13.136;
     -0.1689 -0.2514;
     -17.251 -1.5766;
      0        0];
C = [0 1 0 0;
     0 0 0 1];
D = [0 0;
     0 0];
plant = ss(A,B,C,D);
x0 = zeros(4,1);
```

The manipulated variables are the elevator and flaperon angles, the attack and pitch angles are measured outputs to be regulated.

The open-loop response of the system is unstable.

```
pole(plant)
```

```
ans =

  -7.6636 + 0.0000i
   5.4530 + 0.0000i
  -0.0075 + 0.0556i
  -0.0075 - 0.0556i
```

Design MPC Controller

To obtain an Explicit MPC controller, you must first design a traditional MPC (also referred as Implicit MPC) that is able to achieves your control objectives.

```
% *MV Constraints*
```

Both manipulated variables are constrained between +/- 25 degrees. Since the plant inputs and outputs are of different orders of magnitude, you also use scale factors to faciliate MPC tuning. Typical choices of scale factor are the upper/lower limit or the operating range.

```
MV = struct('Min',{-25,-25},'Max',{25,25},'ScaleFactor',{50,50});
```

OV Constraints

Both plant outputs have constraints to limit undershoots at the first prediction horizon. You also specify scale factors for outputs.

```
OV = struct('Min',{[-0.5;-Inf],[-100;-Inf]},'Max',{[0.5;Inf],[100;Inf]},'ScaleFactor',
```

Weights

The control task is to get zero offset for piecewise-constant references, while avoiding instability due to input saturation. Because both MV and OV variables are already scaled in MPC controller, MPC weights are dimensionless and applied to the scaled MV and OV values. In this example, you penalize the two outputs equally with the same OV weights.

```
Weights = struct('MV',[0 0],'MVRate',[0.1 0.1],'OV',[10 10]);
```

Construct the traditional MPC controller

Create an MPC controller with plant model, sample time and horizons.

```
Ts = 0.05;          % Sampling time
p = 10;             % Prediction horizon
m = 2;              % Control horizon
mpcobj = mpc(plant,Ts,p,m,Weights,MV,OV);
```

Generate Explicit MPC Controller

Explicit MPC executes the equivalent explicit piecewise affine version of the MPC control law defined by the traditional MPC. To generate an Explicit MPC from a traditional MPC, you must specify range for each controller state, reference signal, manipulated variable and measured disturbance so that the multi-parametric quadratic programming problem is solved in the parameter space defined by these ranges.

Obtain a range structure for initialization

Use `generateExplicitRange` command to obtain a range structure where you can specify range for each parameter afterwards.

```
range = generateExplicitRange(mpcobj);
```

```
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->Assuming output disturbance added to measured output channel #2 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Specify ranges for controller states

MPC controller states include states from plant model, disturbance model and noise model in that order. Setting the range of a state variable is sometimes difficult when the state does not correspond to a physical parameter. In that case, multiple runs of open-loop plant simulation with typical reference and disturbance signals are recommended in order to collect data that reflect the ranges of states.

```
range.State.Min(:) = -10000;
range.State.Max(:) = 10000;
```

Specify ranges for reference signals

Usually you know the practical range of the reference signals being used at the nominal operating point in the plant. The ranges used to generate Explicit MPC must be at least as large as the practical range.

```
range.Reference.Min = [-1;-11];
range.Reference.Max = [1;11];
```

Specify ranges for manipulated variables

If manipulated variables are constrained, the ranges used to generate Explicit MPC must be at least as large as these limits.

```
range.ManipulatedVariable.Min = [MV(1).Min; MV(2).Min] - 1;
range.ManipulatedVariable.Max = [MV(1).Max; MV(2).Max] + 1;
```

Construct the Explicit MPC controller

Use `generateExplicitMPC` command to obtain the Explicit MPC controller with the parameter ranges previously specified.

```
mpcobjExplicit = generateExplicitMPC(mpcobj, range);
display(mpcobjExplicit);
```

```
Regions found / unexplored:      483/       0


Explicit MPC Controller
---------------------------------------------
Controller sample time:    0.05 (seconds)
Polyhedral regions:        483
Number of parameters:      10
Is solution simplified:    No
State Estimation:          Default Kalman gain
---------------------------------------------
Type 'mpcobjExplicit.MPC' for the original implicit MPC design.
Type 'mpcobjExplicit.Range' for the valid range of parameters.
Type 'mpcobjExplicit.OptimizationOptions' for the options used in multi-parametric QP c
Type 'mpcobjExplicit.PiecewiseAffineSolution' for regions and gain in each solution.
```

Use `simplify` command with the "exact" method to join pairs of regions whose corresponding gains are the same and whose union is a convex set. This practice can reduce memory footprint of the Explicit MPC controller without sacrifice any performance.

```
mpcobjExplicitSimplified = simplify(mpcobjExplicit, 'exact');
display(mpcobjExplicitSimplified);
```

```
Regions to analyze:      471/     471


Explicit MPC Controller
---------------------------------------------
Controller sample time:    0.05 (seconds)
```

```
Polyhedral regions:        471
Number of parameters:      10
Is solution simplified:    Yes
State Estimation:          Default Kalman gain
---------------------------------------------
Type 'mpcobjExplicitSimplified.MPC' for the original implicit MPC design.
Type 'mpcobjExplicitSimplified.Range' for the valid range of parameters.
Type 'mpcobjExplicitSimplified.OptimizationOptions' for the options used in multi-paran
Type 'mpcobjExplicitSimplified.PiecewiseAffineSolution' for regions and gain in each so
```

The number of piecewise affine region has been reduced.

Plot Piecewise Affine Partition

You can review any 2-D section of the piecewise affine partition defined by the Explicit MPC control law.

Obtain a plot parameter structure for initialization

Use `generatePlotParameters` command to obtain a parameter structure where you can specify which 2-D section to plot afterwards.

```
params = generatePlotParameters(mpcobjExplicitSimplified);
```

Specify parameters for a 2-D plot

In this example, you plot the pitch angle (the 4th state variable) vs. its reference (the 2nd reference signal). All the other parameters must be fixed at a value within its range.

Fix other state variables

```
params.State.Index = [1 2 3 5 6];
params.State.Value = [0 0 0 0 0];
```

Fix other reference signals

```
params.Reference.Index = 1;
params.Reference.Value = 0;
```

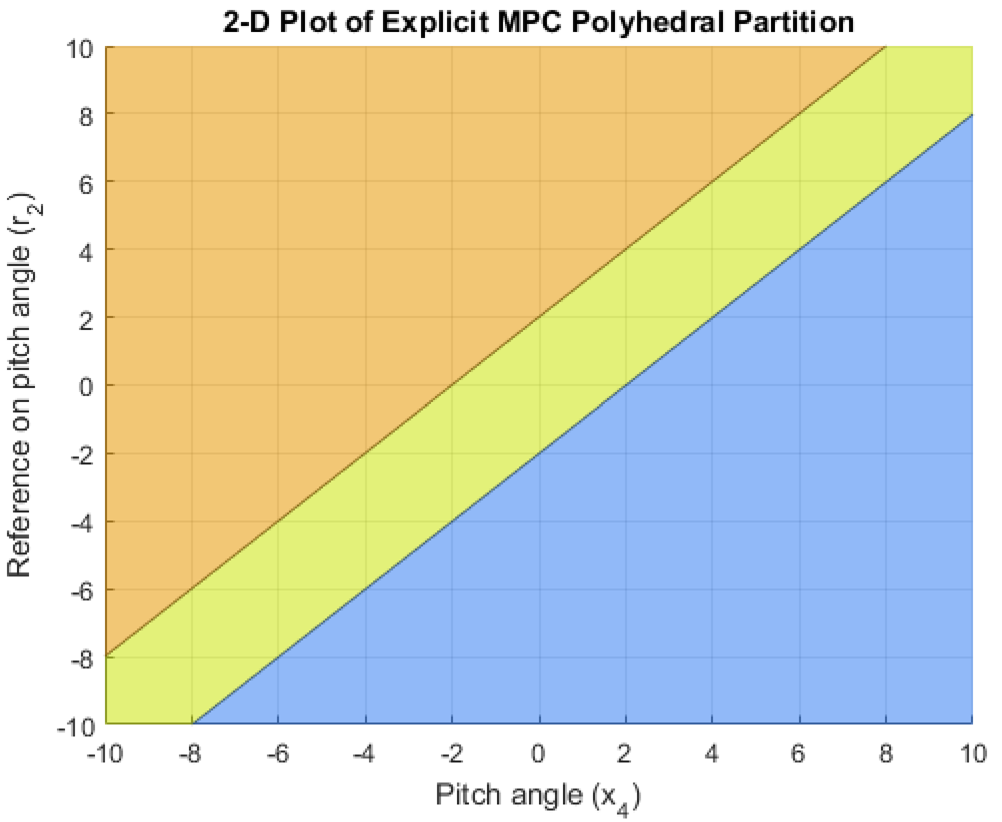
Fix manipulated variables

```
params.ManipulatedVariable.Index = [1 2];
params.ManipulatedVariable.Value = [0 0];
```

Plot the 2-D section

Use `plotSection` command to plot the 2-D section defined previously.

```
plotSection(mpcobjExplicitSimplified, params);
axis([-10 10 -10 10]);
grid;
xlabel('Pitch angle (x_4)');
ylabel('Reference on pitch angle (r_2)');
```



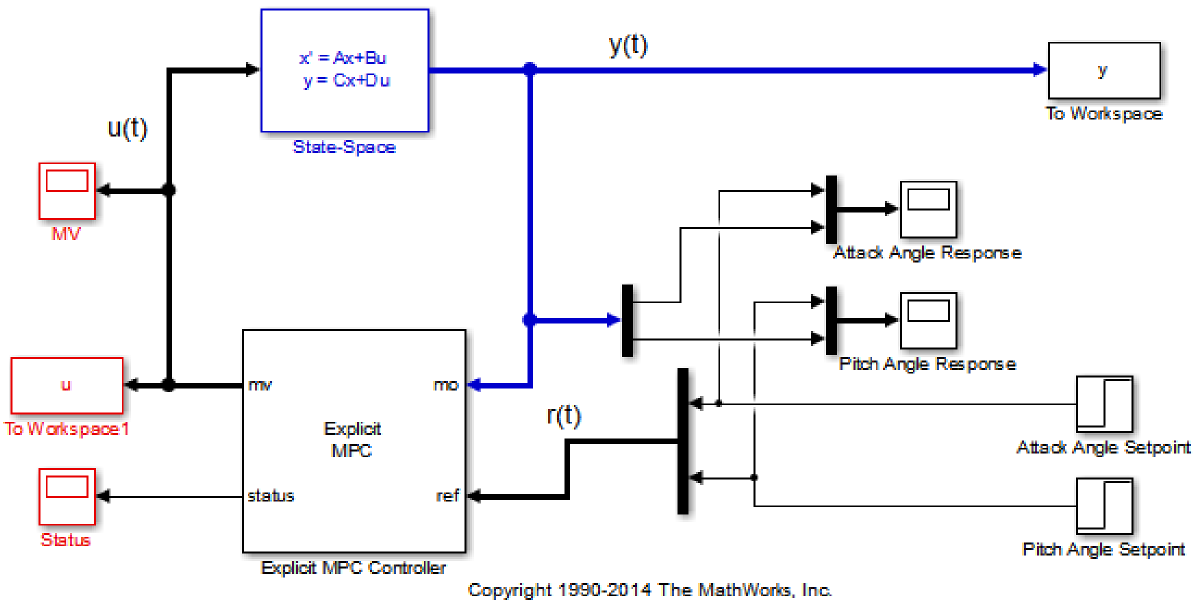
Simulate Using Simulink®

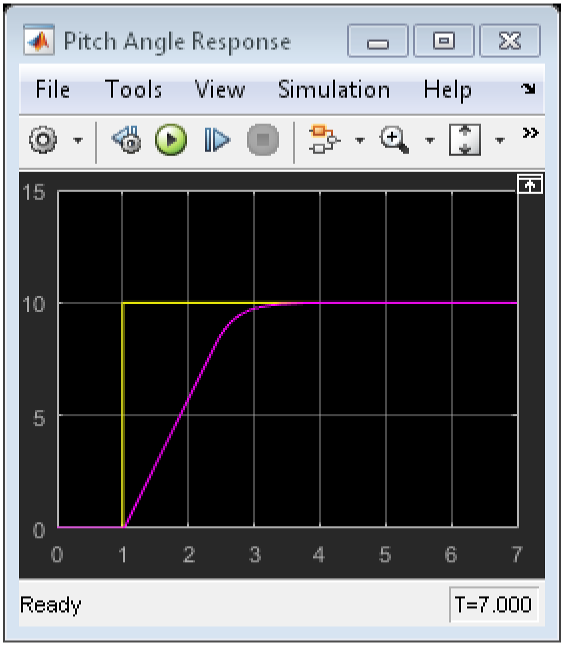
To run this example, Simulink® is required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
```

Simulate closed-loop control of the linear plant model in Simulink, using the Explicit MPC Controller block. Controller "mpcobjExplicitSimplified" is specified in the block dialog.

```
mdl = 'empc_aircraft';
open_system(mdl)
sim(mdl)
```




Pitch Angle Response
File Tools View Simulation Help
15
10
5
0
0 1 2 3 4 5 6 7
Ready
T=7.000

The closed-loop response is identical to the traditional MPC controller designed in the "mpcaircraft" example.

```
bdclose(mdl)
```

Related Examples

- "Explicit MPC Control of a Single-Input-Single-Output Plant" on page 6-9
- "Explicit MPC Control of DC Servomotor with Constraint on Unmeasured Output" on page 6-30

More About

- "Explicit MPC" on page 6-2

Explicit MPC Control of DC Servomotor with Constraint on Unmeasured Output

This example shows how to use Explicit MPC to control DC servomechanism under voltage and shaft torque constraints.

Reference

[1] A. Bemporad and E. Mosca, "Fulfilling hard constraints in uncertain linear systems by reference managing," Automatica, vol. 34, no. 4, pp. 451-461, 1998.

See also MPCMOTOR.

Define DC-Servo Motor Model

The linear open-loop dynamic model is defined in "plant". Variable "tau" is the maximum admissible torque to be used as an output constraint.

```
[plant, tau] = mpcmotormodel;
```

Design MPC Controller

Specify input and output signal types for the MPC controller. The second output, torque, is unmeasurable.

```
plant = setmpcsignals(plant,'MV',1,'MO',1,'UO',2);
```

MV Constraints

The manipulated variable is constrained between +/- 220 volts. Since the plant inputs and outputs are of different orders of magnitude, you also use scale factors to faciliate MPC tuning. Typical choices of scale factor are the upper/lower limit or the operating range.

```
MV = struct('Min',-220,'Max',220,'ScaleFactor',440);
```

OV Constraints

Torque constraints are only imposed during the first three prediction steps to limit the complexity of the explicit MPC design.

```
OV = struct('Min',{Inf, [-tau;-tau;-tau;-Inf]},'Max',{Inf, [tau;tau;tau;Inf]},'ScaleFac
```

Weights

The control task is to get zero tracking offset for the angular position. Since you only have one manipulated variable, the shaft torque is allowed to float within its constraint by setting its weight to zero.

```
Weights = struct('MV',0,'MVRate',0.1,'OV',[0.1 0]);
```

Construct MPC controller

Create an MPC controller with plant model, sample time and horizons.

```
Ts = 0.1;           % Sampling time
p = 10;             % Prediction horizon
m = 2;              % Control horizon
mpcobj = mpc(plant,Ts,p,m,Weights,MV,OV);
```

Generate Explicit MPC Controller

Explicit MPC executes the equivalent explicit piecewise affine version of the MPC control law defined by the traditional MPC. To generate an Explicit MPC from a traditional MPC, you must specify the range for each controller state, reference signal, manipulated variable and measured disturbance so that the multi-parametric quadratic programming problem is solved in the parameter sets defined by these ranges.

Obtain a range structure for initialization

Use `generateExplicitRange` command to obtain a range structure where you can specify the range for each parameter afterwards.

```
range = generateExplicitRange(mpcobj);
```

```
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Specify ranges for controller states

MPC controller states include states from plant model, disturbance model and noise model in that order. Setting the range of a state variable is sometimes difficult when the state does not correspond to a physical parameter. In that case, multiple runs of open-loop plant simulation with typical reference and disturbance signals are recommended in order to collect data that reflect the ranges of states.

```
range.State.Min(:) = -1000;
range.State.Max(:) = 1000;
```

Specify ranges for reference signals

Usually you know the practical range of the reference signals being used at the nominal operating point in the plant. The ranges used to generate Explicit MPC must be at least as large as the practical range. Note that the range for torque reference is fixed at 0 because it has zero weight.

```
range.Reference.Min = [-5;0];
range.Reference.Max = [5;0];
```

Specify ranges for manipulated variables

If manipulated variables are constrained, the ranges used to generate Explicit MPC must be at least as large as these limits.

```
range.ManipulatedVariable.Min = MV.Min - 1;
range.ManipulatedVariable.Max = MV.Max + 1;
```

Construct the Explicit MPC controller

Use `generateExplicitMPC` command to obtain the Explicit MPC controller with the parameter ranges previously specified.

```
mpcobjExplicit = generateExplicitMPC(mpcobj, range);
display(mpcobjExplicit);
```

```
Regions found / unexplored:       75/       0


Explicit MPC Controller
---------------------------------------------
Controller sample time:    0.1 (seconds)
Polyhedral regions:        75
Number of parameters:      6
```

```
Is solution simplified:    No
State Estimation:          Default Kalman gain
---------------------------------------------
Type 'mpcobjExplicit.MPC' for the original implicit MPC design.
Type 'mpcobjExplicit.Range' for the valid range of parameters.
Type 'mpcobjExplicit.OptimizationOptions' for the options used in multi-parametric QP c
Type 'mpcobjExplicit.PiecewiseAffineSolution' for regions and gain in each solution.
```

Plot Piecewise Affine Partition

You can review any 2-D section of the piecewise affine partition defined by the Explicit MPC control law.

Obtain a plot parameter structure for initialization

Use `generatePlotParameters` command to obtain a parameter structure where you can specify which 2-D section to plot afterwards.

```
params = generatePlotParameters(mpcobjExplicit);
```

Specify parameters for a 2-D plot

In this example, you plot the 1th state variable vs. the 2nd state variable. All the other parameters must be fixed at a value within its range.

Fix other state variables

```
params.State.Index = [3 4];
params.State.Value = [0 0];
```

Fix reference signals

```
params.Reference.Index = [1 2];
params.Reference.Value = [pi 0];
```

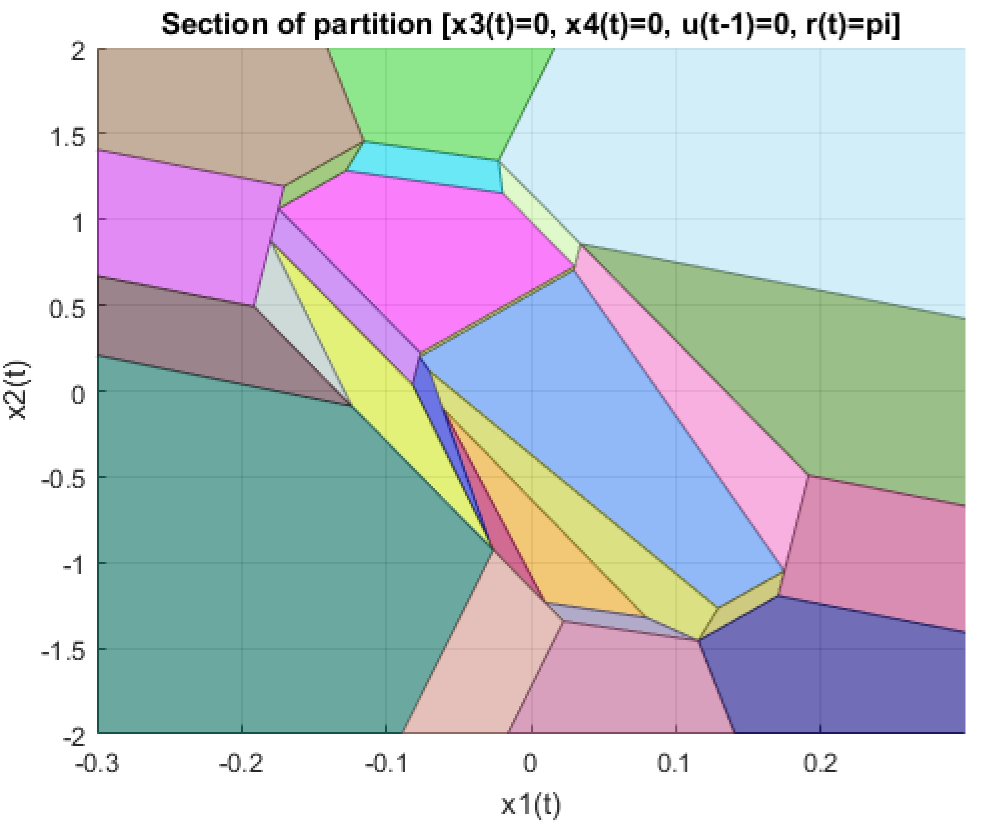
Fix manipulated variables

```
params.ManipulatedVariable.Index = 1;
params.ManipulatedVariable.Value = 0;
```

Plot the 2-D section

Use `plotSection` command to plot the 2-D section defined previously.

```
plotSection(mpcobjExplicit, params);
axis([-.3 .3 -2 2]);
grid
title('Section of partition [x3(t)=0, x4(t)=0, u(t-1)=0, r(t)=pi]')
xlabel('x1(t)');
ylabel('x2(t)');
```



Simulate Using SIM Command

Compare closed-loop simulation between traditional MPC (as referred as Implicit MPC) and Explicit MPC

```
Tstop = 8;                          % seconds
```

```
Tf = round(Tstop/Ts);           % simulation iterations
r = [pi 0];                     % reference signal
[y1,t1,u1] = sim(mpcobj,Tf,r);  % simulation with traditional MPC
[y2,t2,u2] = sim(mpcobjExplicit,Tf,r);   % simulation with Explicit MPC
```

```
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

The simulation results are identical.

```
fprintf('SIM command: Difference between QP-based and Explicit MPC trajectories = %g\n
```

```
SIM command: Difference between QP-based and Explicit MPC trajectories = 5.50433e-12
```

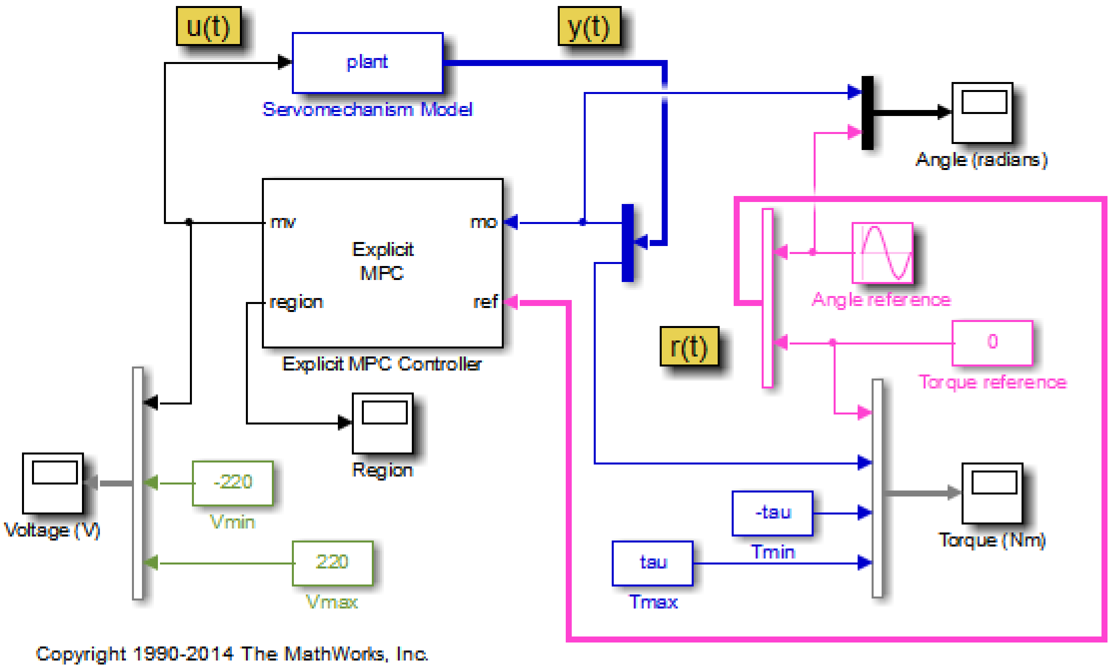
Simulate Using Simulink®

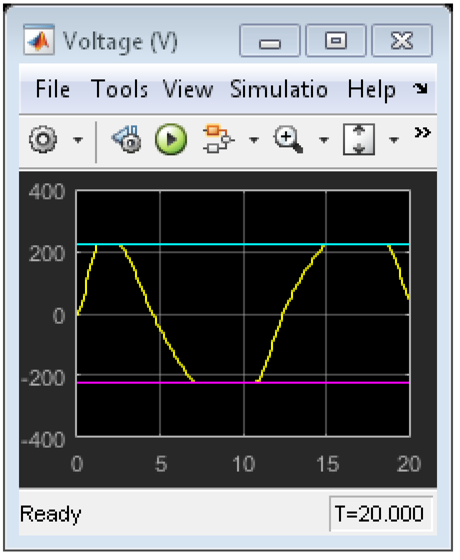
To run this example, Simulink® is required.

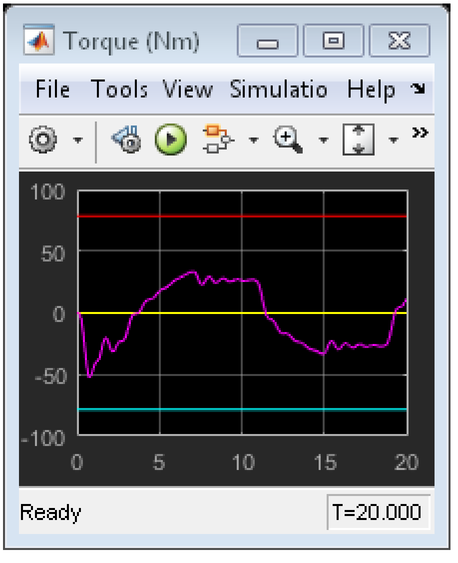
```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
```

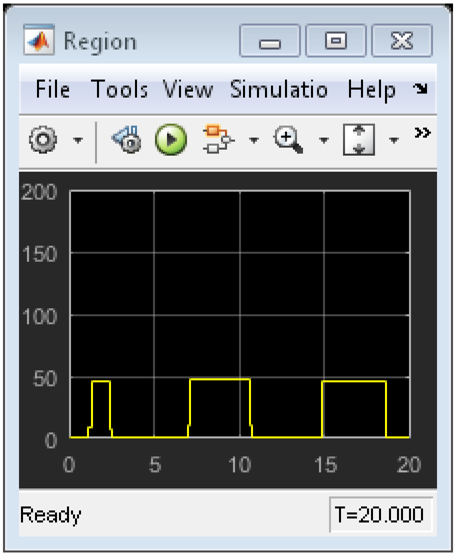
Simulate closed-loop control of the linear plant model in Simulink, using the Explicit MPC Controller block. Controller "mpcobjExplicit" is specified in the block dialog.

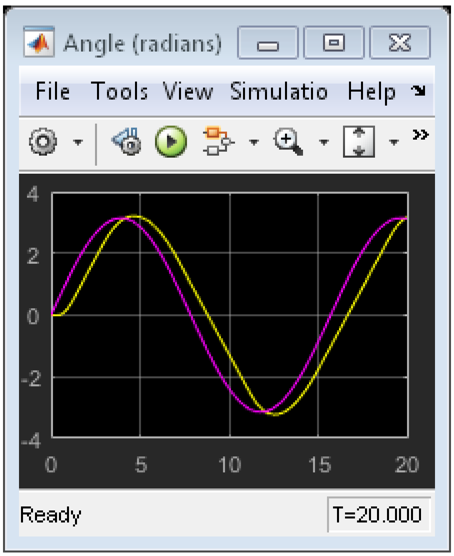
```
mdl = 'empc_motor';
open_system(mdl)
sim(mdl);
```


u(t)
y(t)
plant
Servomechanism Model
Angle (radians)
mv
mo
Explicit
MPC
region
ref
Explicit MPC Controller
Angle reference
r(t)
0
Torque reference
Region
Voltage (V)
-220
Vmin
220
Vmax
-tau
Tmin
tau
Tmax
Torque (Nm)
Copyright 1990-2014 The MathWorks, Inc.


Voltage (V)
File Tools View Simulatio Help
400
200
0
-200
-400
0
5
10
15
20
Ready
T=20.000


Torque (Nm)
File Tools View Simulatio Help
100
50
0
-50
-100
0
5
10
15
20
Ready
T=20.000





The closed-loop response is identical to the traditional MPC controller designed in the "mpcmotor" example.

Control Using Sub-optimal Explicit MPC

To reduce the memory footprint, you can use `simplify` command to reduce the number of piecewise affine solution regions. For example, you can remove regions whose Chebychev radius is smaller than .08. However, the price you pay is that the controler performance now becomes sub-optimal.

Use `simplify` command to generate Explicit MPC with sub-optimal solutions.

```
mpcobjExplicitSimplified = simplify(mpcobjExplicit, 'radius', 0.08);
disp(mpcobjExplicitSimplified);
```

```
Regions to analyze:       75/      75 --> 37 regions deleted.

  explicitMPC with properties:

                        MPC: [1x1 mpc]
                      Range: [1x1 struct]
        OptimizationOptions: [1x1 struct]
    PiecewiseAffineSolution: [1x38 struct]
               IsSimplified: 1
```

The number of piecewise affine regions has been reduced.

Compare closed-loop simulation between sub-optimal Explicit MPC and Explicit MPC.

```
[y3,t3,u3] = sim(mpcobjExplicitSimplified, Tf, r);
```

```
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

The simulation results are not the same.

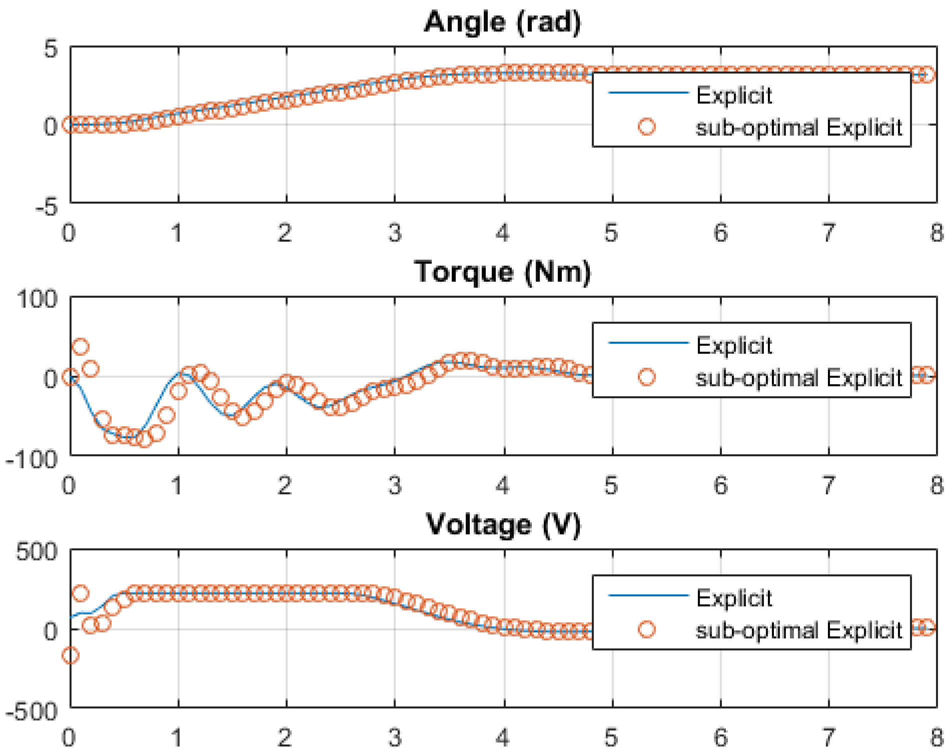
```
fprintf('SIM command: Difference between exact and suboptimal MPC trajectories = %g\n',
```

```
SIM command: Difference between exact and suboptimal MPC trajectories = 439.399
```

Plot results.

```
figure;
```

```
subplot(3,1,1)
plot(t1,y1(:,1),t3,y3(:,1),'o');
grid
title('Angle (rad)')
legend('Explicit','sub-optimal Explicit')
subplot(3,1,2)
plot(t1,y1(:,2),t3,y3(:,2),'o');
grid
title('Torque (Nm)')
legend('Explicit','sub-optimal Explicit')
subplot(3,1,3)
plot(t1,u1,t3,u3,'o');
grid
title('Voltage (V)')
legend('Explicit','sub-optimal Explicit')
```



The simulation result with the sub-optimal Explicit MPC is slightly worse.

```
bdclose(mdl)
```

Related Examples

- "Explicit MPC Control of a Single-Input-Single-Output Plant" on page 6-9
- "Explicit MPC Control of an Aircraft with Unstable Poles" on page 6-21

More About

- "Explicit MPC" on page 6-2

7

Gain Scheduling MPC Design

Gain-Scheduled MPC

The `Multiple MPC Controllers` block for Simulink allows you to switch between a defined set of MPC Controllers. You might need this feature if the plant operating characteristics change in a predictable way, and the change is such that a single prediction model cannot provide adequate accuracy. This approach is comparable to the use of gain scheduling in conventional feedback control.

The individual MPC controllers coordinate to make switching from one to another bumpless, avoiding a sudden change in the manipulated variables when the switch occurs.

You can perform command-line simulations using the `mpcmoveMultiple` command.

More About

- "Design Workflow for Gain Scheduling" on page 7-3
- "Relationship of Multiple MPC Controllers to MPC Controller Block" on page 3-3

Design Workflow for Gain Scheduling

In this section...

General Design Steps

- Define and tune a nominal MPC controller for the most likely (or average) operating conditions. (See "MPC Design".)
- Use simulations to determine an operating condition at which the nominal controller loses robustness. See "Simulation".
- Identify a measurement (or combination of measurements) signaling when the nominal controller should be replaced.
- Determine a plant prediction model to be used at the new condition. Its input and output variables must be the same as in the nominal case.
- Define a new MPC controller based on the new prediction model. Use the nominal controller settings as a starting point, and test and retune controller settings if necessary.
- If two controllers are inadequate to provide robustness over the full operational range, consider adding another. If it appears that you need more than three controllers to provide robustness over the full range, consider using adaptive MPC instead. See "Adaptive MPC Design".
- In your Simulink model, configure the `Multiple MPC Controllers` block. Specify the set of MPC controllers to be used, and specify the switching criterion.
- Test in closed-loop simulation over the full operating range to verify robustness and bumpless switching.

Tips

- Recommended MPC start-up practice is a warm-up period in which the plant operates under manual control while the controller initializes its state estimate. This typically requires 10-20 control intervals. A warm-up is especially important for the Multiple MPC Controllers block. Otherwise, switching between MPC controllers might upset the manipulated variables.

- If you select the Multiple MPC Controllers block's custom state estimation option, all MPC controllers in the set must have the same state dimension. This places implicit restrictions on plant and disturbance models.

See Also

`mpcmoveMultiple` | `Multiple MPC Controllers`

Related Examples

- "Schedule Controllers at Multiple Operating Points"
- "Coordinate Multiple Controllers at Different Operating Points" on page 4-64
- "Gain Scheduled MPC Control of Nonlinear Chemical Reactor" on page 7-5
- "Gain Scheduled MPC Control of Mass-Spring System" on page 7-28

More About

- "Relationship of Multiple MPC Controllers to MPC Controller Block" on page 3-3

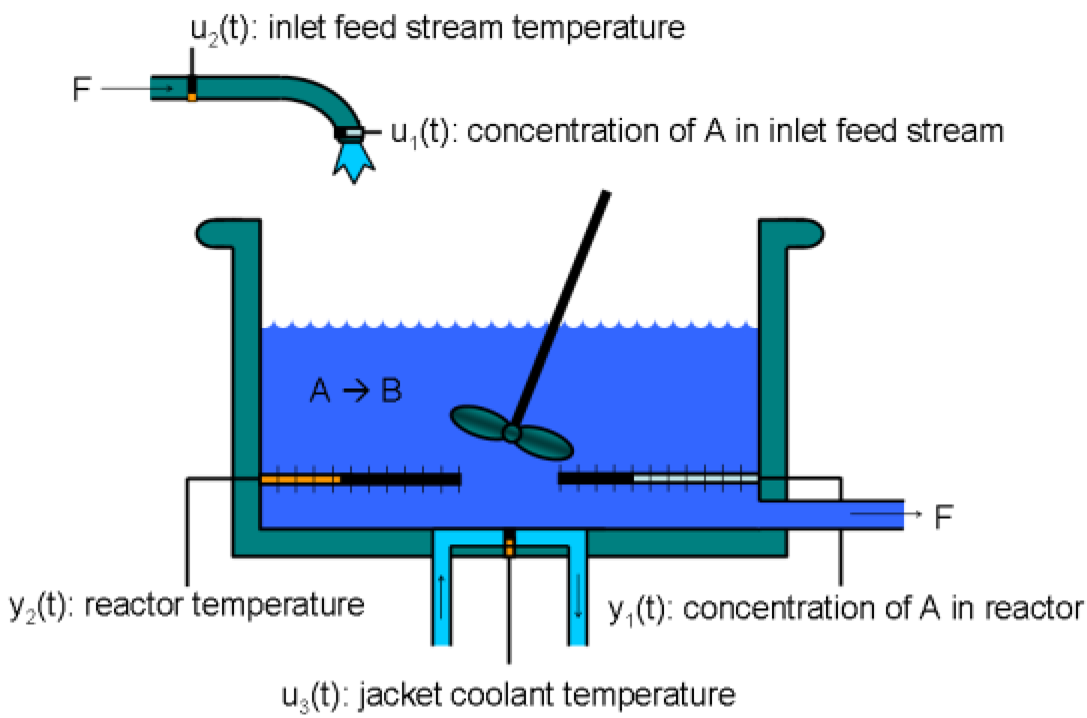
Gain Scheduled MPC Control of Nonlinear Chemical Reactor

This example shows how to use multiple MPC controllers to control a nonlinear continuous stirred tank reactor (CSTR) as it transitions from low conversion rate to high conversion rate.

Multiple MPC Controllers are designed at different operating conditions and then implemented with the Multiple MPC Controller block in Simulink. At run time, a scheduling signal is used to switch controller from one to another.

About the Continuous Stirred Tank Reactor

A Continuously Stirred Tank Reactor (CSTR) is a common chemical system in the process industry. A schematic of the CSTR system is:



This is a jacketed non-adiabatic tank reactor described extensively in Seborg's book, "Process Dynamics and Control", published by Wiley, 2004. The vessel is assumed to be perfectly mixed, and a single first-order exothermic and irreversible reaction, A --> B, takes place. The inlet stream of reagent A is fed to the tank at a constant volumetric rate. The product stream exits continuously at the same volumetric rate and liquid density is constant. Thus the volume of reacting liquid is constant.

The inputs of the CSTR model are:

$$\begin{array}{ll} u_1 = CA_i & \text{Concentration of A in inlet feed stream}[kgmol/m^3] \\ u_2 = T_i & \text{Inlet feed stream temperature}[K] \\ u_3 = T_c & \text{Jacket coolant temperature}[K] \end{array}$$

and the outputs (y(t)), which are also the states of the model (x(t)), are:

$$\begin{array}{ll} y_1 = x_1 = CA & \text{Concentration of A in reactor tank}[kgmol/m^3] \\ y_2 = x_2 = T & \text{Reactor temperature}[K] \end{array}$$

The control objective is to maintain the concentration of reagent A, CA at its desired setpoint, which changes over time when reactor transitions from low conversion rate to high conversion rate. The coolant temperature T_c is the manipulated variable used by the MPC controller to track the reference. The inlet feed stream concentration and temperature are assumed to be constant. The Simulink model `mpc_cstr_plant` implements the nonlinear CSTR plant.

About Gain Scheduled Model Predictive Control

It is well known that the CSTR dynamics are strongly nonlinear with respect to reactor temperature variations and can be open-loop unstable during the transition from one operating condition to another. A single MPC controller designed at a particular operating condition cannot give satisfactory control performance over a wide operating range.

To control the nonlinear CSTR plant with linear MPC control technique, you have a few options:

- If a linear plant model cannot be obtained at run time, first you need to obtain several linear plant models offline at different operating conditions that cover the typical operating range. Next you can choose one of the two approaches to implement MPC control strategy:

(1) Design several MPC controllers offline, one for each plant model. At run time, use Multiple MPC Controller block that switches MPC controllers from one to another based on a desired scheduling strategy, as discussed in this example. Use this approach when the plant models have different orders or time delays.

(2) Design one MPC controller offline at a nominal operating point. At run time, use Adaptive MPC Controller block (updating predictive model at each control interval) together with Linear Parameter Varying (LPV) System block (supplying linear plant model with a scheduling strategy). See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Linear Parameter Varying System" for more details. Use this approach when all the plant models have the same order and time delay.

- If a linear plant model can be obtained at run time, you should use Adaptive MPC Controller block to achieve nonlinear control. There are two typical ways to obtain a linear plant model online:

(1) Use successive linearization. See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Successive Linearization" for more details. Use this approach when a nonlinear plant model is available and can be linearized at run time.

(2) Use online estimation to identify a linear model when loop is closed. See "Adaptive MPC Control of Nonlinear Chemical Reactor Using Online Model Estimation" for more details. Use this approach when linear plant model cannot be obtained from either an LPV system or successive linearization.

Obtain Linear Plant Model at Initial Operating Condition

To run this example, Simulink® and Simulink Control Design® are required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('slcontrol')
    disp('Simulink Control Design(R) is required to run this example.')
    return
end
```

First, a linear plant model is obtained at the initial operating condition, CAi is 10 kgmol/m^3, Ti and Tc are 298.15 K. Functions from Simulink Control Design such as "operspec", "findop", "linearize", are used to generate the linear state space system from the Simulink model.

Create operating point specification.

```
plant_mdl = 'mpc_cstr_plant';
op = operspec(plant_mdl);
```

Feed concentration is known at the initial condition.

```
op.Inputs(1).u = 10;
op.Inputs(1).Known = true;
```

Feed temperature is known at the initial condition.

```
op.Inputs(2).u = 298.15;
op.Inputs(2).Known = true;
```

Coolant temperature is known at the initial condition.

```
op.Inputs(3).u = 298.15;
op.Inputs(3).Known = true;
```

Compute initial condition.

```
[op_point, op_report] = findop(plant_mdl,op);
% Obtain nominal values of x, y and u.
x0 = [op_report.States(1).x;op_report.States(2).x];
y0 = [op_report.Outputs(1).y;op_report.Outputs(2).y];
u0 = [op_report.Inputs(1).u;op_report.Inputs(2).u;op_report.Inputs(3).u];
```

```
 Operating Point Search Report:
---------------------------------

 Operating Report for the Model mpc_cstr_plant.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) mpc_cstr_plant/CSTR/Integrator
      x:           311      dx:      8.12e-11 (0)
(2.) mpc_cstr_plant/CSTR/Integrator1
      x:          8.57      dx:     -6.87e-12 (0)

Inputs:
```

```
----------
(1.) mpc_cstr_plant/CAi
      u:           10
(2.) mpc_cstr_plant/Ti
      u:          298
(3.) mpc_cstr_plant/Tc
      u:          298

Outputs:
----------
(1.) mpc_cstr_plant/T
      y:          311    [-Inf Inf]
(2.) mpc_cstr_plant/CA
      y:         8.57    [-Inf Inf]
```

Obtain linear model at the initial condition.

```
plant = linearize(plant_mdl, op_point);
```

Verify that the linear model is open-loop stable at this condition.

```
eig(plant)
```

```
ans =

   -0.5223
   -0.8952
```

Design MPC Controller for Initial Operating Condition

You design an MPC at the initial operating condition.

```
Ts = 0.5;
```

Specify signal types used in MPC. Assume both reactor temperature and concentration are measurable.

```
plant.InputGroup.UnmeasuredDisturbances = [1 2];
plant.InputGroup.ManipulatedVariables = 3;
plant.OutputGroup.Measured = [1 2];
plant.InputName = {'CAi','Ti','Tc'};
```

```
plant.OutputName = {'T','CA'};
```

Create MPC controller with default prediction and control horizons

```
mpcobj = mpc(plant, Ts);
```

```
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
   for output(s) y1 and zero weight for output(s) y2
```

Set nominal values in the controller. Note that nominal values for unmeasured disturbance must be zero.

```
mpcobj.Model.Nominal = struct('X', x0, 'U', [0;0;u0(3)], 'Y', y0, 'DX', [0 0]);
```

Set scale factors because plant input and output signals have different orders of magnitude

```
Uscale = [10;30;50];
Yscale = [50;10];
mpcobj.DV(1).ScaleFactor = Uscale(1);
mpcobj.DV(2).ScaleFactor = Uscale(2);
mpcobj.MV.ScaleFactor = Uscale(3);
mpcobj.OV(1).ScaleFactor = Yscale(1);
mpcobj.OV(2).ScaleFactor = Yscale(2);
```

The goal will be to track a specified transition in the reactor concentration. The reactor temperature will be measured and used in state estimation but the controller will not attempt to regulate it directly. It will vary as needed to regulate the concentration. Thus, set its MPC weight to zero.

```
mpcobj.Weights.OV = [0 1];
```

Plant inputs 1 and 2 are unmeasured disturbances. By default, the controller assumes integrated white noise with unit magnitude at these inputs when configuring the state estimator. Try increasing the state estimator signal-to-noise by a factor of 10 to improve disturbance rejection performance.

```
D = ss(getindist(mpcobj));
D.b = eye(2)*10;
```

```
setindist(mpcobj, 'model', D);
```

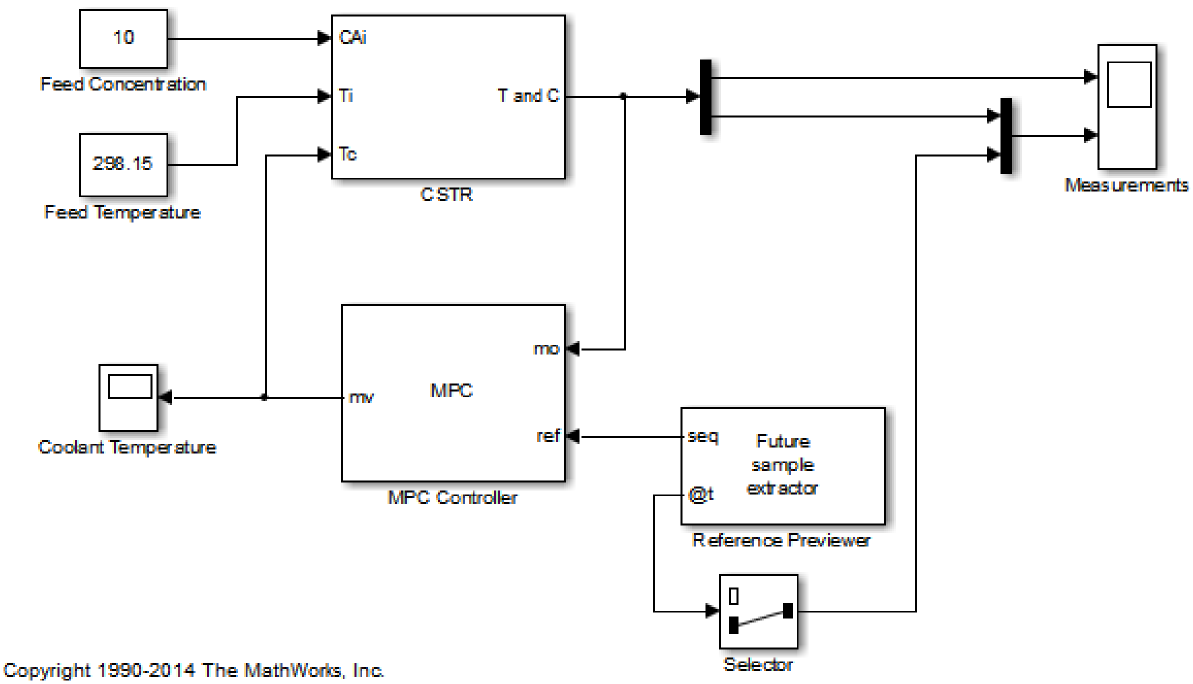
```
-->Converting model to discrete time.
-->The "Model.Disturbance" property of "mpc" object is empty:
   Assuming unmeasured input disturbance #1 is integrated white noise.
   Assuming unmeasured input disturbance #2 is integrated white noise.
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

All other MPC parameters are at their default values.

Test the Controller With a Step Disturbance in Feed Concentration

"mpc_cstr_single" contains a Simulink® model with CSTR and MPC Controller blocks in a feedback configuration.

```
mpc_mdl = 'mpc_cstr_single';
open_system(mpc_mdl)
```



Note that the MPC Controller block is configured to look ahead (preview) the setpoint changes in the future, i.e., anticipating the setpoint transition. This generally improves setpoint tracking.

Define a constant setpoint for the output.

```
CSTR_Setpoints.time = [0; 60];
CSTR_Setpoints.signals.values = [y0 y0]';
```

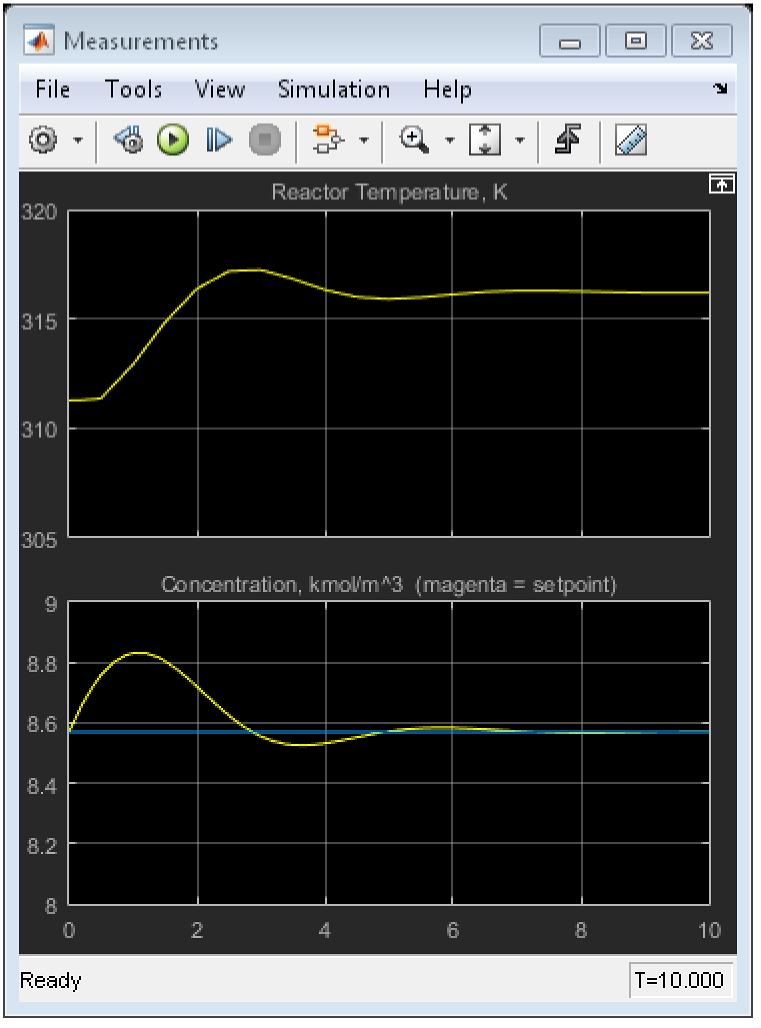
Test the response to a 5% increase in feed concentration.

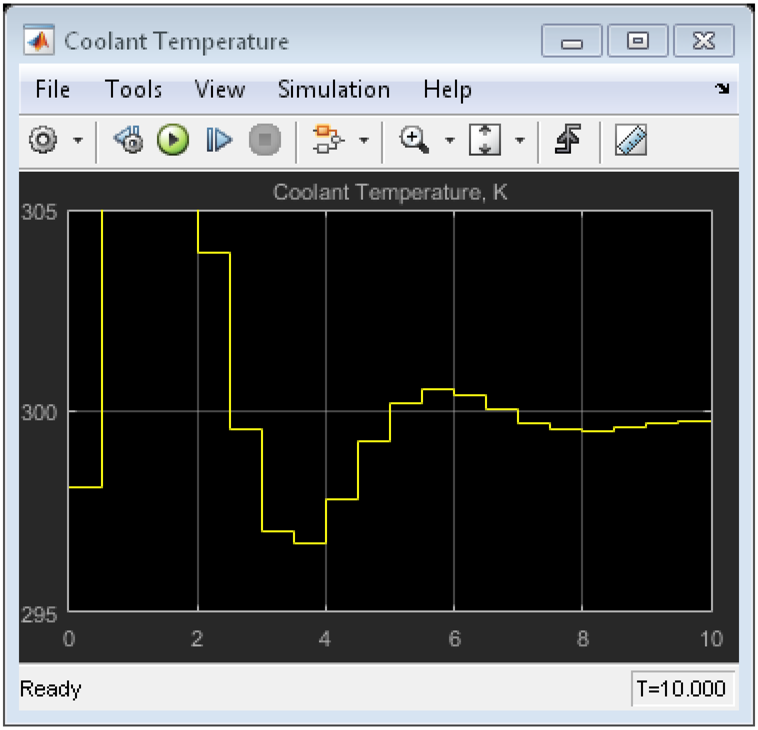
```
set_param([mpc_mdl '/Feed Concentration'], 'Value', '10.5');
```

Set plot scales and simulate the response.

```
open_system([mpc_mdl '/Measurements'])
open_system([mpc_mdl '/Coolant Temperature'])
set_param([mpc_mdl '/Measurements'], 'Ymin', '305~8', 'Ymax', '320~9')
set_param([mpc_mdl '/Coolant Temperature'], 'Ymin', '295', 'Ymax', '305')
sim(mpc_mdl, 10);
```

```
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```


Measurements
File
Tools
View
Simulation
Help
Reactor Temperature, K
320
315
310
305
Concentration, kmol/m^3 (magenta = setpoint)
9
8.8
8.6
8.4
8.2
8
0
2
4
6
8
10
Ready
T=10.000



The closed-loop response is satisfactory.

Simulate Designed MPC Controller Using Full Transition

First, define the desired setpoint transition. After a 10-minute warm-up period, ramp the concentration setpoint downward at a rate of 0.25 per minute until it reaches 2.0 kmol/m^3.

```
CSTR_Setpoints.time = [0 10 11:39]';
CSTR_Setpoints.signals.values = [y0(1)*ones(31,1),[y0(2);y0(2);(y0(2):-0.25:2)';2;2]];
```

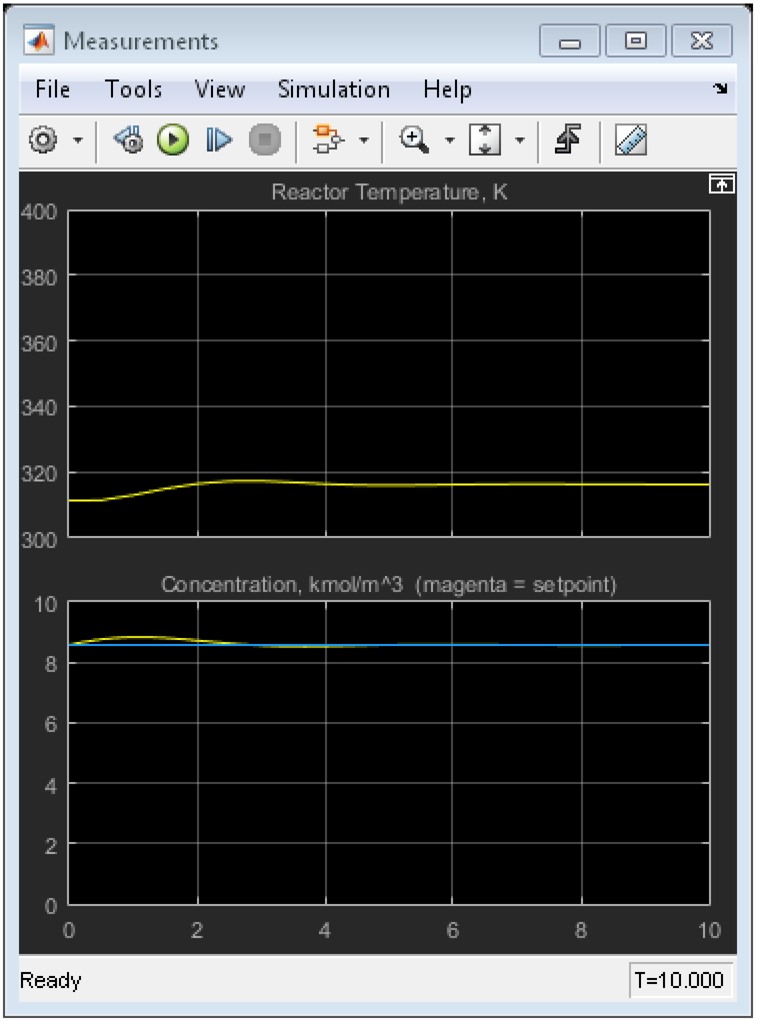
Remove the 5% increase in feed concentration used previously.

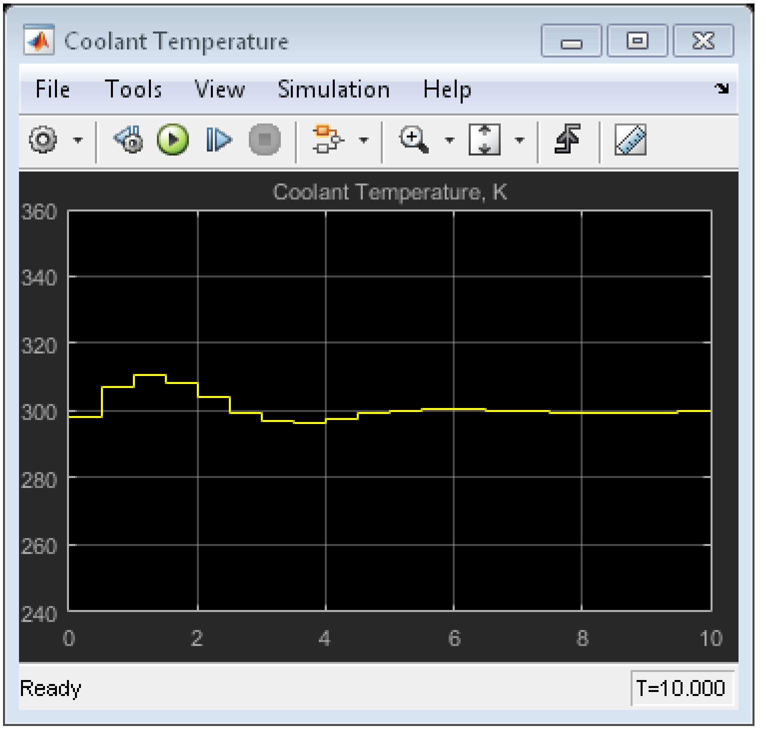
```
set_param([mpc_mdl '/Feed Concentration'], 'Value', '10')
```

Set plot scales and simulate the response.

```
set_param([mpc_mdl '/Measurements'], 'Ymin', '300~0', 'Ymax', '400~10')
```

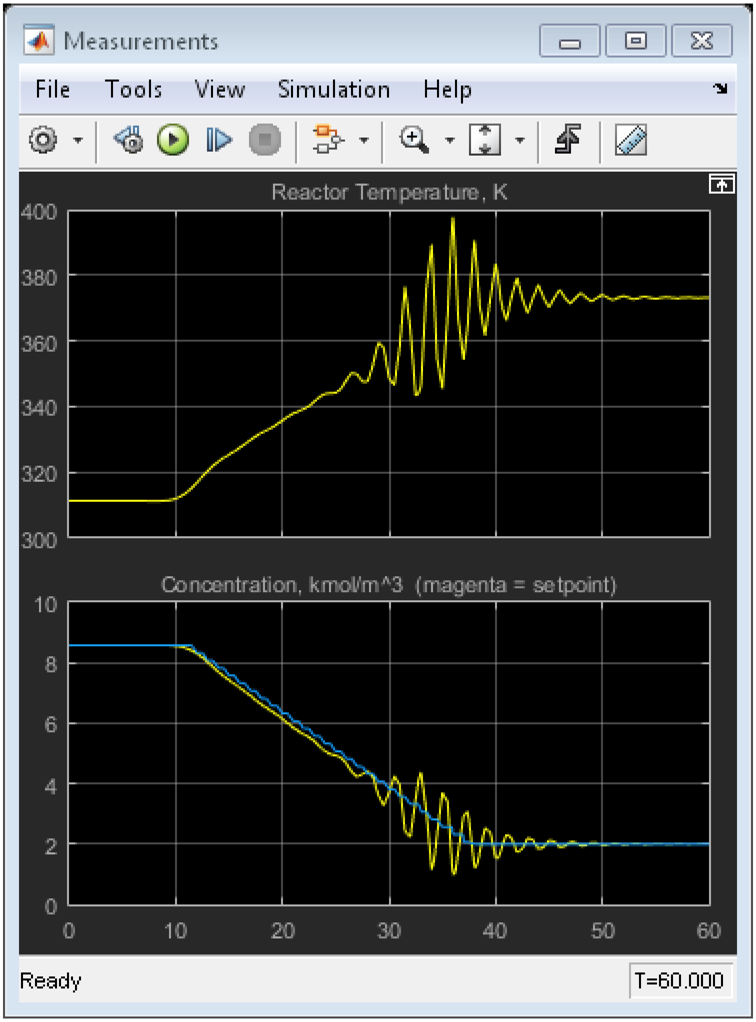
```
set_param([mpc_mdl '/Coolant Temperature'], 'Ymin', '240', 'Ymax', '360')
```

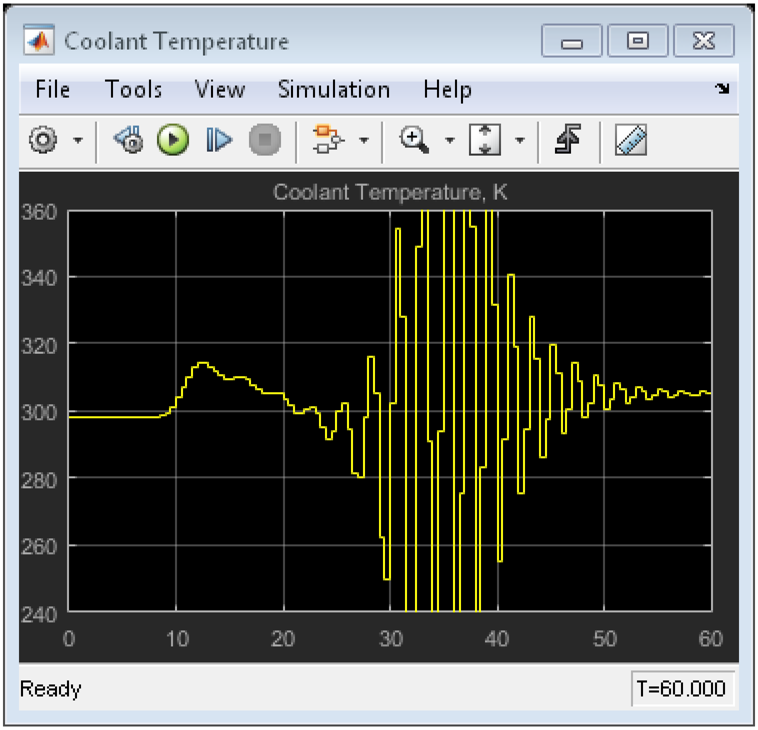




Simulate model.

```
sim(mpc_mdl, 60)
```


Measurements
File Tools View Simulation Help
Reactor Temperature, K
400
380
360
340
320
300
Concentration, kmol/m^3 (magenta = setpoint)
10
8
6
4
2
0
0 10 20 30 40 50 60
Ready
T=60.000



The closed-loop response is unacceptable. Performance along the full transition can be improved if other MPC controllers are designed at different operating conditions along the transition path. In the next two section, two additional MPC controllers are design at intermediate and final transition stages respectively.

Design MPC Controller for Intermediate Operating Condition

Create operating point specification.

```
op = operspec(plant_mdl);
```

Feed concentration is known.

```
op.Inputs(1).u = 10;
op.Inputs(1).Known = true;
```

Feed temperature is known.

```
op.Inputs(2).u = 298.15;
```

```
op.Inputs(2).Known = true;
```

Reactor concentration is known

```
op.Outputs(2).y = 5.5;
op.Outputs(2).Known = true;
```

Find steady state operating condition.

```
[op_point, op_report] = findop(plant_mdl,op);
% Obtain nominal values of x, y and u.
x0 = [op_report.States(1).x;op_report.States(2).x];
y0 = [op_report.Outputs(1).y;op_report.Outputs(2).y];
u0 = [op_report.Inputs(1).u;op_report.Inputs(2).u;op_report.Inputs(3).u];
```

```
 Operating Point Search Report:
---------------------------------

 Operating Report for the Model mpc_cstr_plant.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) mpc_cstr_plant/CSTR/Integrator
      x:           339      dx:      3.42e-08 (0)
(2.) mpc_cstr_plant/CSTR/Integrator1
      x:           5.5      dx:     -2.87e-09 (0)

Inputs:
----------
(1.) mpc_cstr_plant/CAi
      u:            10
(2.) mpc_cstr_plant/Ti
      u:           298
(3.) mpc_cstr_plant/Tc
      u:           298    [-Inf Inf]

Outputs:
----------
(1.) mpc_cstr_plant/T
      y:           339    [-Inf Inf]
(2.) mpc_cstr_plant/CA
      y:           5.5    (5.5)
```

Obtain linear model at the initial condition.

```
plant_intermediate = linearize(plant_mdl, op_point);
```

Verify that the linear model is open-loop unstable at this condition.

```
eig(plant_intermediate)
```

```
ans =

    0.4941
   -0.8357
```

Specify signal types used in MPC. Assume both reactor temperature and concentration are measurable.

```
plant_intermediate.InputGroup.UnmeasuredDisturbances = [1 2];
plant_intermediate.InputGroup.ManipulatedVariables = 3;
plant_intermediate.OutputGroup.Measured = [1 2];
plant_intermediate.InputName = {'CAi','Ti','Tc'};
plant_intermediate.OutputName = {'T','CA'};
```

Create MPC controller with default prediction and control horizons

```
mpcobj_intermediate = mpc(plant_intermediate, Ts);
```

```
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
   for output(s) y1 and zero weight for output(s) y2
```

Set nominal values, scale factors and weights in the controller

```
mpcobj_intermediate.Model.Nominal = struct('X', x0, 'U', [0;0;u0(3)], 'Y', y0, 'DX', [0
Uscale = [10;30;50];
Yscale = [50;10];
mpcobj_intermediate.DV(1).ScaleFactor = Uscale(1);
mpcobj_intermediate.DV(2).ScaleFactor = Uscale(2);
mpcobj_intermediate.MV.ScaleFactor = Uscale(3);
mpcobj_intermediate.OV(1).ScaleFactor = Yscale(1);
```

```
mpcobj_intermediate.OV(2).ScaleFactor = Yscale(2);
mpcobj_intermediate.Weights.OV = [0 1];
D = ss(getindist(mpcobj_intermediate));
D.b = eye(2)*10;
setindist(mpcobj_intermediate, 'model', D);
```

```
-->Converting model to discrete time.
-->The "Model.Disturbance" property of "mpc" object is empty:
   Assuming unmeasured input disturbance #1 is integrated white noise.
   Assuming unmeasured input disturbance #2 is integrated white noise.
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Design MPC Controller for Final Operating Condition

Create operating point specification.

```
op = operspec(plant_mdl);
```

Feed concentration is known.

```
op.Inputs(1).u = 10;
op.Inputs(1).Known = true;
```

Feed temperature is known.

```
op.Inputs(2).u = 298.15;
op.Inputs(2).Known = true;
```

Reactor concentration is known

```
op.Outputs(2).y = 2;
op.Outputs(2).Known = true;
```

Find steady state operating condition.

```
[op_point, op_report] = findop(plant_mdl,op);
% Obtain nominal values of x, y and u.
x0 = [op_report.States(1).x;op_report.States(2).x];
y0 = [op_report.Outputs(1).y;op_report.Outputs(2).y];
u0 = [op_report.Inputs(1).u;op_report.Inputs(2).u;op_report.Inputs(3).u];
```

```
 Operating Point Search Report:
---------------------------------
```

```
 Operating Report for the Model mpc_cstr_plant.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) mpc_cstr_plant/CSTR/Integrator
      x:          373      dx:      5.57e-11 (0)
(2.) mpc_cstr_plant/CSTR/Integrator1
      x:            2      dx:      -4.6e-12 (0)

Inputs:
----------
(1.) mpc_cstr_plant/CAi
      u:           10
(2.) mpc_cstr_plant/Ti
      u:          298
(3.) mpc_cstr_plant/Tc
      u:          305    [-Inf Inf]

Outputs:
----------
(1.) mpc_cstr_plant/T
      y:          373    [-Inf Inf]
(2.) mpc_cstr_plant/CA
      y:            2    (2)
```

Obtain linear model at the initial condition.

```
plant_final = linearize(plant_mdl, op_point);
```

Verify that the linear model is again open-loop stable at this condition.

```
eig(plant_final)
```

```
ans =

  -1.1077 + 1.0901i
  -1.1077 - 1.0901i
```

Specify signal types used in MPC. Assume both reactor temperature and concentration are measurable.

```
plant_final.InputGroup.UnmeasuredDisturbances = [1 2];
plant_final.InputGroup.ManipulatedVariables = 3;
plant_final.OutputGroup.Measured = [1 2];
plant_final.InputName = {'CAi','Ti','Tc'};
plant_final.OutputName = {'T','CA'};
```

Create MPC controller with default prediction and control horizons

```
mpcobj_final = mpc(plant_final, Ts);
```

```
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
   for output(s) y1 and zero weight for output(s) y2
```

Set nominal values, scale factors and weights in the controller

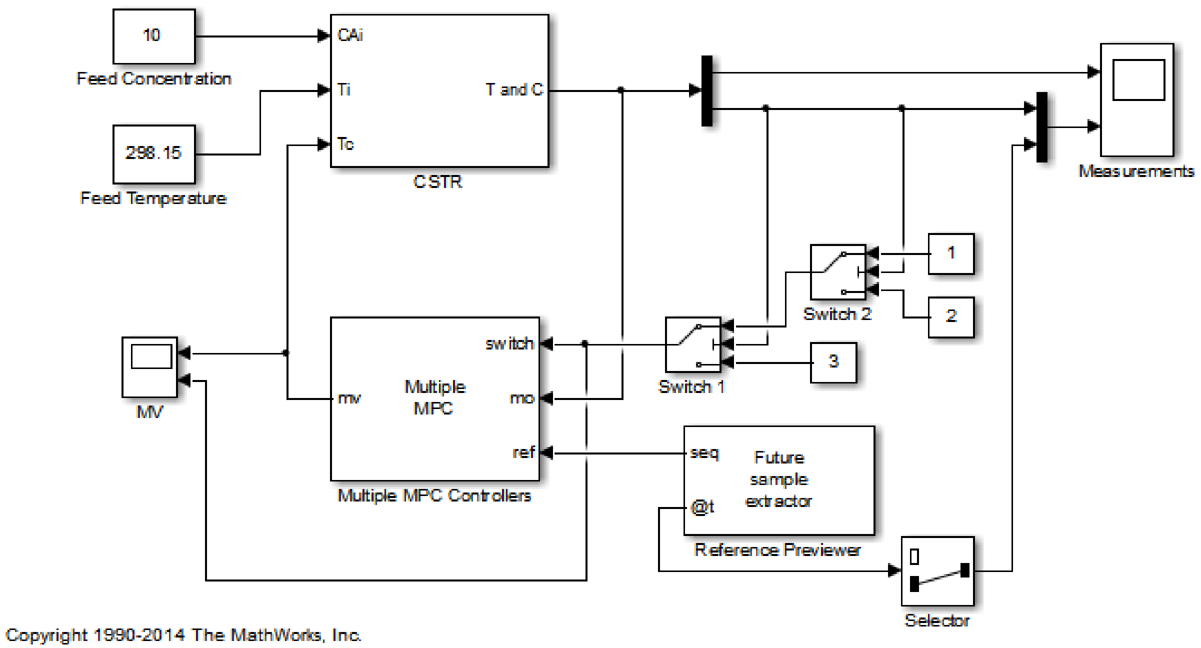
```
mpcobj_final.Model.Nominal = struct('X', x0, 'U', [0;0;u0(3)], 'Y', y0, 'DX', [0 0]);
Uscale = [10;30;50];
Yscale = [50;10];
mpcobj_final.DV(1).ScaleFactor = Uscale(1);
mpcobj_final.DV(2).ScaleFactor = Uscale(2);
mpcobj_final.MV.ScaleFactor = Uscale(3);
mpcobj_final.OV(1).ScaleFactor = Yscale(1);
mpcobj_final.OV(2).ScaleFactor = Yscale(2);
mpcobj_final.Weights.OV = [0 1];
D = ss(getindist(mpcobj_final));
D.b = eye(2)*10;
setindist(mpcobj_final, 'model', D);
```

```
-->Converting model to discrete time.
-->The "Model.Disturbance" property of "mpc" object is empty:
   Assuming unmeasured input disturbance #1 is integrated white noise.
   Assuming unmeasured input disturbance #2 is integrated white noise.
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```

Control the CSTR Plant With the Multiple MPC Controllers Block

The following model uses the Multiple MPC Controllers block to implement three MPC controllers across the operating range.

```
mmpc_mdl = 'mpc_cstr_multiple';
open_system(mmpc_mdl);
```



Note that it has been configured to use the three controllers in a sequence: mpcobj, mpcobj_intermediate and mpcobj_final.

```
open_system([mmpc_mdl '/Multiple MPC Controllers']);
```

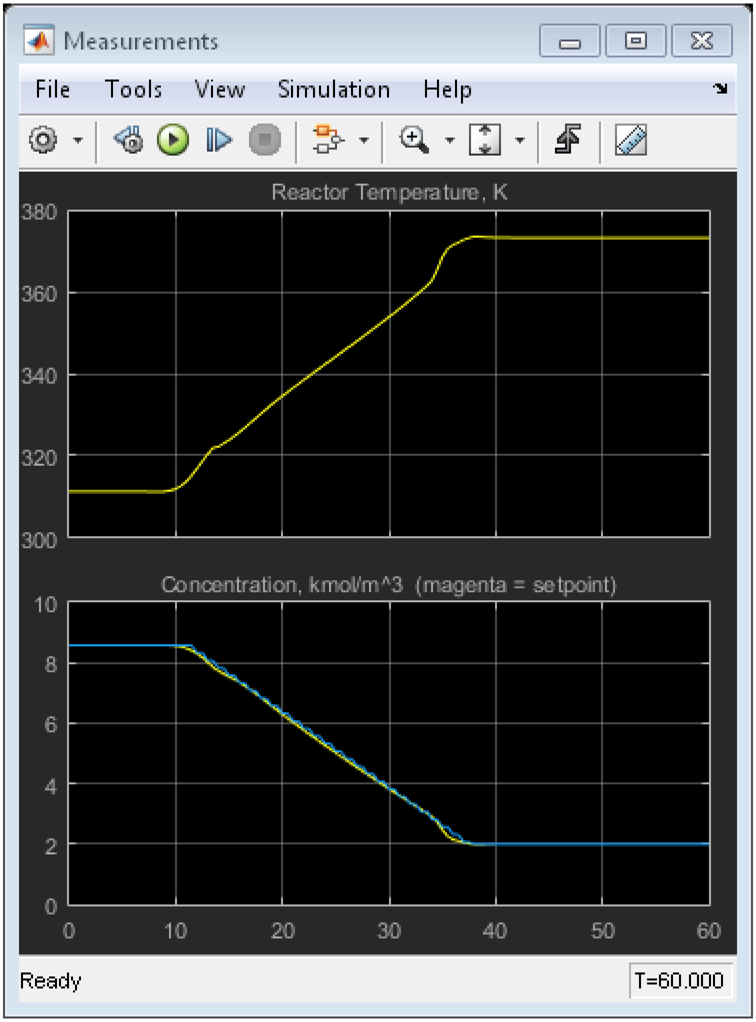
Note also that the two switches specify when to switch from one controller to another. The rules are: 1. If CSTR concentration >= 8, use "mpcobj" 2. If 3 <= CSTR concentration < 8, use "mpcobj_intermediate" 3. If CSTR concentration < 3, use "mpcobj_final"

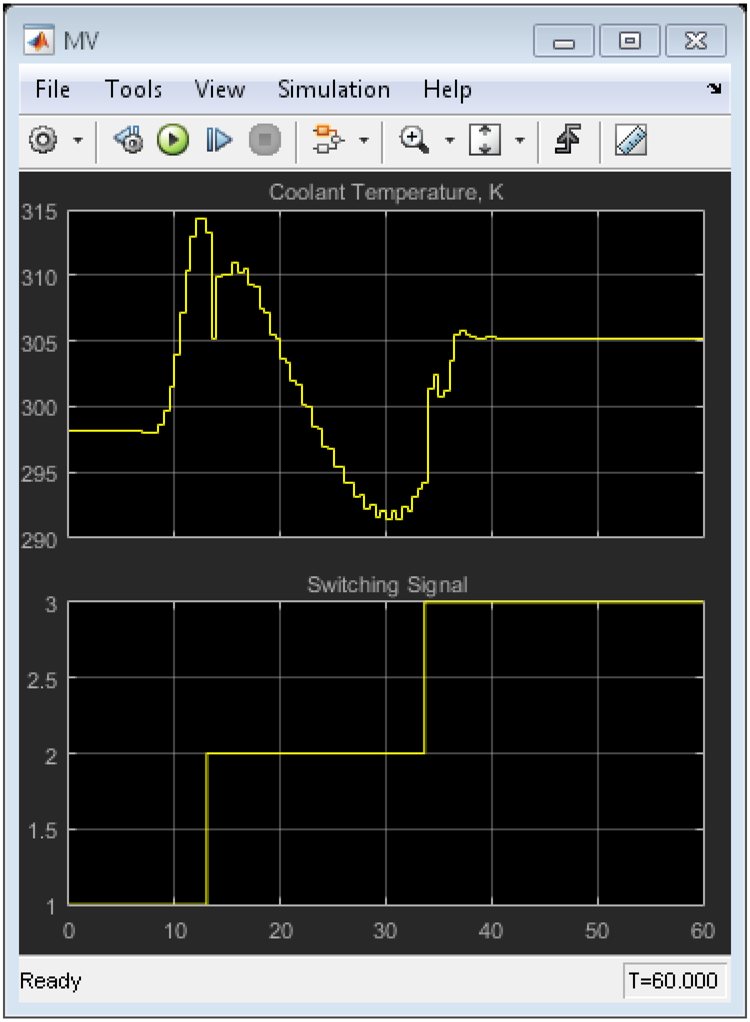
Simulate with the Multiple MPC Controllers block

```
open_system([mmpc_mdl '/Measurements']);
open_system([mmpc_mdl '/MV']);
sim(mmpc_mdl)
```

```
-->Converting model to discrete time.
```

```
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #2.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```





The transition is now well controlled. The major improvement is in the transition through the open-loop unstable region. The plot of the switching signal shows when controller transitions occur. The MV character changes at these times because of the change in dynamic characteristics introduced by the new prediction model.

```
bdclose(plant_mdl)
bdclose(mpc_mdl)
```

```
bdclose(mmpc_mdl)
```

Related Examples

- “Schedule Controllers at Multiple Operating Points”
- “Coordinate Multiple Controllers at Different Operating Points” on page 4-64
- “Gain Scheduled MPC Control of Mass-Spring System” on page 7-28

More About

- “Design Workflow for Gain Scheduling” on page 7-3

Gain Scheduled MPC Control of Mass-Spring System

This example shows how to use an Multiple MPC Controllers block to implement gain scheduled MPC control of a nonlinear plant.

System Description

The system is composed by two masses M1 and M2 connected to two springs k1 and k2 respectively. The collision is assumed completely inelastic. Mass M1 is pulled by a force F, which is the manipulated variable. The objective is to make mass M1's position y1 track a given reference r.

The dynamics are twofold: when the masses are detached, M1 moves freely. Otherwise, M1+M2 move together. We assume that only M1 position and a contact sensor are available for feedback. The latter is used to trigger switching the MPC controllers. Note that position and velocity of mass M2 are not controllable.

```
                         /-----\     k1                 ||
                  F <--- | M1  |----/\/\/\-------------[|| wall
    ||                   | |---/                        ||
    ||     k2            \-/      /----\                ||
wall||]--/\/\/\-------------------| M2 |                ||
    ||                            \----/                ||
    ||                                                  ||
----yeq2------------------ y1 ------ y2 ----------------yeq1----> y axis
```

The model is a simplified version of the model proposed in the following reference:

A. Bemporad, S. Di Cairano, I. V. Kolmanovsky, and D. Hrovat, "Hybrid modeling and control of a multibody magnetic actuator for automotive applications," in Proc. 46th IEEE® Conf. on Decision and Control, New Orleans, LA, 2007.

Model Parameters

```
M1=1;       % mass
M2=5;       % mass
k1=1;       % spring constant
k2=0.1;     % spring constant
b1=0.3;     % friction coefficient
b2=0.8;     % friction coefficient
yeq1=10;    % wall mount position
yeq2=-10;   % wall mount position
```

State Space Models

states: position and velocity of mass M1; manipulated variable: pull force F measured disturbance: a constant value of 1 which provides calibrates spring force to the right value measured output: position of mass M1

State-space model of M1 when masses are not in contact.

```
A1=[0 1;-k1/M1 -b1/M1];
B1=[0 0;-1/M1 k1*yeq1/M1];
C1=[1 0];
D1=[0 0];
sys1=ss(A1,B1,C1,D1);
sys1=setmpcsignals(sys1,'MD',2);
```

```
-->Assuming unspecified input signals are manipulated variables.
```

State-space model when the two masses are in contact.

```
A2=[0 1;-(k1+k2)/(M1+M2) -(b1+b2)/(M1+M2)];
B2=[0 0;-1/(M1+M2) (k1*yeq1+k2*yeq2)/(M1+M2)];
C2=[1 0];
D2=[0 0];
sys2=ss(A2,B2,C2,D2);
sys2=setmpcsignals(sys2,'MD',2);
```

```
-->Assuming unspecified input signals are manipulated variables.
```

Design MPC Controllers

Common parameters

```
Ts=0.2;      % sampling time
p=20;        % prediction horizon
m=1;         % control horizon
```

Define MPC object for mass M1 detached from M2.

```
MPC1=mpc(sys1,Ts,p,m);
MPC1.Weights.OV=1;
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

Define constraints on the manipulated variable.

```
MPC1.MV=struct('Min',0,'Max',Inf,'RateMin',-1e3,'RateMax',1e3);
```

Define MPC object for mass M1 and M2 stuck together.

```
MPC2=mpc(sys2,Ts,p,m);
MPC2.Weights.OV=1;
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming defau
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming d
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1
```

Define constraints on the manipulated variable.

```
MPC2.MV=MPC1.MV;
```

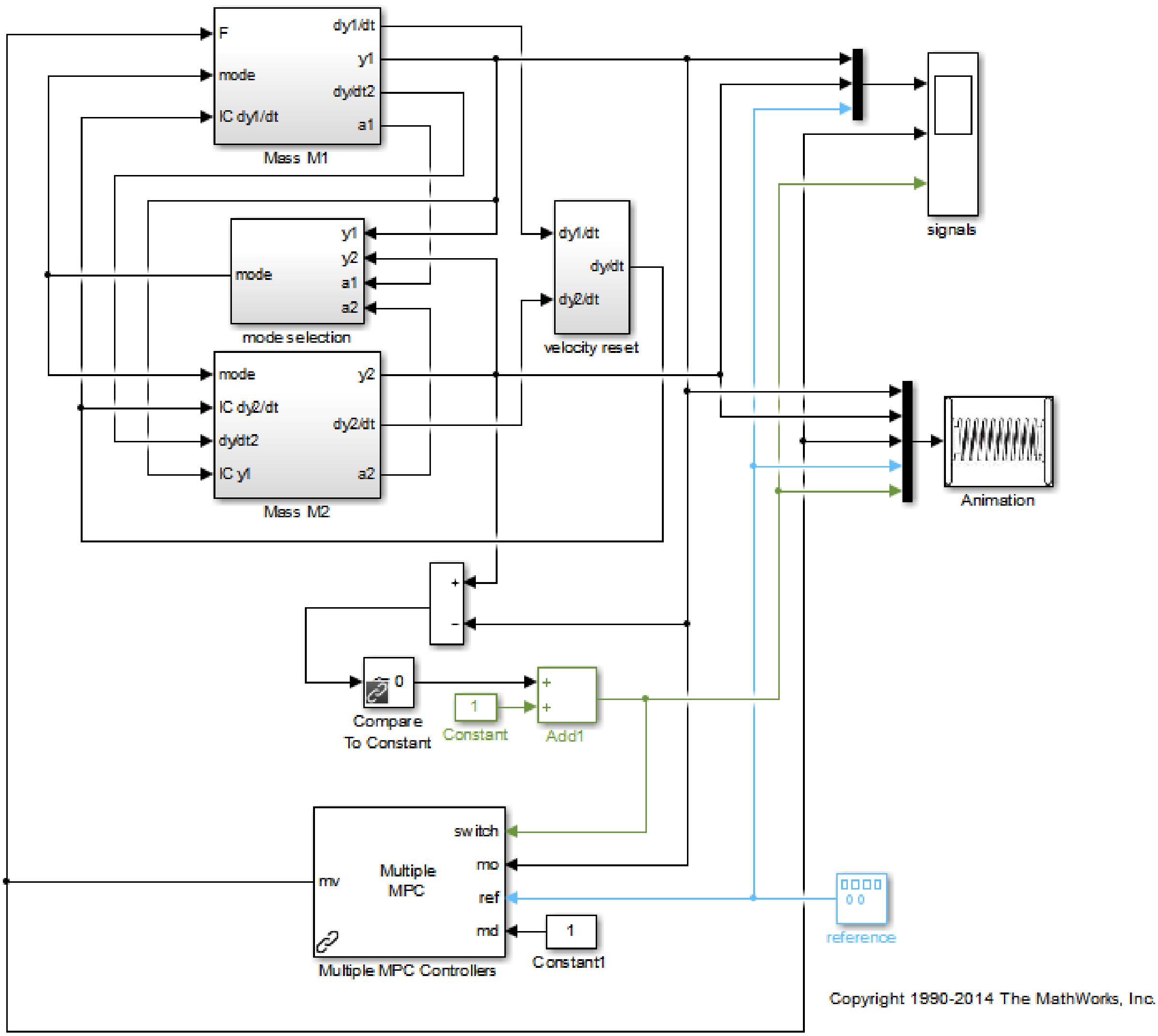
Simulate Gain Scheduled MPC in Simulink®

To run this example, Simulink® is required.

```
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
mdl = 'mpc_switching';
```

Simulate gain scheduled MPC control with Multiple MPC Controllers block.

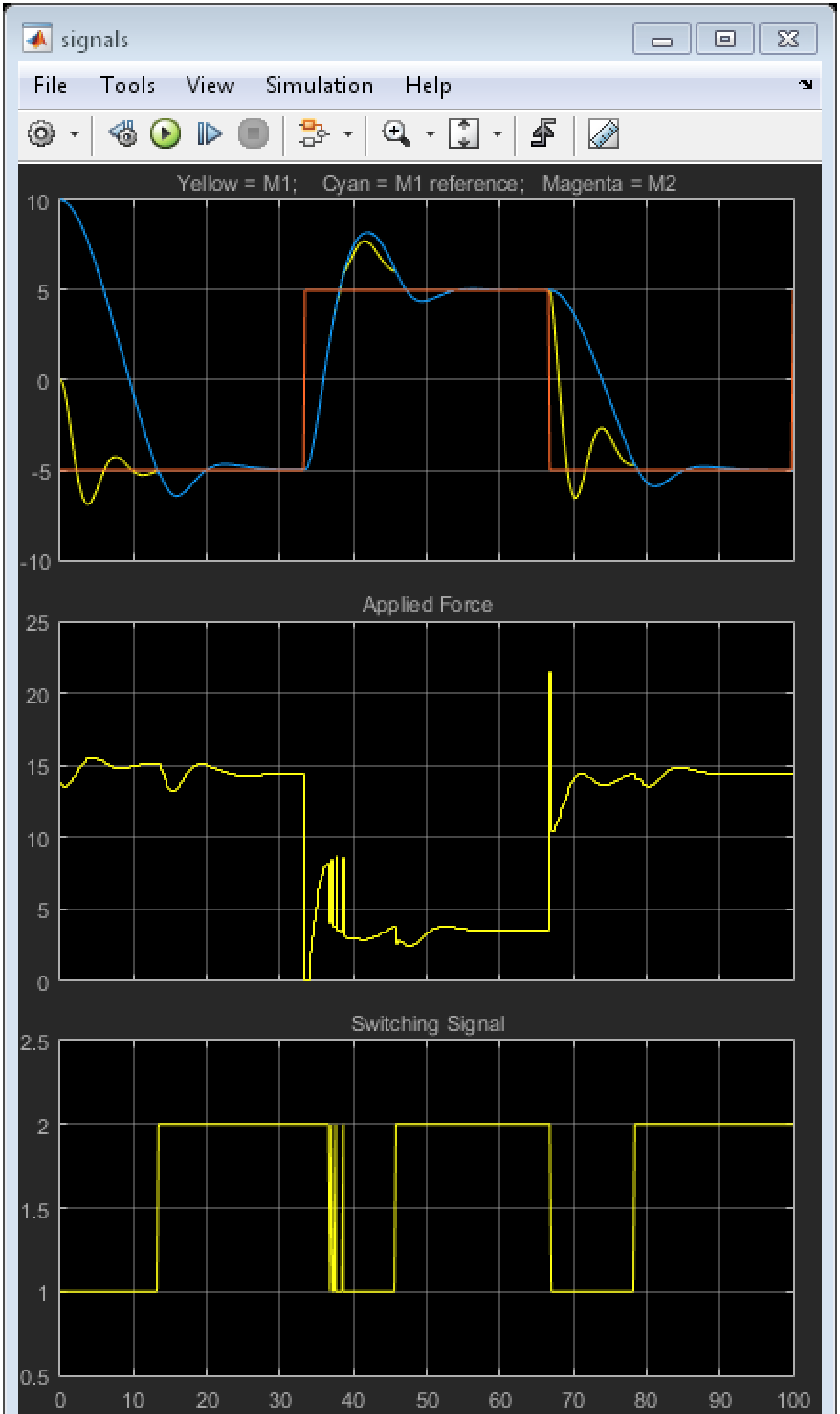
```
y1initial=0;    % Initial positions
y2initial=10;
open_system(mdl);
if exist('animationmpc_switchoff','var') && animationmpc_switchoff
    close_system([mdl '/Animation']);
    clear animationmpc_switchoff
end
```

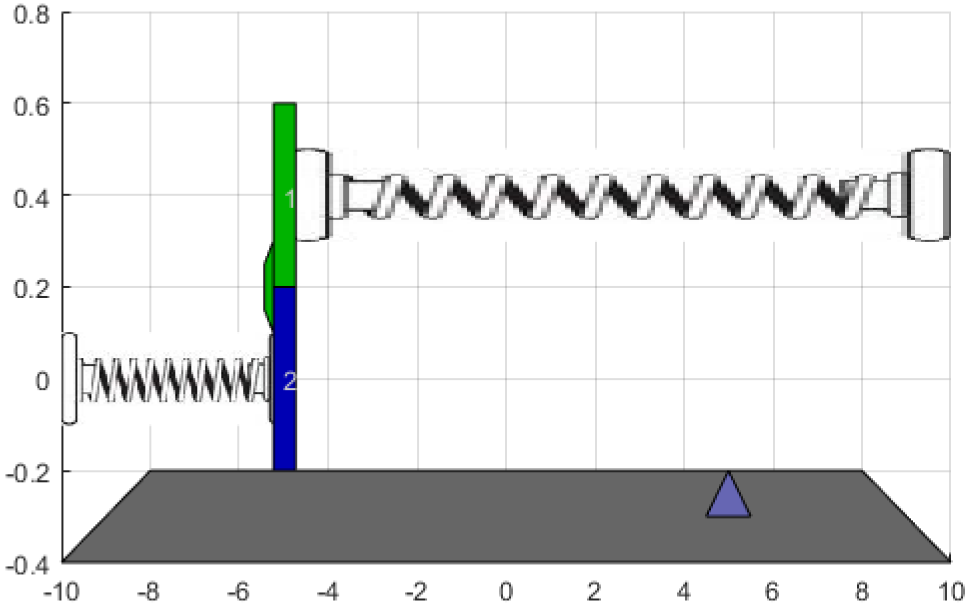



```
disp('Start simulation by switching control between MPC1 and MPC2 ...');
disp('Control performance is satisfactory.');
open_system([mdl '/signals']);
sim(mdl);
```

```
Start simulation by switching control between MPC1 and MPC2 ...
```

```
Control performance is satisfactory.
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on eac
```


signals
File Tools View Simulation Help
Yellow = M1; Cyan = M1 reference; Magenta = M2
Applied Force
Switching Signal

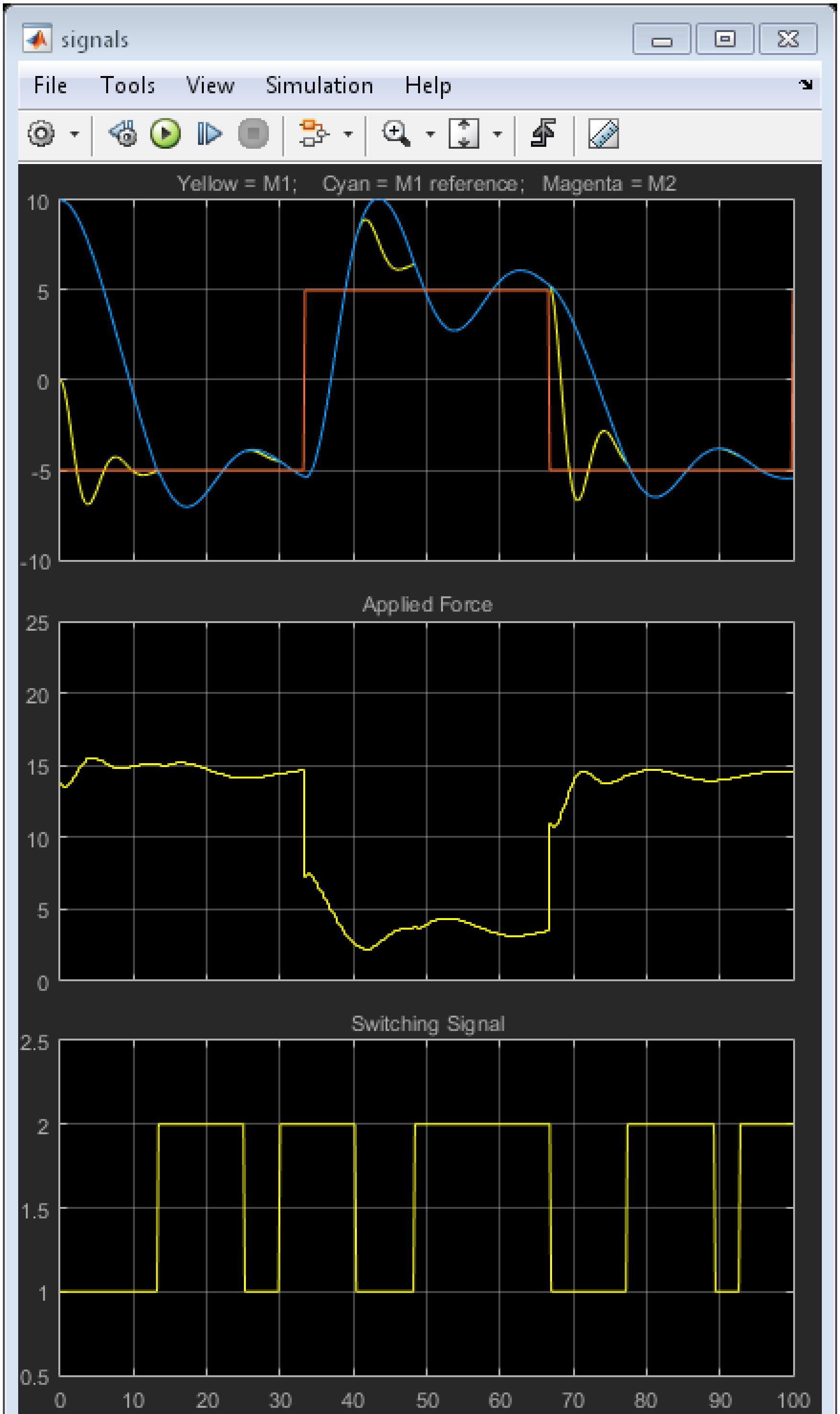


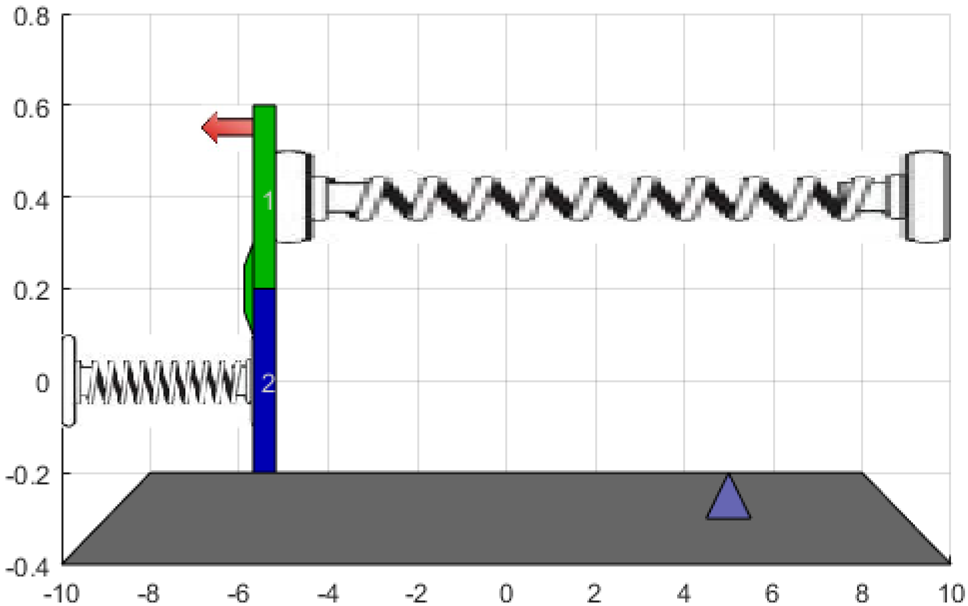
Use of two controllers provides good performance under all conditions.

Repeat Simulation Using MPC1 Only (Assumes Masses Never in Contact)

```
disp('Now repeat simulation by using only MPC1 ...');
disp('When two masses stick together, control performance deteriorates.');
MPC2save=MPC2;
MPC2=MPC1; %#ok<*NASGU>
sim(mdl);
```

```
Now repeat simulation by using only MPC1 ...
When two masses stick together, control performance deteriorates.
```


signals
File Tools View Simulation Help
Yellow = M1; Cyan = M1 reference; Magenta = M2
10
5
0
-5
-10
Applied Force
25
20
15
10
5
0
Switching Signal
2.5
2
1.5
1
0.5
0 10 20 30 40 50 60 70 80 90 100

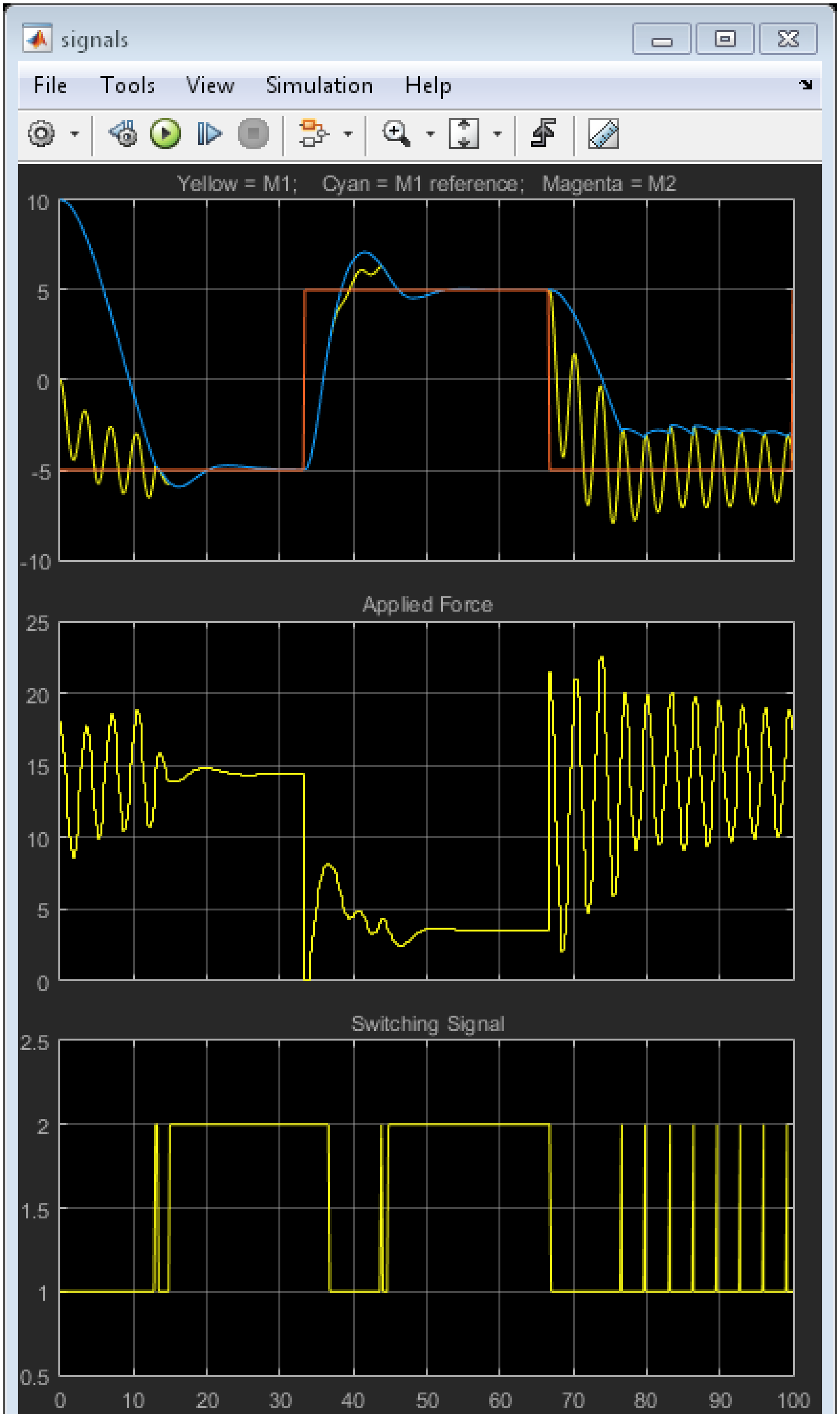


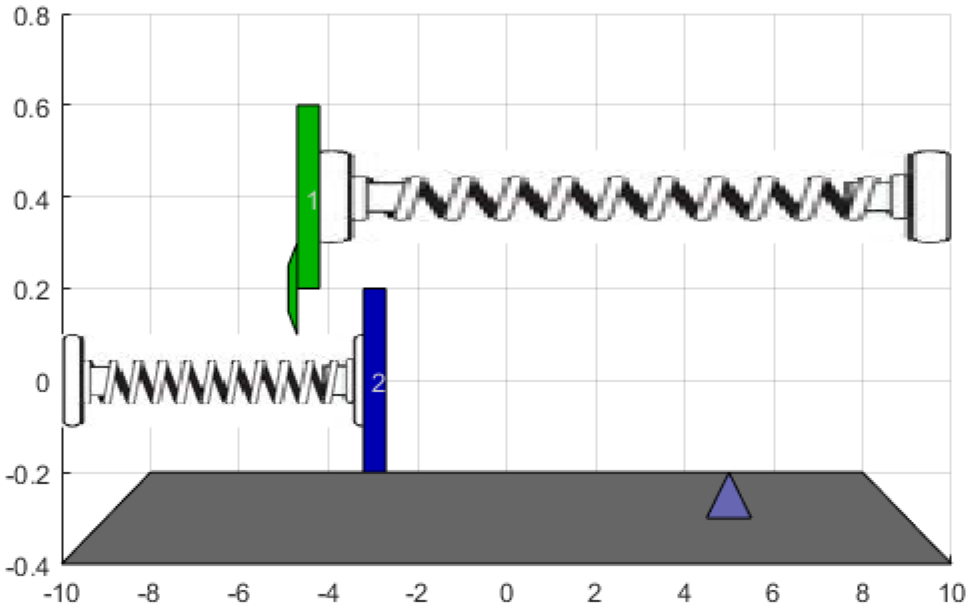
In this case, performance degrades whenever the two masses join.

Repeat Simulation Using MPC2 Only (Assumes Masses Always in Contact)

```
disp('Now repeat simulation by using only MPC2 ...');
disp('When two masses are detached, control performance deteriorates.');
MPC1=MPC2save;
MPC2=MPC1;
sim(mdl);
```

```
Now repeat simulation by using only MPC2 ...
When two masses are detached, control performance deteriorates.
```


signals
File Tools View Simulation Help
Yellow = M1; Cyan = M1 reference; Magenta = M2
Applied Force
Switching Signal
10
5
0
-5
-10
25
20
15
10
5
0
2.5
2
1.5
1
0.5
0 10 20 30 40 50 60 70 80 90 100



In this case, performance degrades when the masses separate, causing the controller to apply excessive force.

```
bdclose(mdl)
close(findobj('Tag','mpc_switching_demo'))
```

Related Examples

- "Schedule Controllers at Multiple Operating Points"
- "Coordinate Multiple Controllers at Different Operating Points" on page 4-64
- "Gain Scheduled MPC Control of Nonlinear Chemical Reactor" on page 7-5

More About

- "Design Workflow for Gain Scheduling" on page 7-3

Reference for MPC Designer App

This chapter is the reference manual for the Model Predictive Control Toolbox MPC Designer app.

Generate MATLAB Code from MPC Designer

This topic shows how to generate MATLAB code for creating and simulating model predictive controllers designed in the MPC Designer app. Generated MATLAB scripts are useful when you want to programmatically reproduce designs that you obtained interactively.

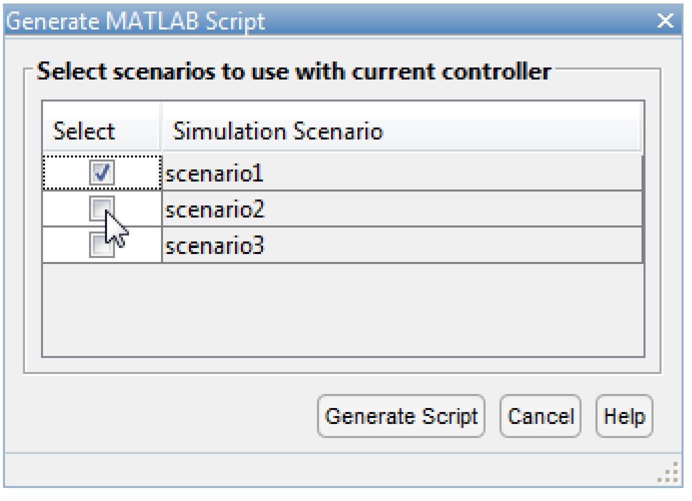
To create a MATLAB script:

1. In the MPC Designer app, interactively design and tune your model predictive controller.
2. On the **Tuning** tab, in the **Analysis** section, click the **Export Controller** arrow ▼.

   Alternatively, **Export Controller** is on the **MPC Designer** tab, in the **Result** section.

   Note: If you opened MPC Designer from Simulink, click the **Update and Simulate** arrow ▼.

   ---
3. Under **Export Controller** or **Update and Simulate**, click `Generate Script` .
4. In the Generate MATLAB Script dialog box, select one or more simulation scenarios to include in the generated script.



5 Click **Generate Script** to create the MATLAB script for creating the current MPC controller and running the selected simulation scenarios. The generated script opens in the MATLAB Editor.

In addition to generating a script, the app exports the following to the MATLAB workspace:

- A copy of the plant used to create the controller, that is the controller internal plant model
- Copies of the plants used in any simulation scenarios that do not use the default internal plant model
- The reference and disturbance signals specified for each simulation scenario

See Also

`mpc`

Related Examples

- "Generate Simulink Model from MPC Designer" on page 8-4

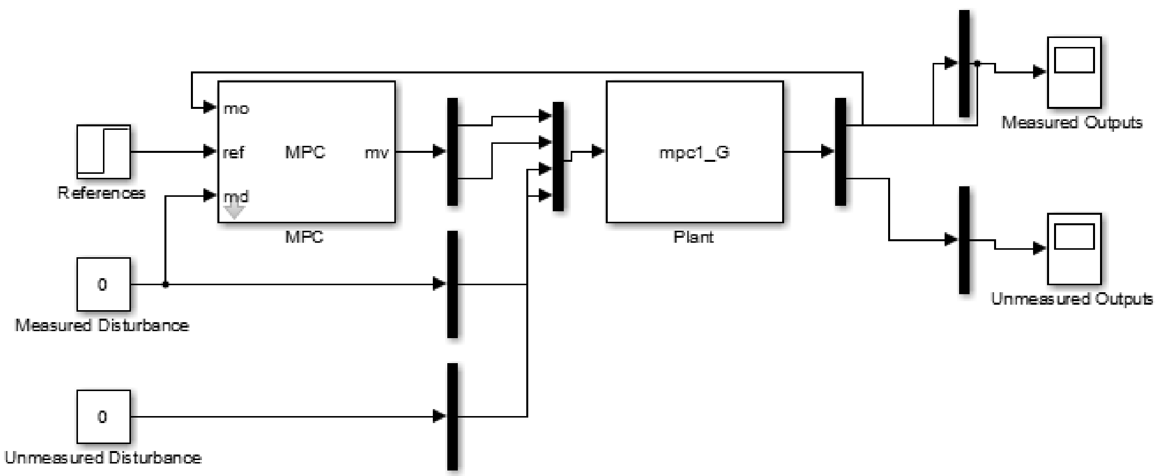
Generate Simulink Model from MPC Designer

This topic shows how to generate a Simulink model that uses the current model predictive controller to control its internal plant model.

To create a Simulink model:

1. In the MPC Designer app, interactively design and tune your model predictive controller.
2. On the **Tuning** tab, in the **Analysis** section, click the **Export Controller** arrow ▾.

   Alternatively, **Export Controller** is on the **MPC Designer** tab, in the **Result** section.
3. Under **Export Controller**, click `Generate Simulink Model`.



The app exports the current MPC controller and its internal plant model to the MATLAB workspace and creates a Simulink model that contains an MPC Controller block and a Plant block

Also, default step changes in the output setpoints are added to the References block.

Use the generated model to validate your controller design. The generated model serves as a template for moving easily from the MATLAB design environment to the Simulink environment.

You can also use the Simulink model to generate code and deploy it for real-time control applications. For more information, see “Generate Code and Deploy Controller to Real-Time Targets” on page 3-5.

See Also

`MPC Controller` | MPC Designer

Related Examples

- “Generate MATLAB Code from MPC Designer” on page 8-2

More About

- “Generate Code and Deploy Controller to Real-Time Targets” on page 3-5
- “Design MPC Controller in Simulink”

Compare Multiple Controller Responses Using MPC Designer

This example shows how to compare multiple controller responses using the MPC Designer app. In particular, controllers with different output constraint configuration are compared.

Define Plant Model

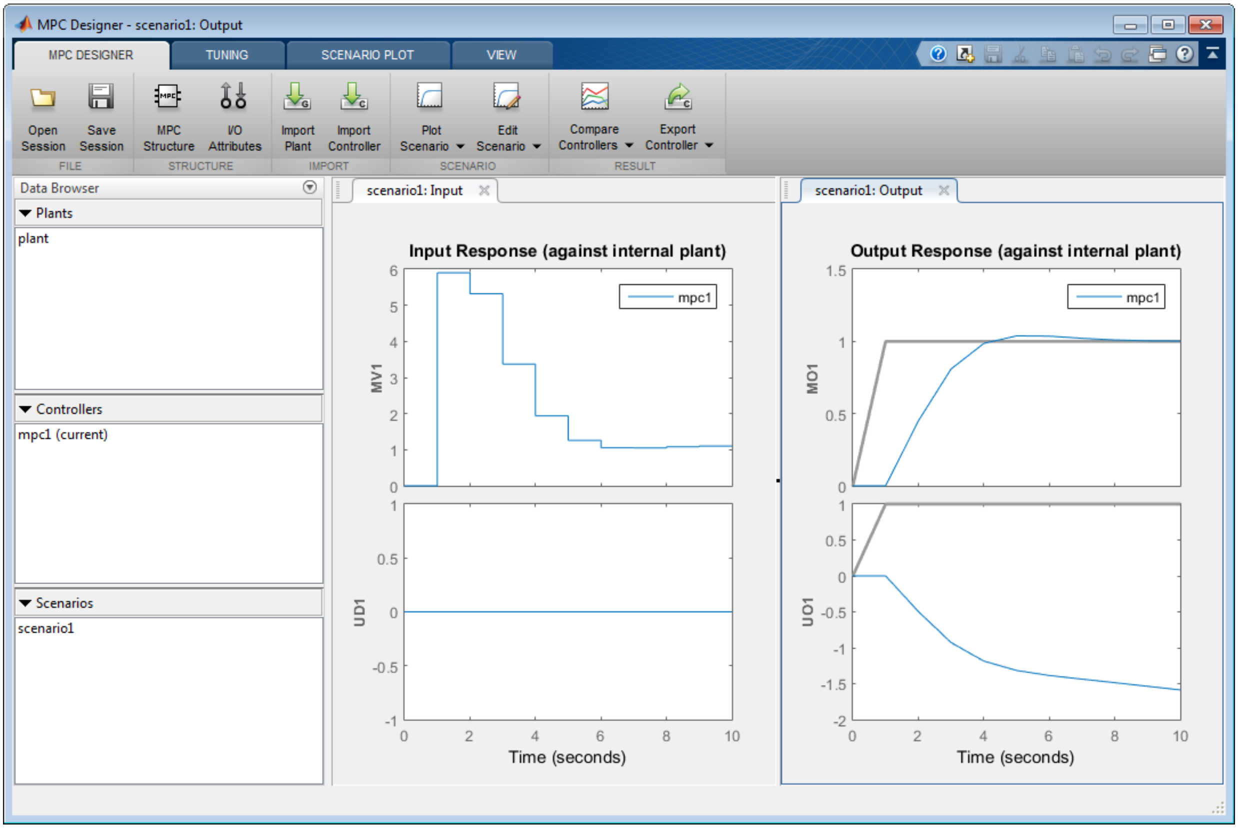
Create a state-space model of your plant, and specify the MPC signal types.

```
A = [-0.0285 -0.0014; -0.0371 -0.1476];
B = [-0.0850 0.0238; 0.0802 0.4462];
C = [0 1; 1 0];
D = zeros(2,2);

plant = ss(A,B,C,D);
plant = setmpcsignals(plant,'MV',1,'UD',2,'MO',1,'UO',2);
```

Open MPC Designer App

Open the MPC Designer app, and import the plant model.

```
mpcDesigner(plant)
```



The app adds the specified plant to the **Data Browser** along with a default controller, `mpc1`, and a default simulation scenario, `scenario1`.

Define Simulation Scenario

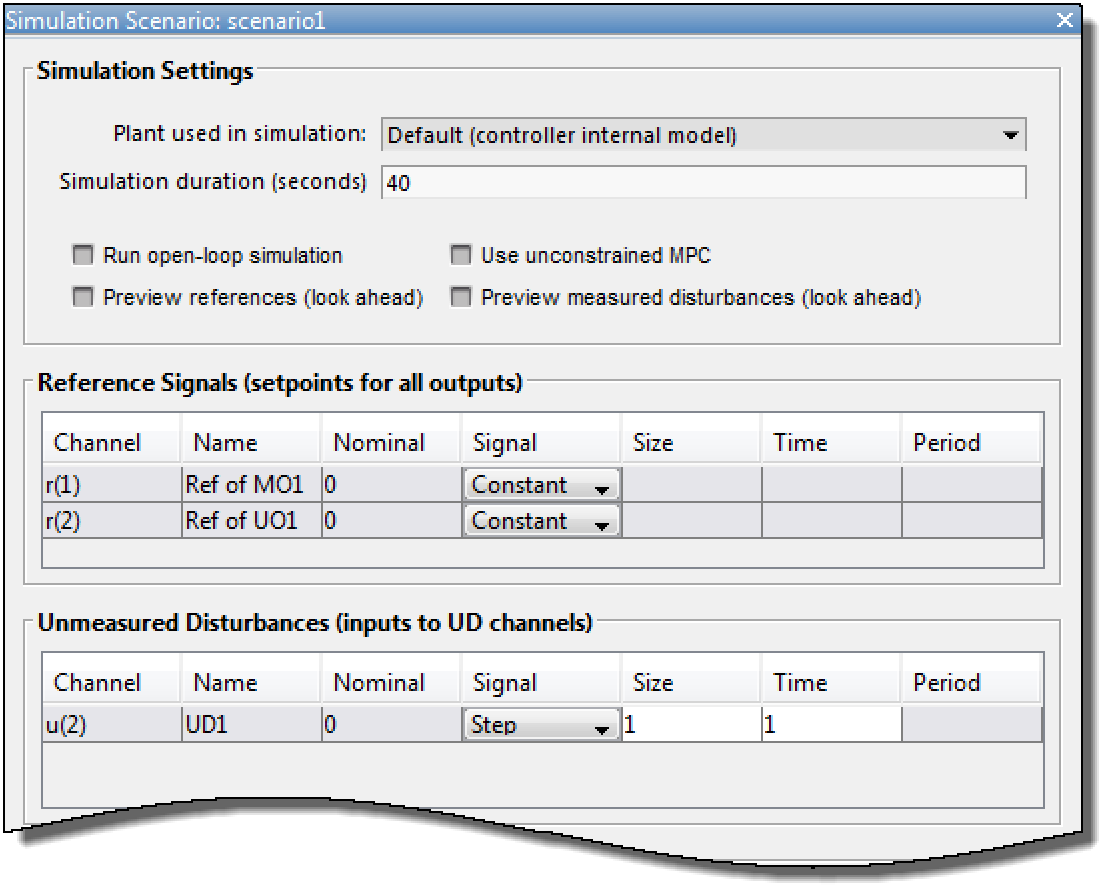
Configure a disturbance rejection simulation scenario.

In the MPC Designer app, on the MPC Designer tab, click **Edit Scenario** > **scenario1**.

In the Simulation Scenario dialog box, specify a **Simulation duration** of 40 seconds.

In the **Reference Signals** table, in the **Signal** drop-down lists, select `Constant` to hold the setpoints of both outputs at their nominal values.

In the **Unmeasured Disturbances** table, in the **Signal** drop-down list, select `Step`. Use the default **Time** and **Step** values.



This scenario simulates a unit step change in the unmeasured input disturbance at a time of 1 second.

Click **OK**.

The app runs the updated simulation scenario and updates the controller response plots. In the **Output Response** plots, the default controller returns the measured output, **MO1**, to its nominal value, however the control action causes an increase in the unmeasured output, **UO1**.

Create Controller with Hard Output Constraints

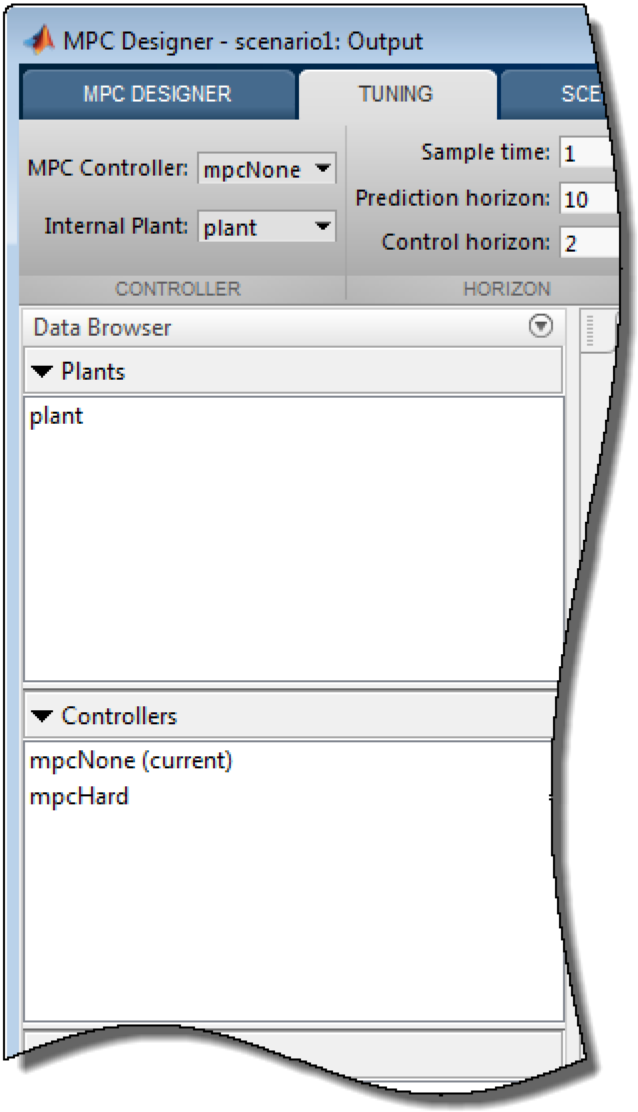
Suppose that the control specifications indicate that such an increase in the unmeasured disturbance is undesirable. To limit the effect of the unmeasured disturbance, create a controller with a hard output constraint.

Note: In practice, using hard output constraints is not recommended. Such constraints can create an infeasible optimization problem when the output variable moves outside of the constraint bounds due to a disturbance.

In the **Data Browser**, in the **Controllers** section, right-click `mpc1`, and select **Copy**.

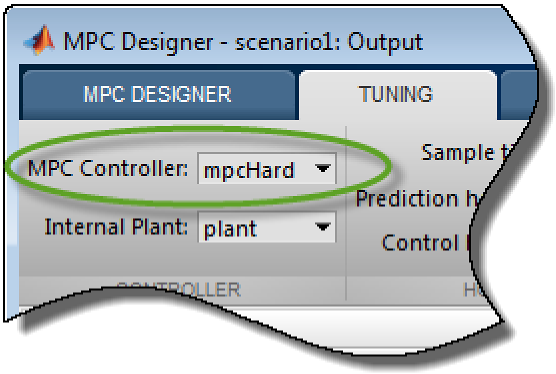
The app creates a copy of the default controller and adds it to the **Data Browser**.

Double-click each controller and rename them as follows.



Right-click the `mpcHard` controller, and select **Tune (make current)**. The app adds the `mpcHard` controller response to the **Input Response** and **Output Response** plots.

On the **Tuning** tab, in the **Controller** section, `mpcHard` is selected as the current **MPC Controller** being tuned.

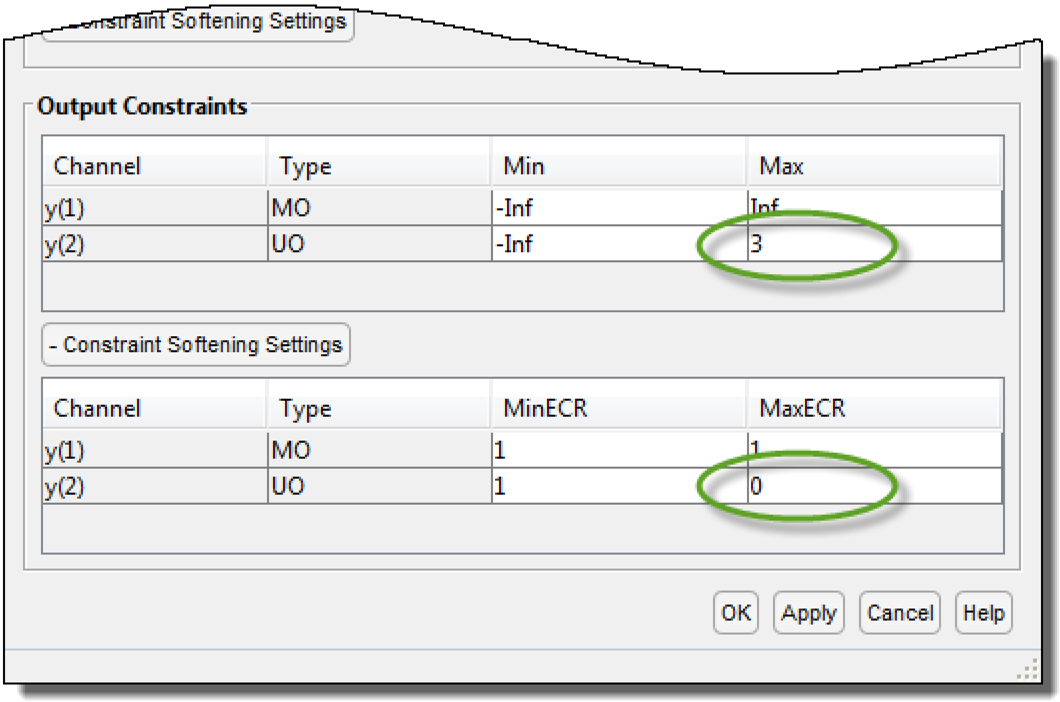


In the **Design** section, click **Constraints**.

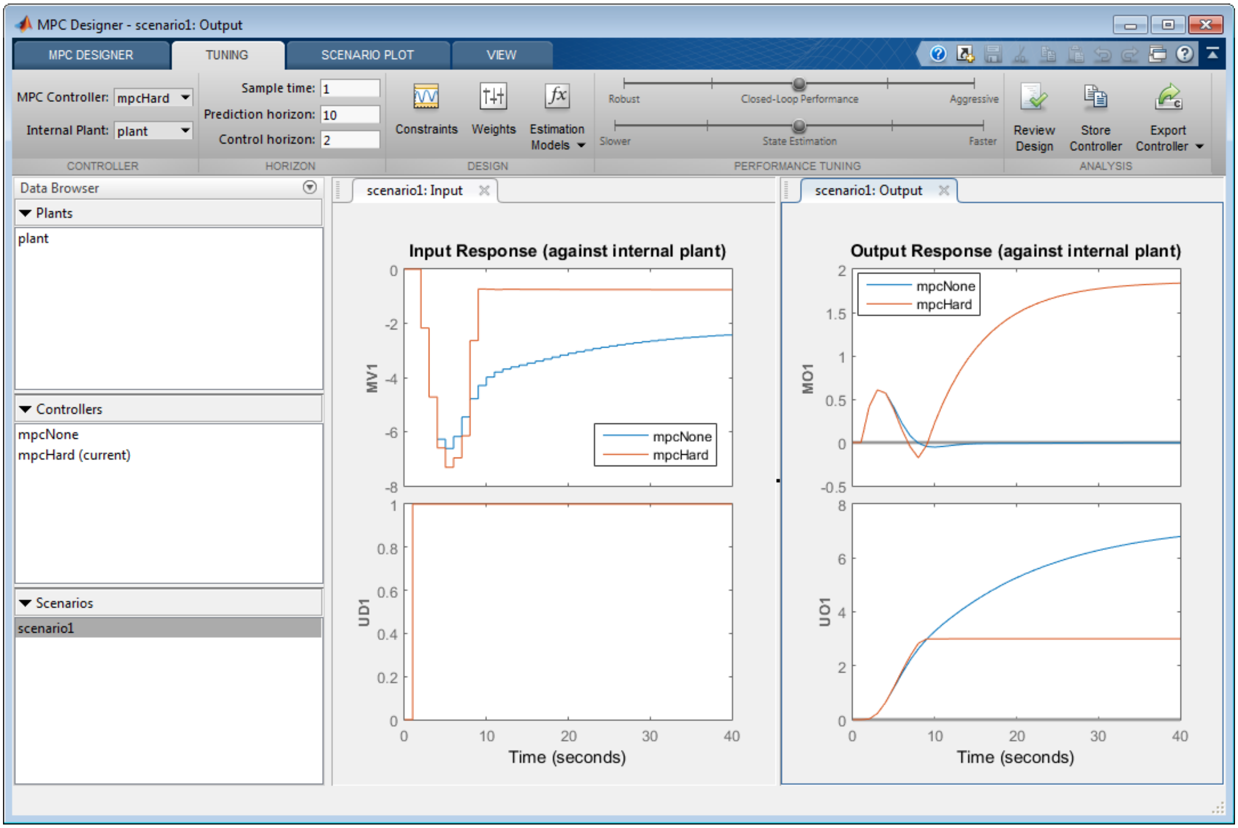
In the Constraints dialog box, in the **Output Constraints** section, in the **Max** column, specify a maximum output constraint of `3` for the unmeasured output (UO).

By default, all output constraints are soft, that is the controller can allow violations of the constraint when computing optimal control moves.

To make the unmeasured output constraint hard, click **Constraint Softening Settings**, and enter a **MaxECR** value of `0` for the UO. This setting places a strict limit on the controller output that cannot be violated.


Constraint Softening Settings
Output Constraints
Channel
Type
Min
Max
y(1)
MO
-Inf
Inf
y(2)
UO
-Inf
3
- Constraint Softening Settings
Channel
Type
MinECR
MaxECR
y(1)
MO
1
1
y(2)
UO
1
0
OK
Apply
Cancel
Help

Click **OK**.



The response plots update to reflect the new `mpcHard` configuration. In the **Output Response** plot, in the **UO1** plot, the `mpcHard` response is limited to a maximum of 3. As a trade-off, the controller cannot return the **MO1** response to its nominal value.

Tip If the plot legends are blocking the response signals, you can drag the legends to different locations.

Create Controller with Soft Output Constraints

Suppose the deviation of **MO1** from its nominal value is too large. You can soften the output constraint for a compromise between the two control objectives: **MO1** output tracking and **UO1** constraint satisfaction.

On the **Tuning** tab, in the **Analysis** section, click **Store Controller** to save a copy of `mpcHard` in the **Data Browser**.

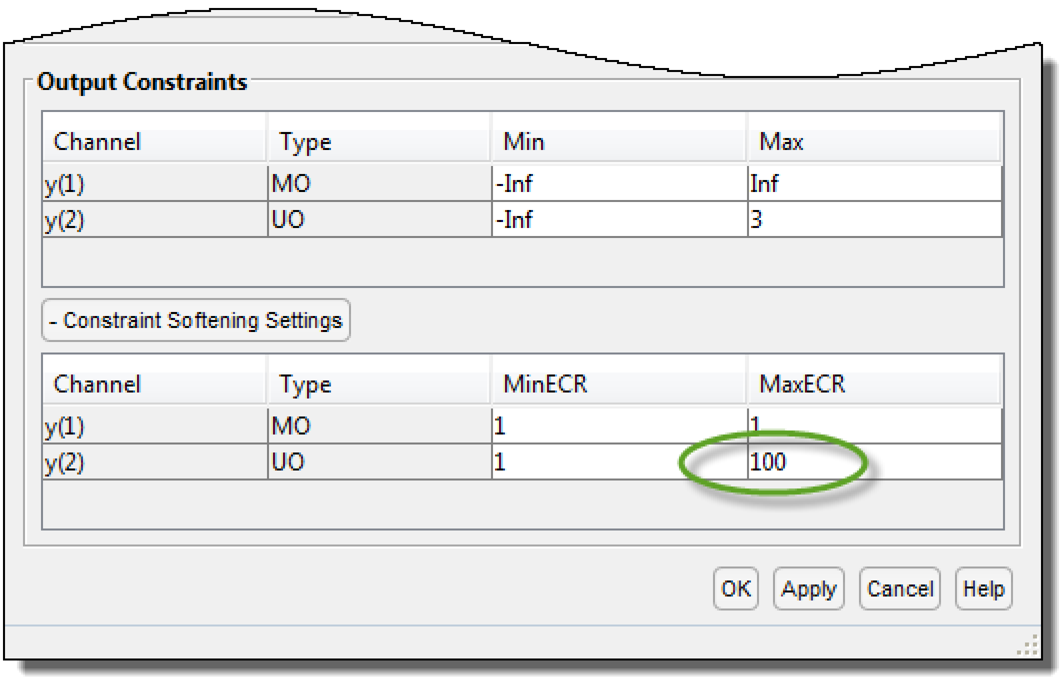
In the **Data Browser**, in the **Controllers** section, rename `mpcHard_Copy` to `mpcSoft`.

On the **Tuning** tab, in the **Controller** section, in the **MPC Controller** drop-down list, select `mpcSoft` as the current controller.

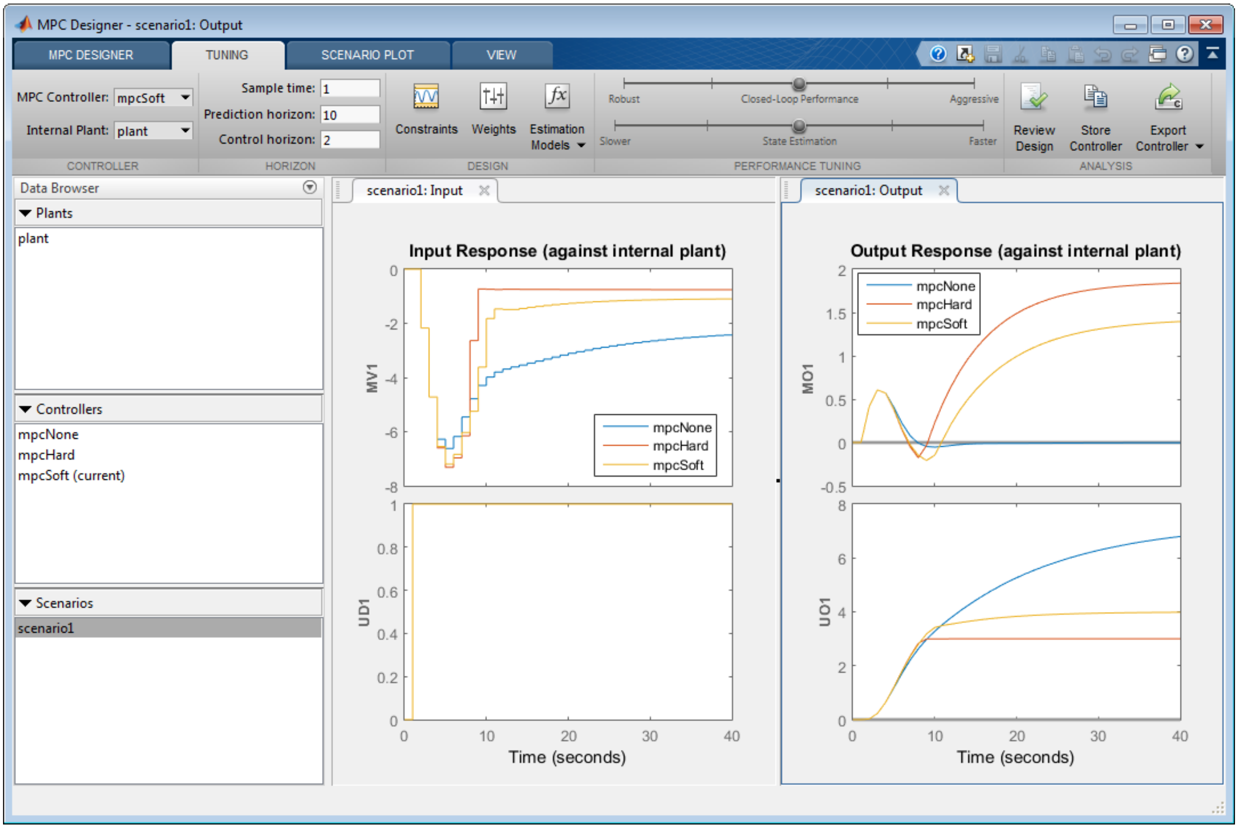
The app adds the `mpcSoft` controller response to the **Input Response** and **Output Response** plots.

In the **Design** section, click **Constraints**.

In the Constraints dialog box, in the **Output Constraints** section, enter a **MaxECR** value of `100` for the UO to soften the constraint.



Click **OK**.

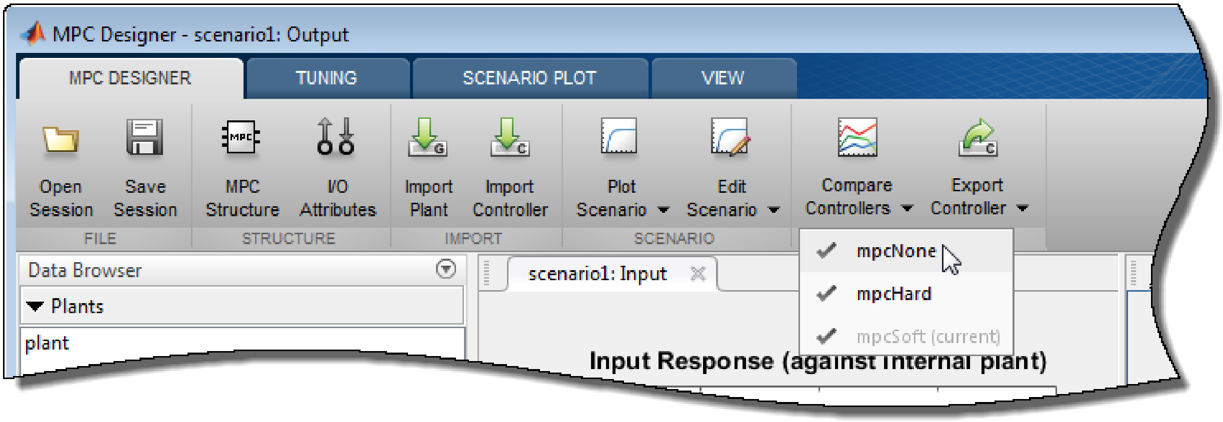


The response plots update to reflect the new `mpcSoft` configuration. In the **Output Response** plot, `mpcSoft` shows a compromise between the previous controller responses.

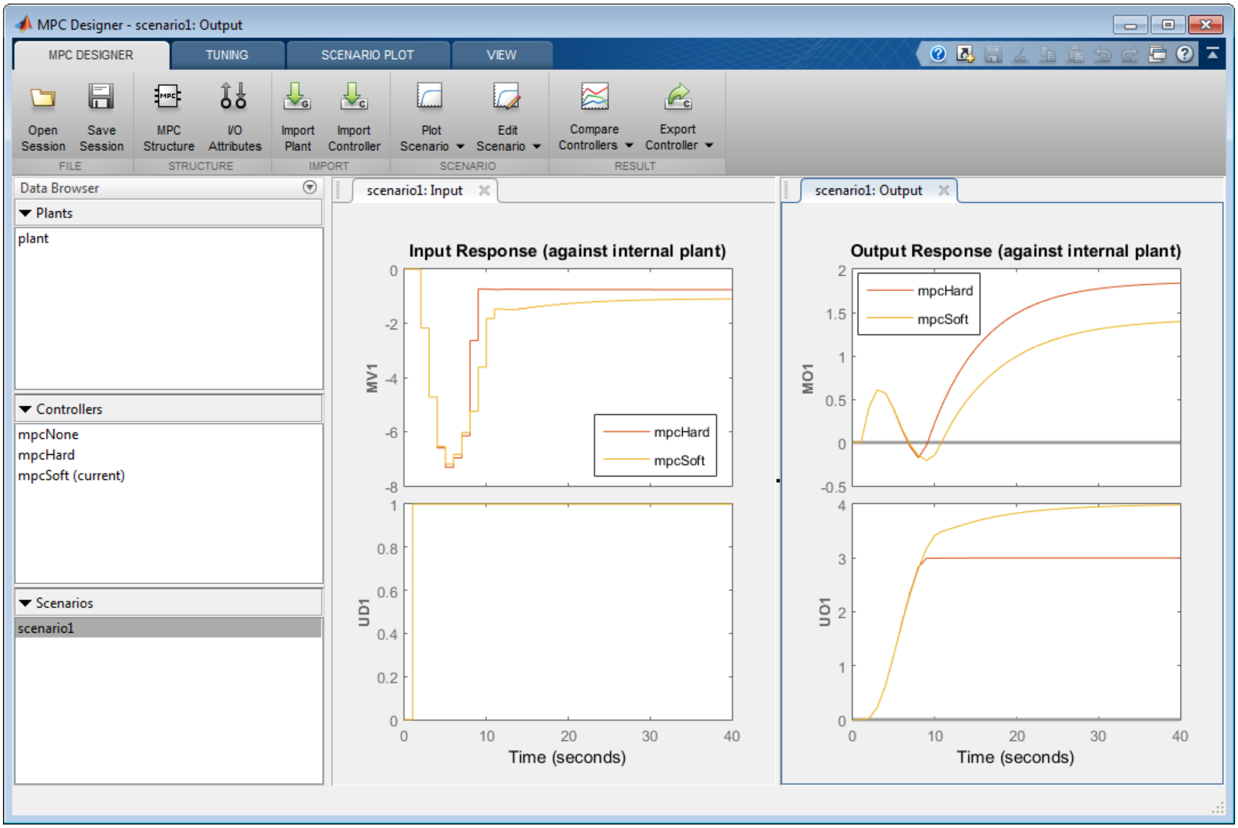
Remove Default Controller Response Plot

To compare the two constrained controllers only, you can remove the default unconstrained controller from the input and output response plots.

On the **MPC Designer** tab, in the **Result** section, click **Compare Controllers** > **mpcNone**.



The app removes the `mpcNone` responses from the **Input Response** and **Output Response** plots.



You can toggle the display of any controller in the **Data Browser** except for controller currently being tuned. Under **Compare Controllers**, the controllers with displayed responses are indicated with check marks.

See Also

MPC Designer

Related Examples

- "Design Controller Using MPC Designer"
- "Design MPC Controller in Simulink"

More About

- "Specifying Constraints" on page 1-10